LECTURE NOTES

ON

ELECTRICAL CIRCUIT ANALYSIS-I

ACADEMIC YEAR 2021-22

I B.Tech.–II SEMESTER(R20)

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DEPARTMENT OF HUMANITIES AND BASIC SCIENCES

V S M COLLEGE OF ENGINEERING
RAMCHANDRAPURAM
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533255

VSM COLLEGE OF ENGINEERING
RAMACHANDRAPRUM-533255
DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

Course Title	Year-Sem	Branch	Contact Periods/Week	Sections
Electrical circuit analysis-I	I-II	Electrical & electronics Engineering	6	-

COURSE OUTCOMES: Students are able to

1. Various electrical networks in presence of active and passive elements.Electrical networks with network topology concepts(K1)
2. Any magnetic circuit with various dot conventions (K5)
3. Understand Any R, L, C network with sinusoidal excitation (K2)
4. Analyze Any R, L, network with variation of any one of the parameters i.e., R, L, C and f. (K4)
5. Determine Electrical networks by using principles of network theorems.(K3)

Unit/ item No.	Outcomes		Topic	Number of periods	Total periods	Book Reference	Delivery Method
1	**CO1:** Various electrical networks in presence of active and passive elements.Electrical networks with network topology concepts		**Introduction to Electrical Circuits**		15	T1,T3, R7	Chalk & Talk, PPT, Active Learning & Tutorial
		1.1	Basic Concepts of passive elements of R, L, C and their V-I relations	2			
		1.2	Sources (dependent and independent), Kirchhoff's laws	2			
		1.3	Network reduction techniques (series, parallel, series - parallel, star-to-delta and delta-to-star transformation).	2			
		1.4	source transformation technique	2			
		1.5	nodal analysis and mesh analysis to DC networks with dependent and independent voltage and current sources	2			
		1.6	Star-Delta transformation technique	2			
		1.7	delta-to-star transformation	2			
		1.8	Problems on above topics	1			
2	**CO2:** Any magnetic circuit with various dot conventions		**Magnetic Circuits**		12	T1,T3, R7	Chalk & Talk, Active Learning & Tutorial
		2.1	Basic definition of MMF, flux and reluctance,	2			
		2.2	analogy between electrical and magnetic circuits	2			
		2.3	Faraday's laws of electromagnetic induction	2			
		2.4	concept of self and mutual inductance, Dot convention	2			
		2.5	coefficient of coupling and composite	2			

			magnetic circuit				
		2.6	analysis of series and parallel magnetic circuits.	2			
3	**CO3:** Understand Any R, L, C network with sinusoidal excitation	**Single Phase A.C Systems**			12	T1,T3, R7	Chalk & Talk, Active Learning & Tutorial
		3.1	Periodic waveforms (determination of rms, average value and form factor), concept of phasor, phase angle and phase difference	2			
		3.2	waveforms and phasor diagrams for lagging, leading networks	2			
		3.3	complex and polar forms of representations. node and mesh analysis.	2			
		3.4	Steady state analysis of R, L and C circuits, power factor and its significance	2			
		3.5	real, reactive and apparent power, waveform of instantaneous power and complex power.	2			
		3.6	Problems on above topics	2			
4	**CO4:** Analyze Any R, L, network with variation of any one of the parameters i.e., R, L, C and f	**Resonance - Locus Diagrams**			8	T1,T3, R7	Chalk & Talk, Active Learning & Tutorial
		4.1	series and parallel resonance	2			
		4.2	selectively band width and Quality factor	**2**			
		4.3	locus diagram- RL, RC, RLC with R, L and C variables.	**2**			
		4.4	Problems on above topics	**2**			
5	**CO5:** Determine Electrical networks by using principles of network theorems	5.1	**Network theorems (DC & AC Excitations)** Superposition theorem	2	12	T3, R7	Chalk & Talk, Active Learning & Tutorial
		5.2	Thevenin's theorem, Norton's theorem	2			
		5.3	Maximum Power Transfer theorem	2			
		5.4	Reciprocity theorem, Millman's theorem and compensation theorem.	2			
		5.5	Design of Filters	2			
		5.6	Problems on above topics	2			
				TOTAL	59		

LIST OF TEXT BOOKS AND AUTHORS

Text Books:
T1. Engineering Circuit Analysis by William Hayt and Jack E.Kemmerley,McGraw Hill Company,6 th edition
T2. Network synthesis: Van Valkenburg; Prentice-Hall of India Private Ltd
T3. Network analysis and synthesis BY sudhakar and syam mohan

Reference Books:
R1. Fundamentals of Electrical Circuits by Charles K.Alexander and Mathew N.O.Sadiku, McGraw Hill Education (India)
R2. Introduction to circuit analysis and design by TildonGlisson. Jr, Springer Publications.
R3. Circuits by A.Bruce Carlson , Cengage Learning Publications
R4. Network Theory Analysis and Synthesis by SmarajitGhosh, PHI publications
R5. Networks and Systems by D. Roy Choudhury, New Age International publishers
R6. Electric Circuits by David A. Bell, Oxford publications
R7. Circuit Theory (Analysis and Synthesis) by A.Chakrabarthi,DhanpatRai&Co.

Faculty Member   Head of the Department   PRINCIPAL

VSM COLLEGE OF ENGINEERING

RAMACHANDRAPURAM

I Year-II Semester **ELECTRICAL CIRCUIT ANALYSIS-I**

UNIT-I

Introduction to Electrical Circuits

Basic Concepts of passive elements of R, L, C and their V-I relations, Sources (dependent and independent), Kirchhoff's laws, Network reduction techniques (series, parallel, series - parallel, star-to-delta and delta-to-star transformation), source transformation technique, nodal analysis and mesh analysis to DC networks with dependent and independent voltage and current sources., node and mesh analysis.

UNIT-II

Magnetic Circuits

Basic definition of MMF, flux and reluctance, analogy between electrical and magnetic circuits, Faraday's laws of electromagnetic induction – concept of self and mutual inductance, Dot convention – coefficient of coupling and composite magnetic circuit, analysis of series and parallel magnetic circuits.

UNIT-III

Single Phase A.C Systems

Periodic waveforms (determination of rms, average value and form factor), concept of phasor, phase angle and phase difference – waveforms and phasor diagrams for lagging, leading networks, complex and polar forms of representations. node and mesh analysis.

Steady state analysis of R, L and C circuits, power factor and its significance, real, reactive and apparent power, waveform of instantaneous power and complex power.

UNIT-IV

Resonance - Locus Diagrams

series and parallel resonance, selectively band width and Quality factor, locus diagram- RL, RC, RLC with R, L and C variables.

UNIT-V

Network theorems (DC & AC Excitations)

Superposition theorem, Thevenin's theorem, Norton's theorem, Maximum Power Transfer theorem, Reciprocity theorem, Millman's theorem and compensation theorem.

Text Books:

1. Engineering Circuit Analysis by William Hayt and Jack E. Kemmerley, 6th edition

McGraw Hill Company, 2012.

2. Network Analysis: Van Valkenburg; Prentice-3rd edition, Hall of India Private Ltd, 2015.

Introduction to Electrical Circuits

- **Introduction to Electrical Circuits** : Network elements classification, Electric charge and current, Electric energy and potential, Resistance parameter – series and parallel combination, Inductance parameter – series and parallel combination, Capacitance parameter – series and parallel combination. Energy sources: Ideal, Non-ideal, Independent and dependent sources, Source transformation, **Kirchoff's laws**, **Mesh analysis** and **Nodal analysis** problem solving with resistances only including dependent sources also. (Text Books: 1,2,3, Reference Books: 3).

TEXT BOOKS

- **Network Analysis – ME Van Valkenburg, Prentice Hall of India, 3rd Edition, 2000.**

- **Electric Circuit Analysis by Hayt and Kimmarle, TMH**

Introduction

- Today's engineering graduates are no longer employed solely to work on the technical design aspects of engineering problems.

- Their efforts now extend beyond the creation of better computers and communication systems etc.....

- To contribute to the solution of engineering problems an engineer must acquire many skills, one of which is a knowledge of electric and electronic circuits analysis.

- They take a fundamental understanding of various scientific principles, combine this with practical knowledge often expressed in mathematical terms and with little creativity arrive at a solution.

➢Electric Circuit

- **Electric circuit can be defined as an interconnection between components or electrical devices for the purpose of communicating or transferring energy from one point to another.**

- **The components of electric circuit are always referred to as circuit elements**

Circuit element definition

- It is important to differentiate between the physical device itself and the mathematical model which we will use to analyze its behavior in a circuit. The model is only an approximation.

- Expression use the circuit element to refer to the mathematical model.

- All simple circuit elements that we will consider can be classified according to the relationship of the current through the element to the voltage across the element.

- Dependant sources are used a great deal in electronics to model both DC and AC behavior of transistors, especially in amplifier circuits.

➢The derived unit commonly used in electric circuit theory

Quantity	Unit	Symbol
electric charge	coulomb	C
electric potential	volt	V
resistance	ohm	Ω
conductance	siemens	S
inductance	henry	H
capacitance	farad	F
frequency	hertz	Hz
force	newton	N
energy, work	joule	J
power	watt	W
magnetic flux	weber	Wb
magnetic fiux density	tesla	T

Electric Charge

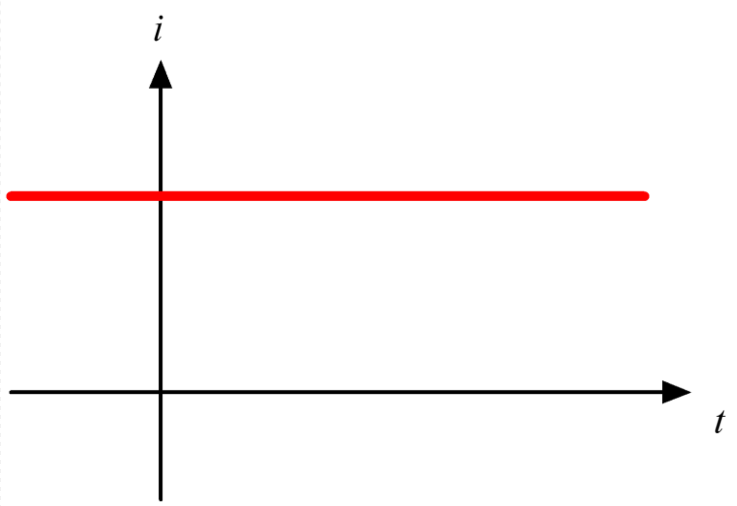
- Electric charge is an electrical property of the atomic particles of which matter consists measured in coulombs (C).
- Electric charge create electric field of force.
- The charge **Q** on one electron is negative and equal in magnitude to $1.602\times10^{-19}C$ which is called as electronic charge.

CURRENT

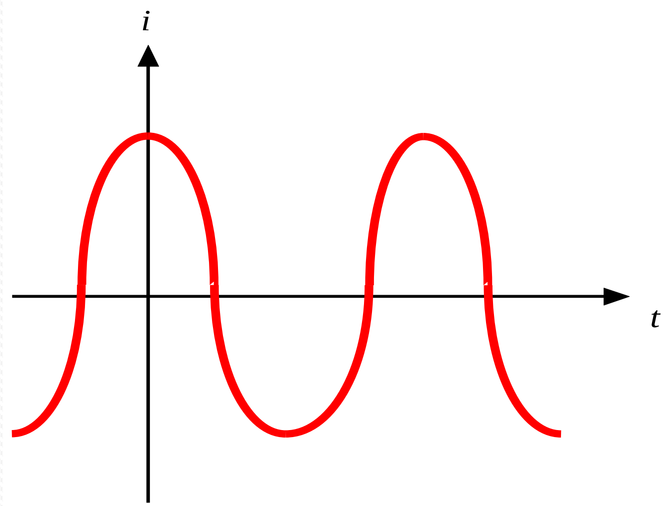
- Current is defined as the movement of charge in a specified direction.

- Electric current i = dq/dt.
- An Ampere = Coulomb per Second

➢Types Of Current



Direct current



Alternating current

- **A direct current (dc) is a current that remains constant with time.**
- **An alternating current (ac) is a current that varies sinusoidally with time. (reverse direction).**

Example

- A conductor has a constant current of 5 A.
- How many electrons pass a fixed point on the conductor in one minute?

Solution

- Total no. of charges, pass in 1 min is given by

5 A = (5 C/s)(60 s/min) = 300 C/min

$$\frac{300\ \text{C/min}}{1.602x10^{-19}\ \text{C/electron}} = 1.87x10^{21}\ \text{electrons}/\text{min}$$

➢ Voltage

- ➢ Voltage is the electric pressure or force that causes current.
- ➢ It is a potential energy difference between two points.
- ➢ It is also known as an electromotive force (EMF).
- ➢ A Volt = Joule per Second

➢ Resistance

- ❖ Resistance is the opposition a material offers to current.

➢ Resistance is determined by

- ❖ Type of material (resistivity)
- ❖ Temperature of material
- ❖ Cross-sectional area
- ❖ Length of material

➢ Resistance Relationships

❖ Resistance $= \dfrac{\text{Resistivity x length}}{\text{area}}$ $\qquad R = \dfrac{KL}{A}$

➢ Example:

❖ $R = \dfrac{KL}{A} = \dfrac{1.4 \text{ x}10^{-6} \text{ W} \cdot \text{ cm} \;\; \text{x} \;\; 5 \text{ x}10^{4} \text{ cm}}{2 \text{ cm}^2} = 5 \text{ W}$

➢ **ohm = Volt per Ampere**

➢ POWER

➢ **Power is the rate of using energy or doing work.**

❖ **Work (*W*): consists of a force moving through a distance.**

❖ **Energy (*W*): is the capacity to do work.**

❖ **Joule (J) : is the base unit for both energy and work.**

❖ **A watt = Joule per second.**

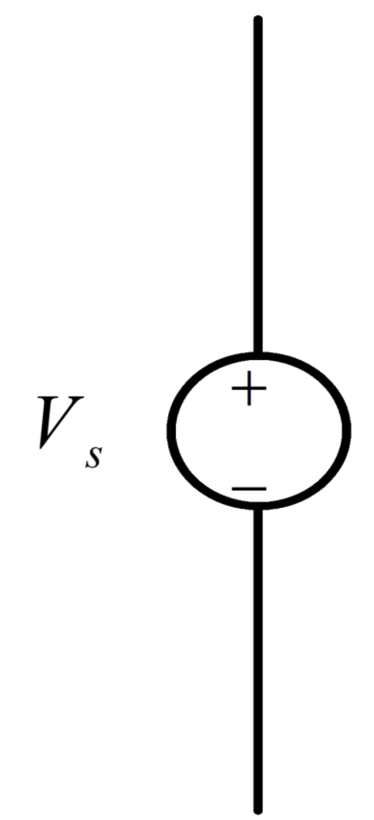
❖ **Power = 200 Watts**

Active Element And Passive Element

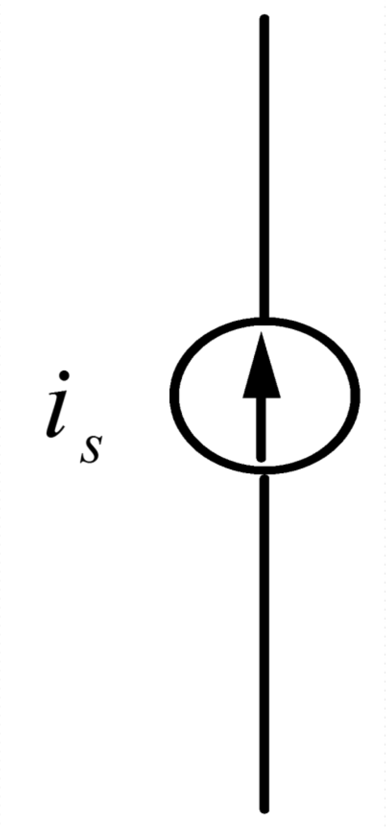
- **Active Element–** elements capable of generating electrical energy.
  - Current source
  - Voltage source
- **Passive Element–** elements not capable of generating electrical energy.
  - Resistor (dissipates energy)
  - Capacitor and Inductor (can store or release energy)

Independent Source

- **Voltage Source** maintains a specified voltage between its terminals but has no control on the current passing through it. The symbol of the independent voltage source is a plus-minus sign enclosed by a circle.

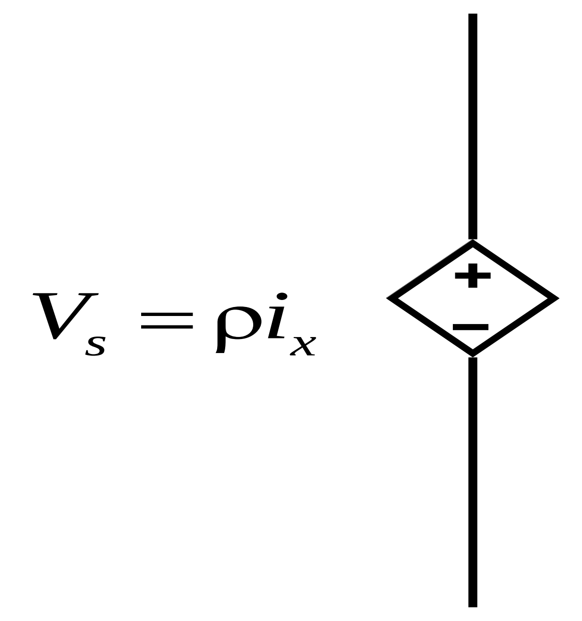


- **Current Source** maintains a specified current through its terminals but has no control on the voltage across its terminals. The symbol of the independent current source is an arrow enclosed by a circle.

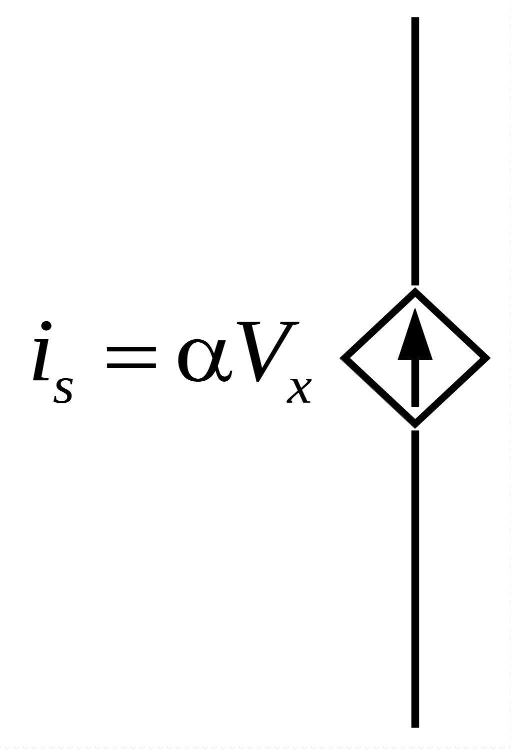


Dependent Source

- The voltage source has a specified voltage between its terminals but it is dependable on some other variable defined somewhere in the circuit.
- The symbol for the dependent voltage source is a plus-minus sign enclosed by a diamond shape.



- **This kind of current source has a specified current between its terminals but it is dependent on some other variable defined somewhere in the circuit.**
- **The symbol for the dependent current source is an arrow enclosed by a diamond shape.**



➢ Controlled sources

Current controlled current source : **Current ratio** $\alpha = \frac{i_2}{i_1}$

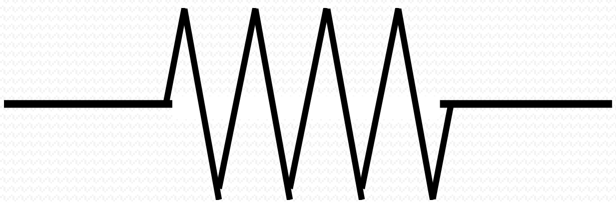
Voltage controlled current source : **Transconductance** $g_m = \frac{i_2}{v_1}$

Voltage controlled voltage source : **Voltage ratio** $\mu = \frac{v_2}{v_1}$

Current controlled voltage source : **Transresistance** $r_m = \frac{v_2}{i_1}$

➢ Circuit symbol of Resistor

Resistor



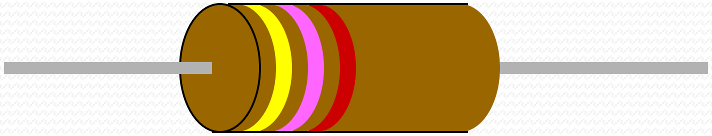
R

UNIT: Ohm (Ω)

Resistor is passive element that dissipates electrical energy.

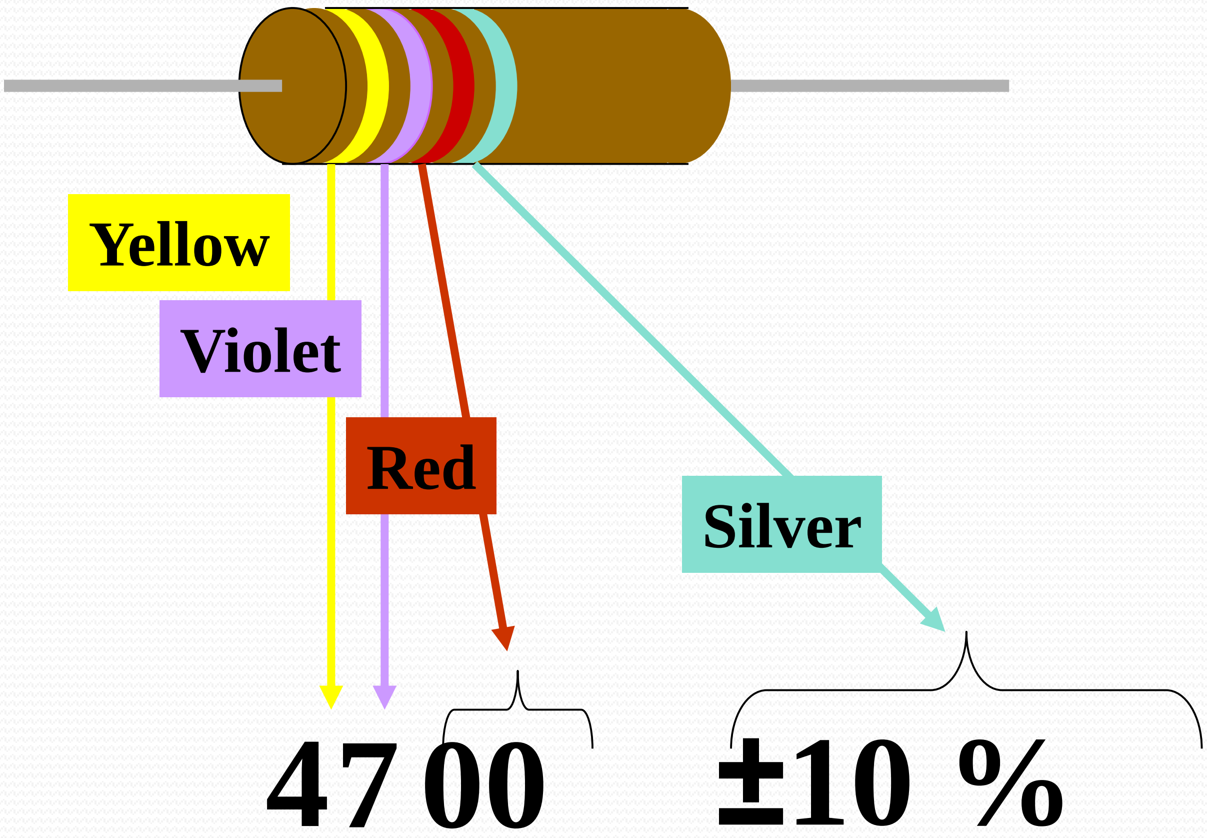
Linear resistor is the resistor that obeys Ohm's law.

➢ Resistor colour code

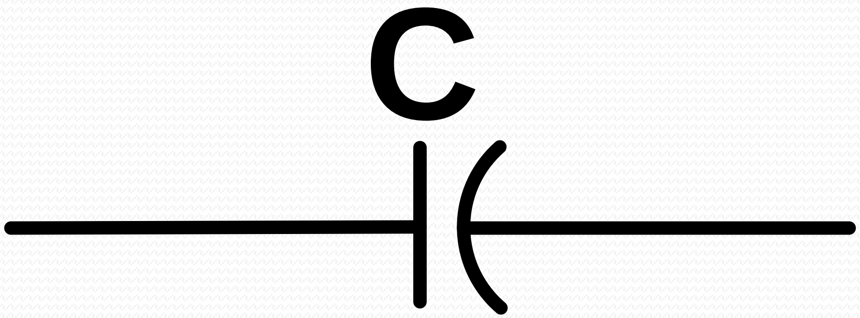


Color	Color Name	1st Digit 1st Stripe	2nd Digit 2nd Stripe	Multiplier 3rd Stripe	Tolerance 4th Stripe
	Black	0	0	x1	-
	Brown	1	1	x10	-
	Red	2	2	x100	-
	Orange	3	3	x1,000	-
	Yellow	4	4	x10,000	-
	Green	5	5	x100,000	-
	Blue	6	6	x1,000,000	-
	Violet	7	7	-	-
	Gray	8	8	-	-
	White	9	9	-	-
	Gold	-	-	-	5%
	Silver	-	-	-	10%

➢ Resistor Colour Codes



➢ Capacitor



UNIT: Farad (F)

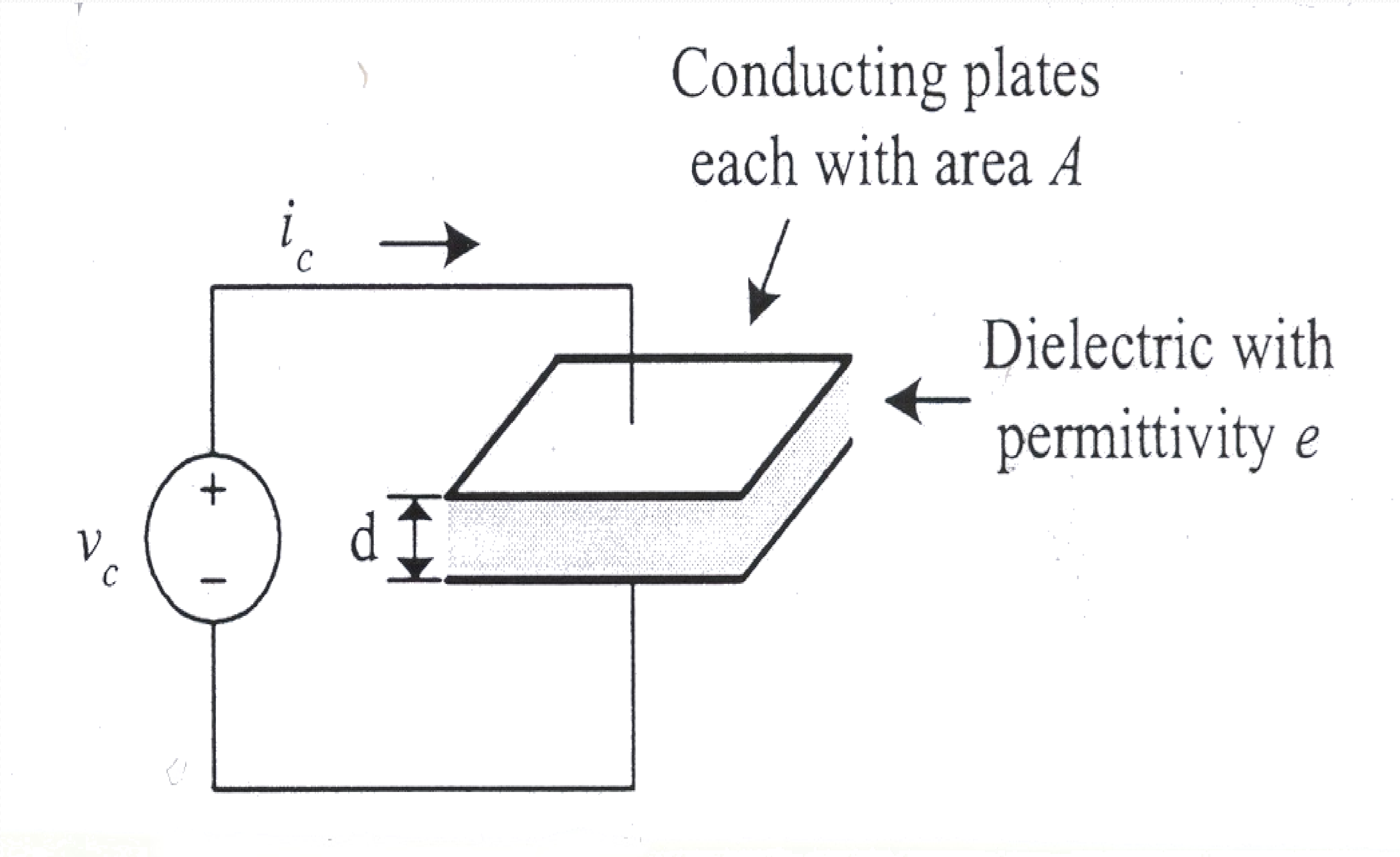
❖ Electrical component that consists of two conductors separated by an insulator or dielectric material.

❖ Its behavior based on phenomenon associated with electric fields, which the source is voltage.

❖ A time-varying electric fields produce a current flow in the space occupied by the fields.

❖ Capacitance is the circuit parameter which relates the displacement current to the voltage.

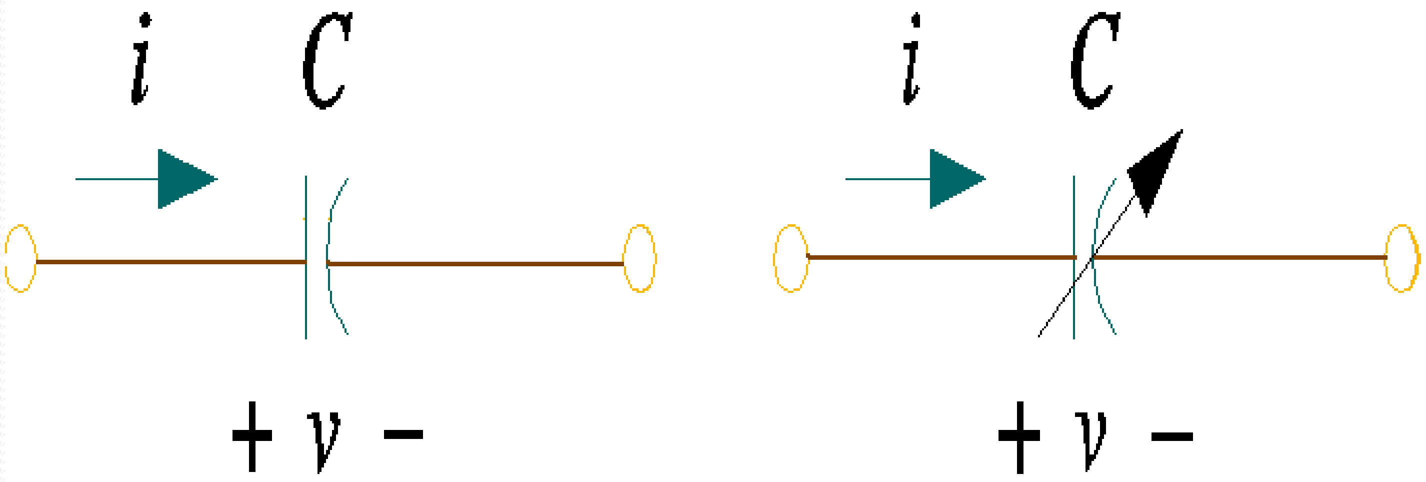
➢ A capacitor with an applied voltage



Plates – aluminum foil

Dielectric – air/ceramic/paper/mica

➢ Circuit symbols for capacitors



(a) Fixed capacitor (b) Variable capacitor

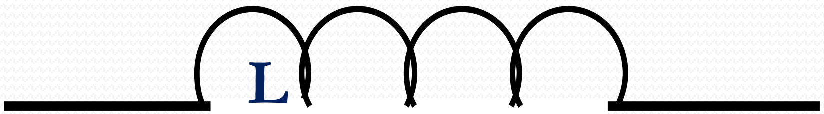
➢ Circuit parameters

- The amount of charge stored, $q = CV$.
- C is capacitance in Farad, ratio of the charge on one plate to the voltage difference between the plates.
- But it does not depend on q or V but capacitor's physical dimensions i.e.,

$$C = \frac{\varepsilon A}{d}$$

ε= permeability of dielectric in Wb/Am
A = surface area of plates in m^2
d = distance between the plates m

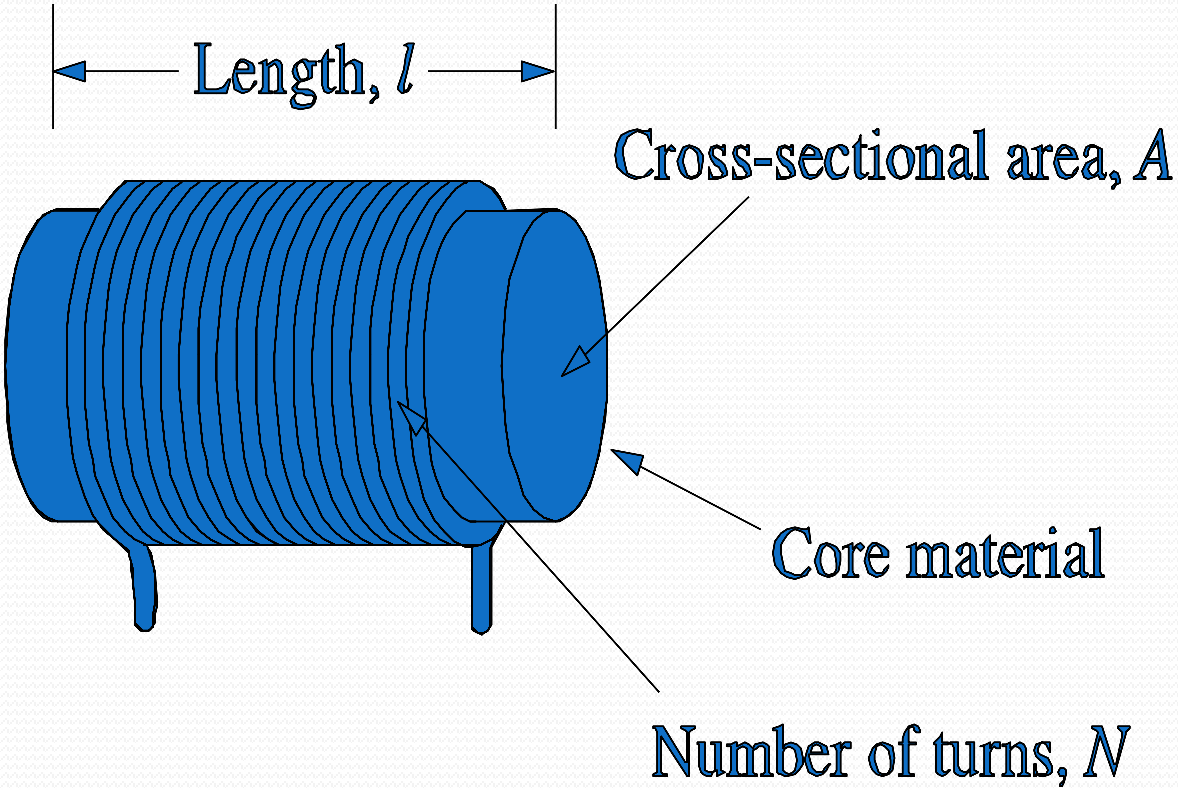
➢ Inductor



UNIT: Henry (H)

- ❖ Electrical component that opposes any change in electrical current.
- ❖ Composed of a coil or wire wound around a non-magnetic core/magnetic core.
- ❖ Its behavior based on phenomenon associated with magnetic fields, which the source is current.
- ❖ A time-varying magnetic fields induce voltage in any conductor linked by the fields.
- ❖ Inductance is the circuit parameter which relates the induced voltage to the current.

Typical form of an inductor

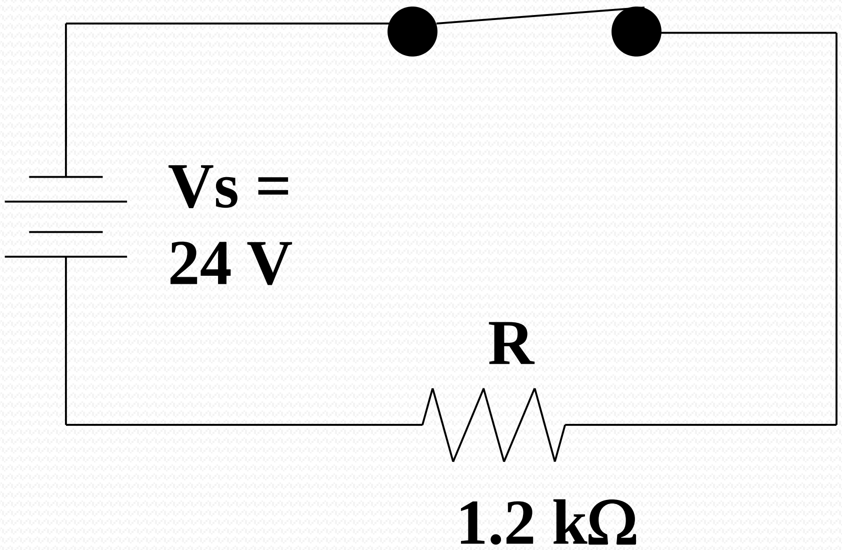


OHM'S LAW

- **Georg Simon Ohm (1787-1854) formulated the relationships among voltage, current, and resistance as follows:**
- **The current in a circuit is directly proportional to the applied voltage and inversely proportional to the resistance of the circuit.**

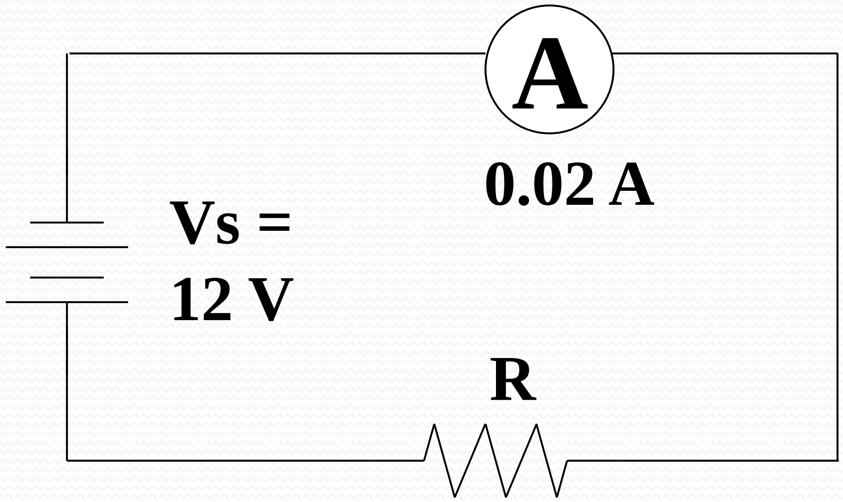
$$V = IR$$

➢ Calculating Current



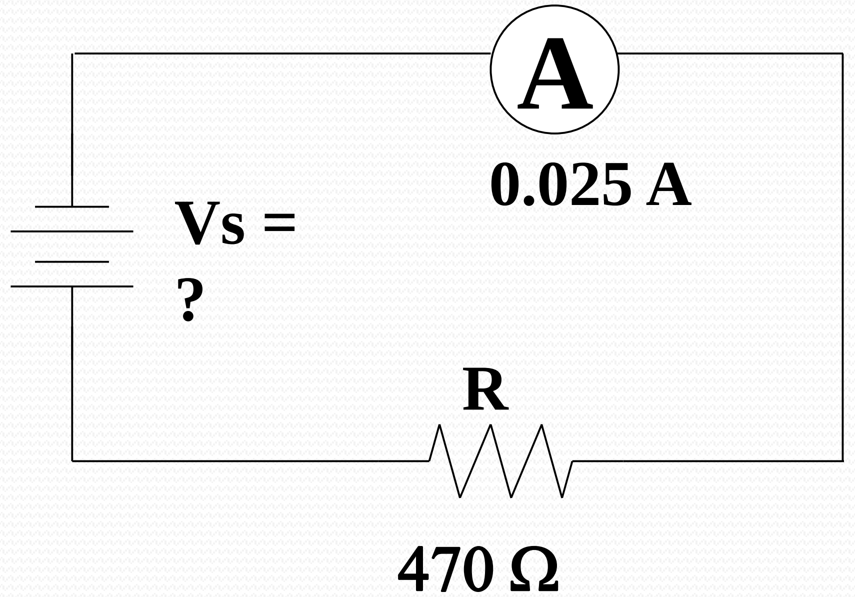
$$I = \frac{V}{R} = \frac{24\ \mathrm{V}}{1200\ \mathrm{K\Omega}} = 0.02\ \mathrm{A} = 20\ \mathrm{mA}$$

➢ Calculating Resistance



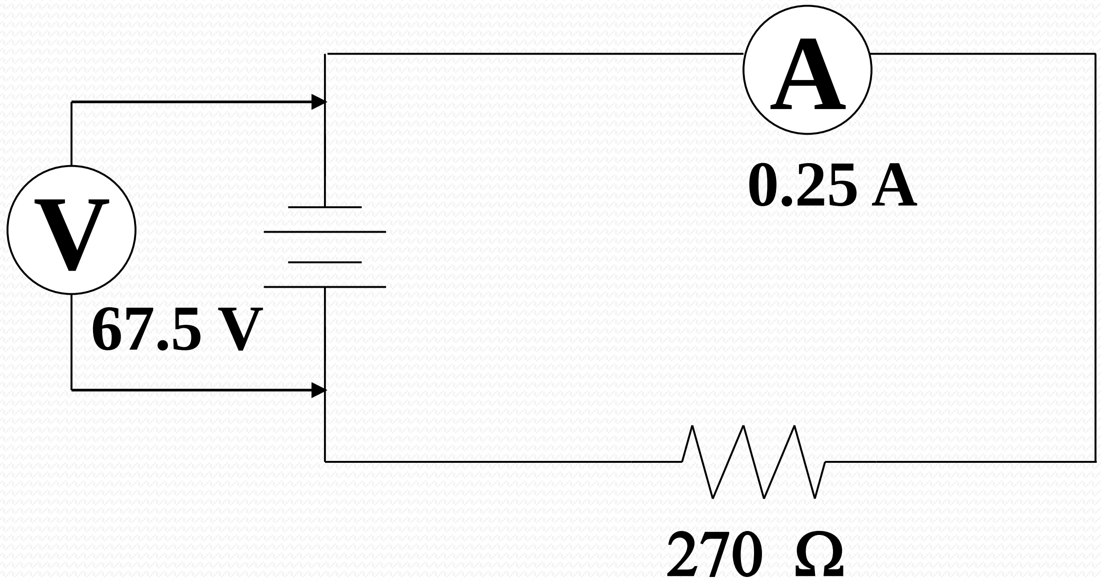
$$R = \frac{V}{I} = \frac{12\ \mathrm{V}}{0.02\ \mathrm{A}} = 600\ \mathrm{OHM}$$

➢ Calculating Voltage



$$V = IR = 0.025\text{ A} \times 470\text{ W} = 11.75\text{ V}$$

➢ Calculating Power



$$P = IV = 0.25\text{ A} \times 67.5\text{ V} = 16.9\text{ W}$$

$$P = I^2R = 0.25\text{ A} \times 0.25\text{ A} \times 270\text{ W} = 16.9\text{ W}$$

$$P = V^2/R = (67.5\text{ V} \times 67.5\text{ V}) / 270\text{ W} = 16.9\text{ W}$$

➢ KIRCHHOFF'S Law

- ➢ Gustav Robert Kirchhoff (1824 – 1887)

- ➢ Models relationship between:
  - ❖ circuit element and currents (KCL)
  - ❖ circuit element and voltages (KVL)

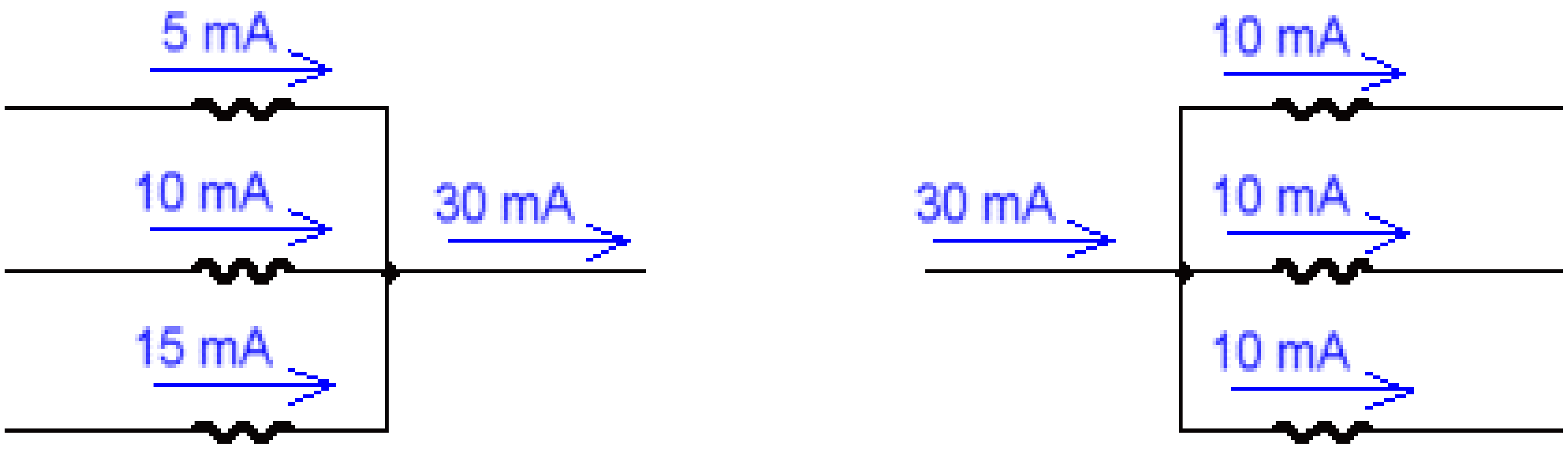
- ➢ He introduces two laws:
  - ❖ Kirchhoff Current Law (KCL)
  - ❖ Kirchhoff Voltage Law (KVL)

Kirchhoff's Current Law (KCL)

- Current entering node = current exiting
- Convention: +i is exit.ing, -i is entering
- For any circuit node:

$$\sum i = 0$$

➢ Kirchhoff’s Current Law (KCL)



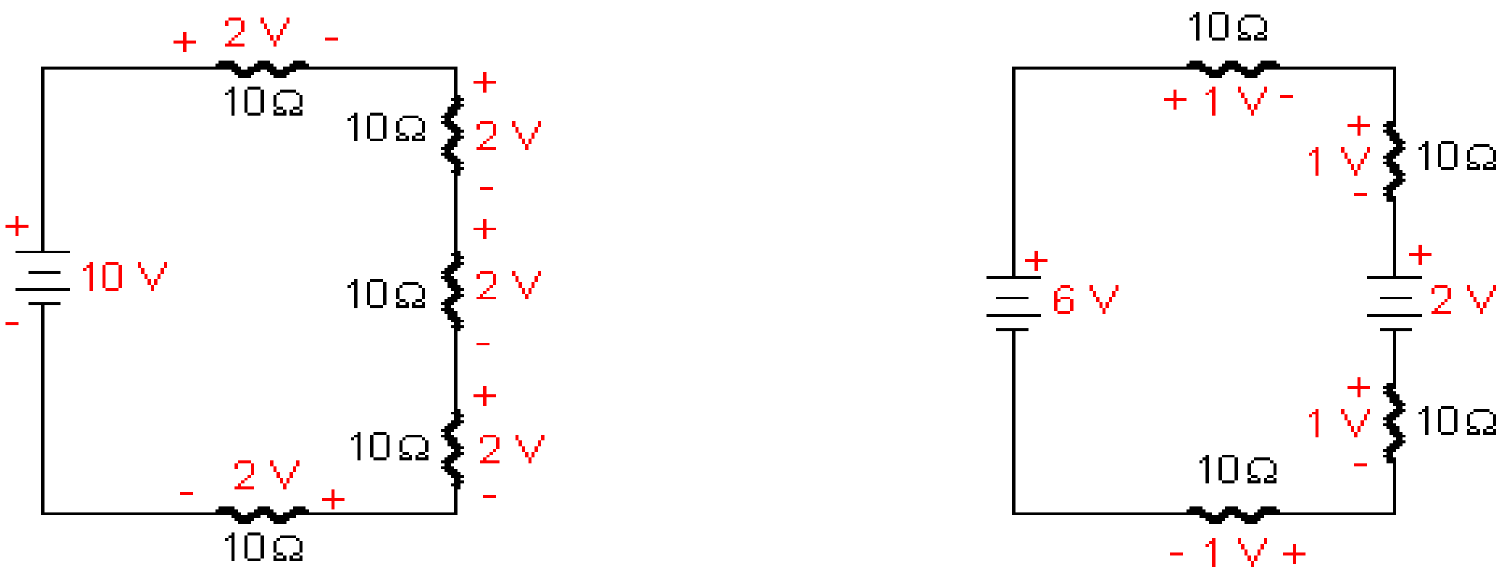
Kirchhoff’s Current Law (KCL) states that the algebraic sum of current entering a node must be equal to that of leaving the same node.

Kirchhoff's Voltage Law (KVL)

- voltage increases = voltage decreases
- Convention: hit minus (-) side first, write negative
- For any circuit loop:

$$\boxed{\sum v = 0}$$

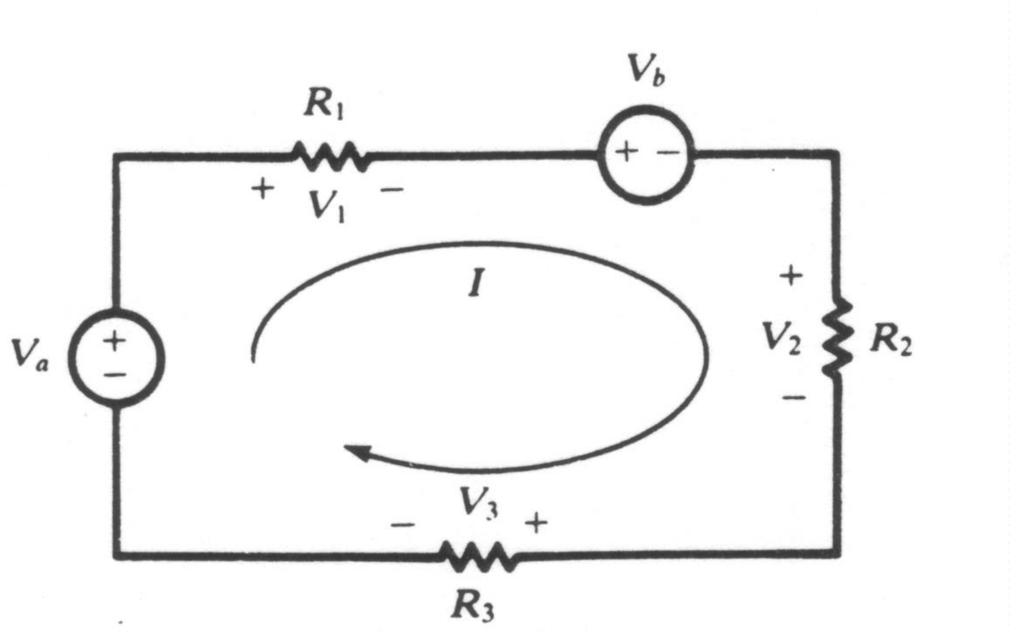
Kirchhoff's Voltage Law (KVL)



- Kirchhoff's Voltage Law states that the algebraic sum of voltage drop in a loop must be equal to that of voltage rise in the same loop.
- Stated it in a different way is that the algebraic sum of all voltages around a loop must be zero.

Example

- Applying the KVL equation for the circuit of the figure below.



$$v_a - v_1 - v_b - v_2 - v_3 = 0$$

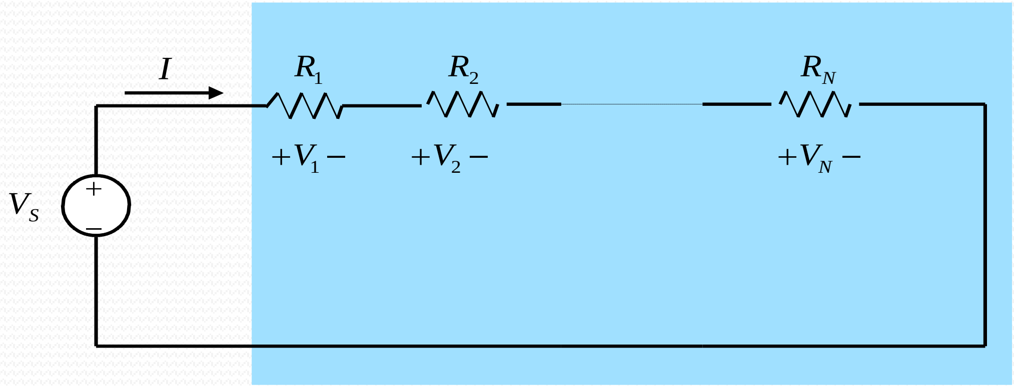
$$V_1 = IR_1 \quad v_2 = IR_2 \quad v_3 = IR_3$$

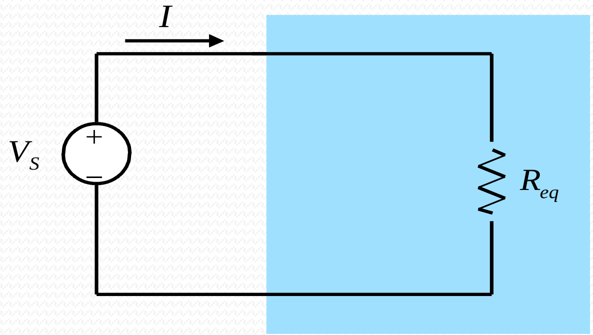
$$\Rightarrow v_a - v_b = I(R_1 + R_2 + R_3)$$

$$I = \frac{v_a - v_b}{R_1 + R_2 + R_3}$$

➢ Series And Parallel Circuit

❖ Resistor below is arranged in series connection:





❖ The equivalent resistance for any number of resistors in series connection is the sum of each individual resistor.

$$R_{eq} = R_1 + R_2 + \ldots\ldots\ldots + R_N$$

Current in Series Circuit

Current in series circuit is the same as in each circuit element.

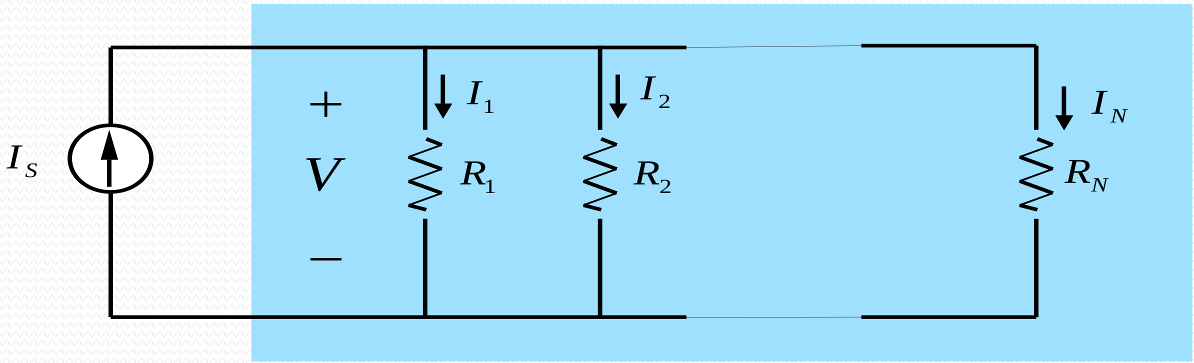
$$I = I_1 = I_2 = I_N$$

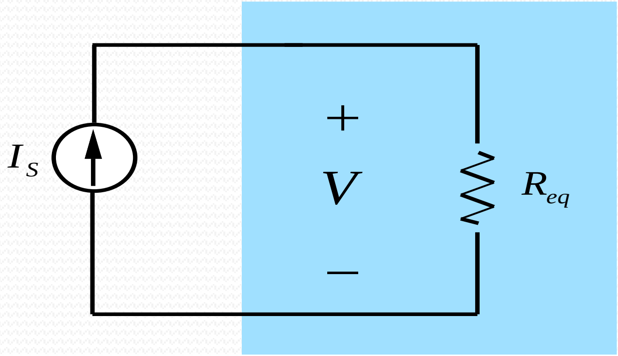
Voltage In Series Circuit

Voltage (V_T) in series circuit is the total voltage of each element circuit.

$$V_T = V_1 + V_2 + .. + V_N$$

Resistor below is arranged in parallel connection:





The equivalent resistance for any number of resistors in parallel connection is obtained by taking the reciprocal of the sum of the reciprocal of each single resistor in the circuit.

Equivalent resistance:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots\ldots\ldots + \frac{1}{R_N}$$

$$R_{eq} = \frac{1}{{}^{1}\!/_{R_1} + {}^{1}\!/_{R_2} + \ldots\ldots\ldots + {}^{1}\!/_{R_N}}$$

For the circuit which have two resistors in parallel connection:

$$R_{eq} = R_1 \| R_2$$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$= \frac{R_1 R_2}{R_1 + R_2}$$

➢ Current In Parallel Circuit

❖ **Current in series circuit is equal to the total current for each element circuit**

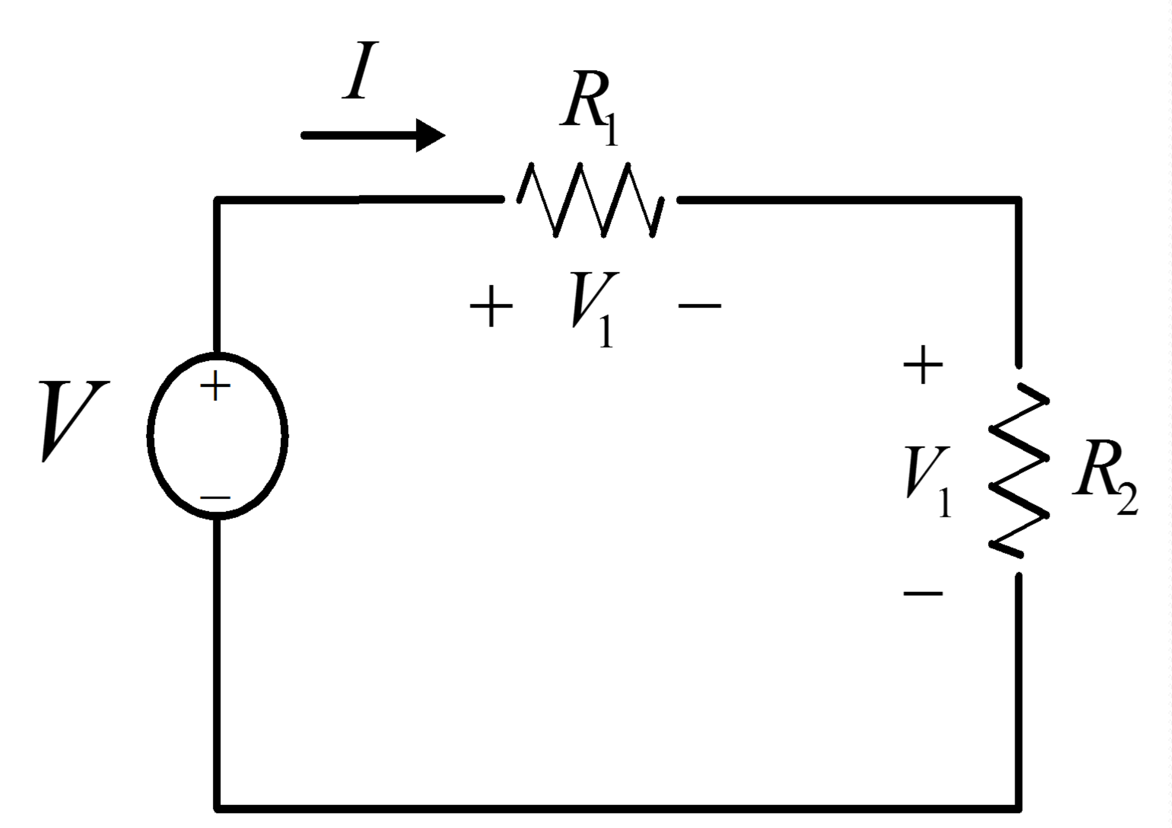
$$I = I_1 + I_2 + .. + I_N$$

➢ Voltage In Parallel Circuit

❖ **Voltage (V_T) in series circuit is the same as for each element circuit**

$$V_T = V_1 = V_2 = V_N$$

➢ Voltage Divider



Whenever voltage has to be divided among resistors in series use voltage divider rule principle.

- **By applying Ohm's Law:**

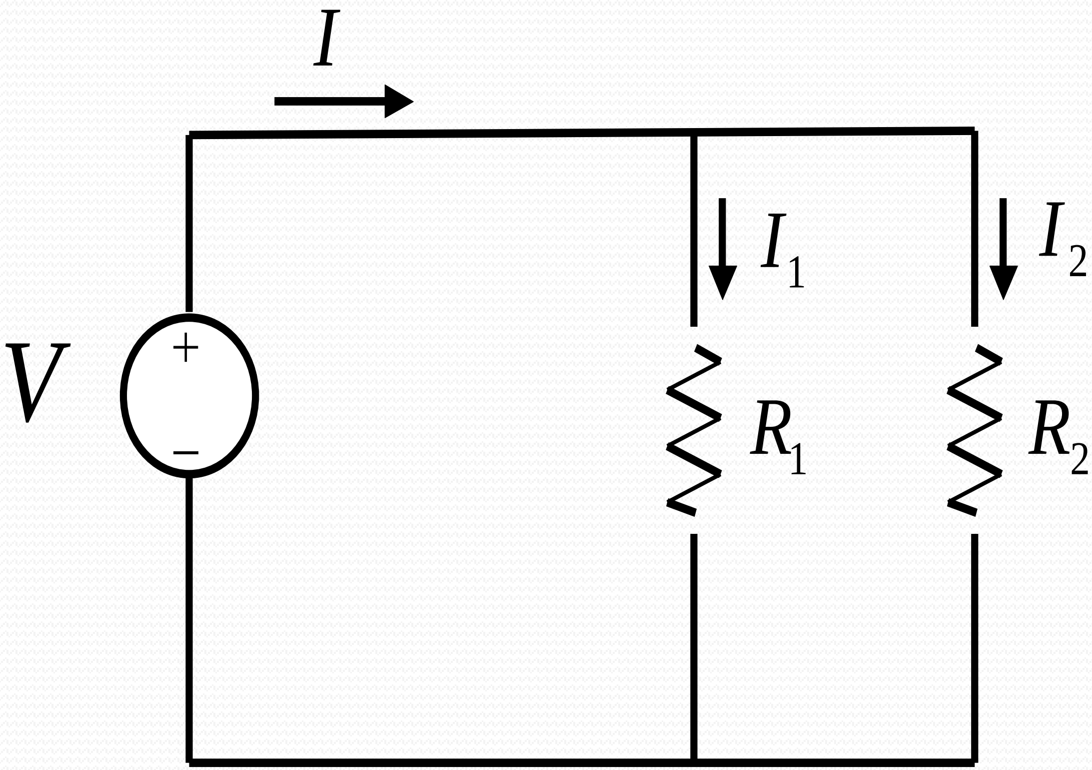
$$V_2 = R_2 I$$

$$I = \frac{V}{R_1 + R_2}$$

- **Voltage at resistor R2:**

$$V_2 = R_2\left(\frac{V}{R_1 + R_2}\right) = V\left(\frac{R_2}{R_1 + R_2}\right)$$

➢ Current Divider



Whenever current has to be divided among resistors in parallel, use current divider rule principle.

- **By applying Ohm's Law:**

$$V = I_1 R_1 = I_2 R_2 = I\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

So, to find current, I_1 and I_2 :

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I$$

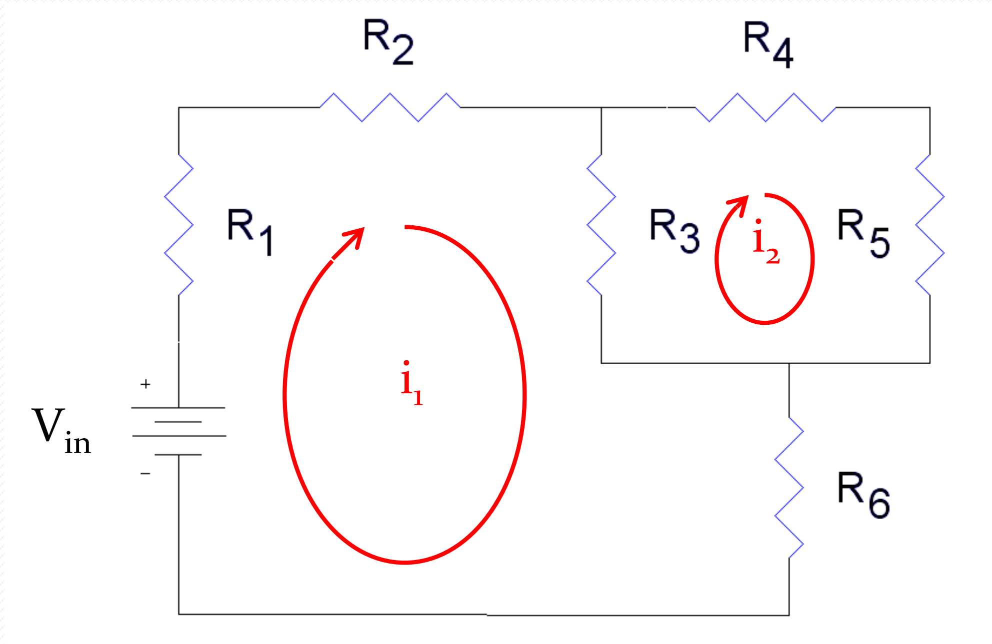
$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I$$

Mesh Analysis

- Mesh – the smallest loop around a subset of components in a circuit
- Technique to find voltage drops around a loop using the currents that flow within the loop, Kirchoff's Voltage Law, and Ohm's Law.
- Multiple meshes are defined so that every component in the circuit belongs to one or more meshes

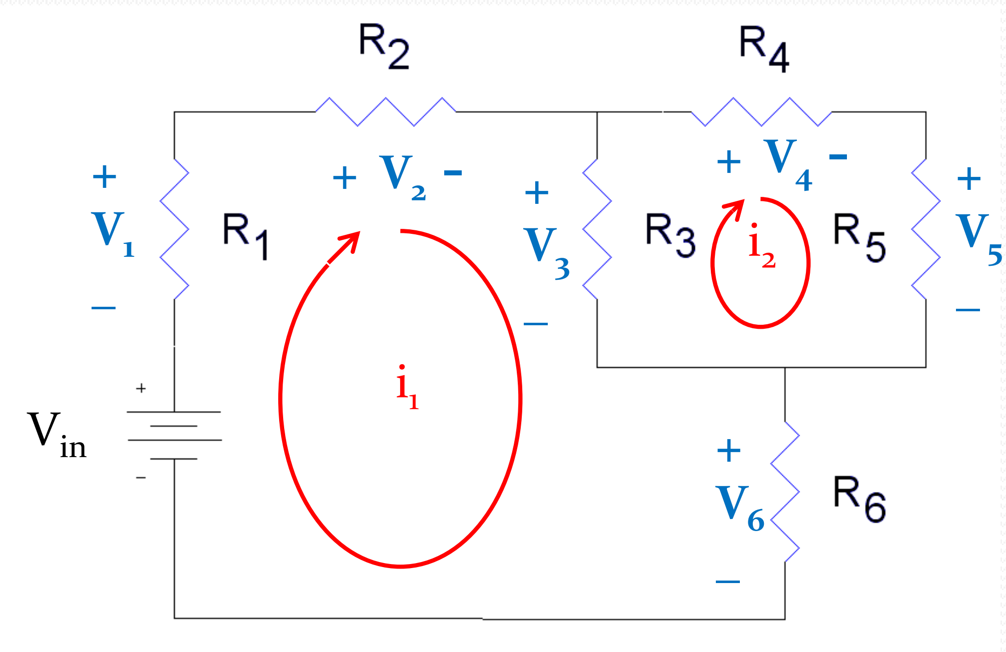
➢ Steps in Mesh Analysis

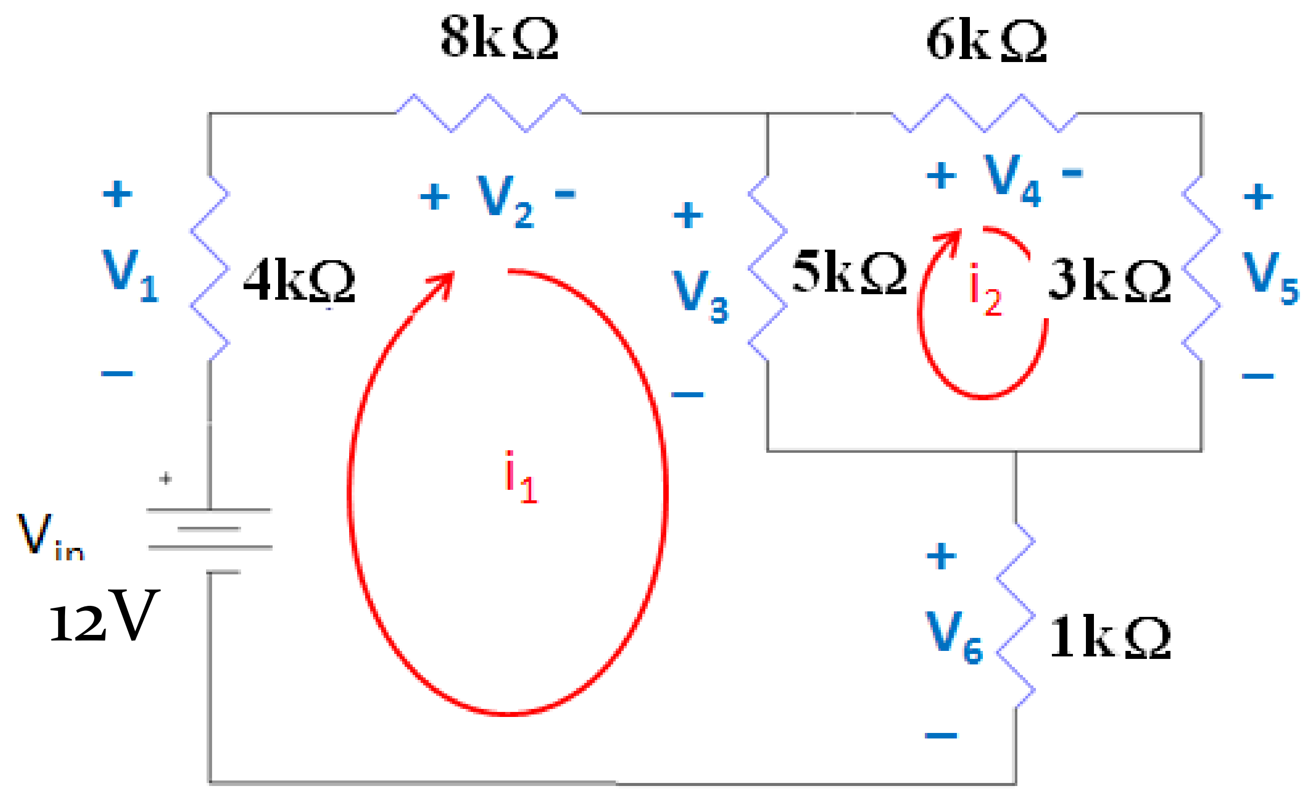
1. Identify all of the meshes in the circuit
2. Label the currents flowing in each mesh
3. Label the voltage across each component in the circuit
4. Write the voltage loop equations using Kirchoff's Voltage Law.
5. Use Ohm's Law to relate the voltage drops across each component to the sum of the currents flowing through them.
6. Solve for the mesh currents
7. Once the mesh currents are known, calculate the voltage across all of the components.


R2
R4
R1
R3
R5
i1
i2
Vin
+
–
R6

Step 3

- Label the voltage across each component in the circuit





$$-V_{in} - V_1 + V_2 + V_3 + V_6 = 0$$

$$-V_3 + V_4 + V_5 = 0$$

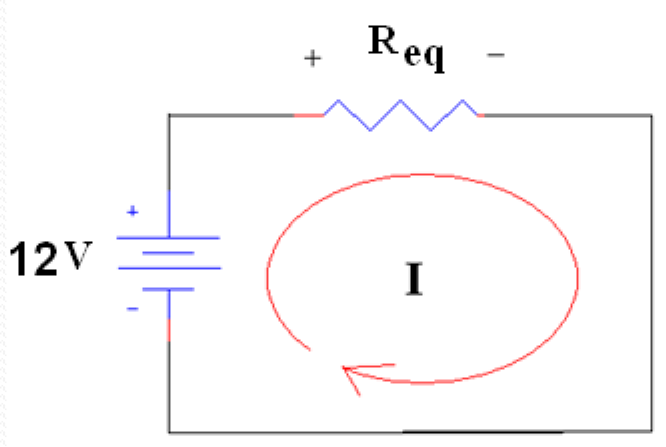
➢Check

❖ None of the mesh currents should be larger than the current that flows through the equivalent resistor in series with the 12V supply.

$$R_{eq} = 4k\Omega + 8k\Omega + [5k\Omega \| (6k\Omega + 3k\Omega)] + 1k\Omega$$

$$R_{eq} = 16.2k\Omega$$

$$I_{eq} = 12V / R_{eq} = 740\mu A$$

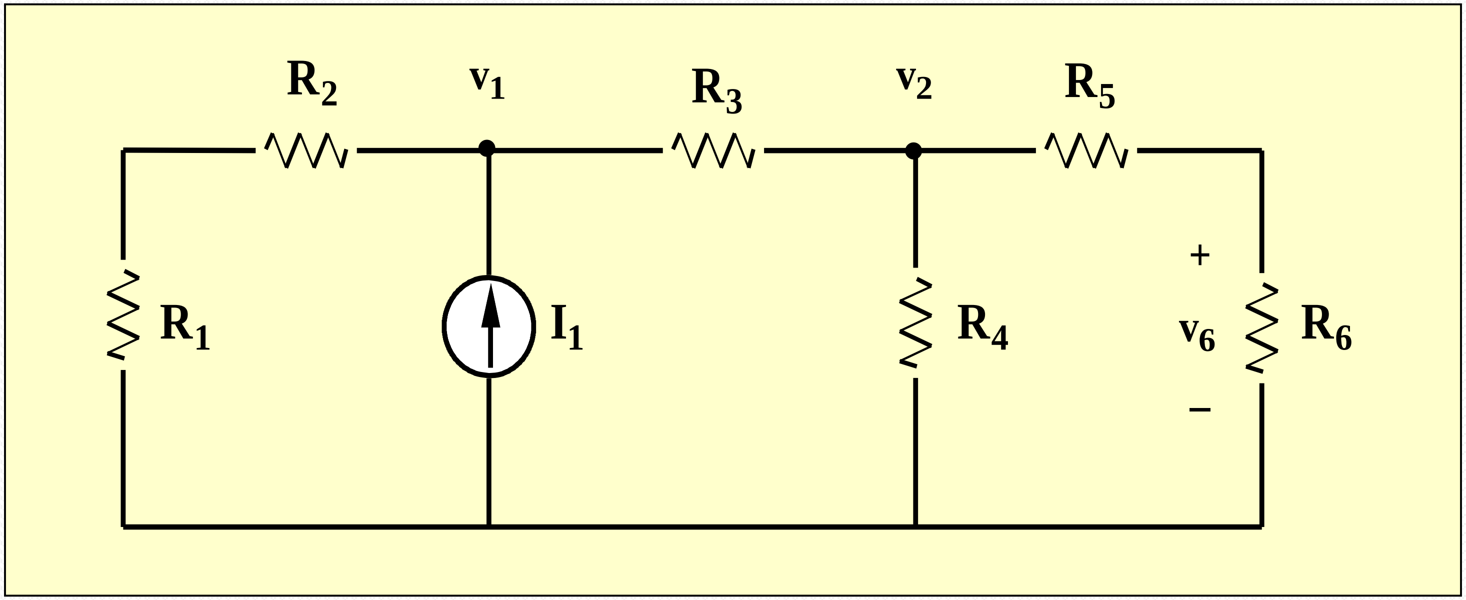


Nodal Analysis

- Technique to find currents at a node using Ohm's Law and the potential differences betweens nodes.

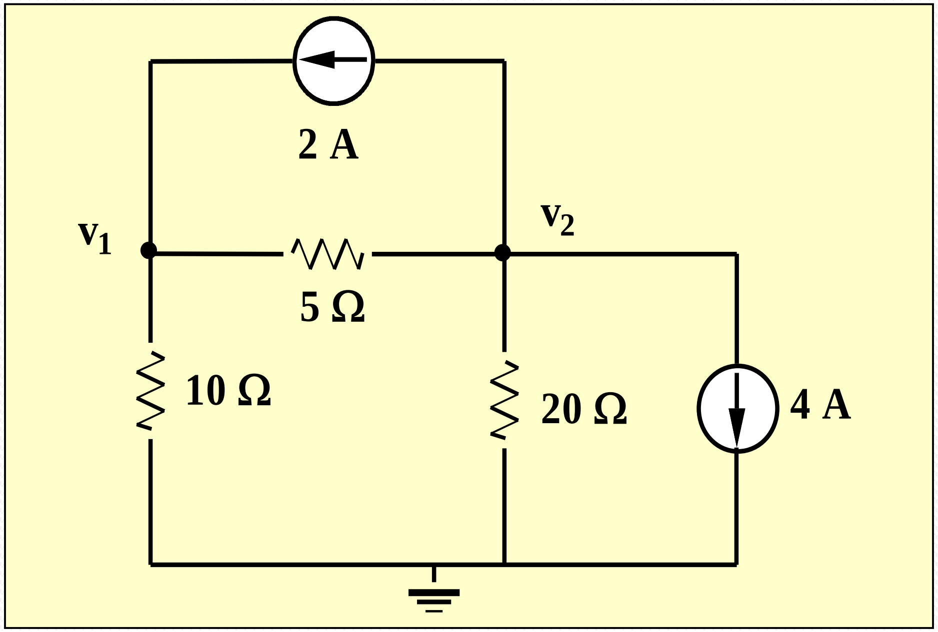
Steps in Nodal Analysis

1. Pick one node as a reference node
2. Label the voltage at the other nodes
3. Label the currents flowing through each of the components in the circuit
4. Use Kirchoff's Current Law
5. Use Ohm's Law to relate the voltages at each node to the currents flowing in and out of them.
6. Solve for the node voltage
7. Once the node voltages are known, calculate the currents.



$$\frac{V_1}{R_1 + R_2} + \frac{V_1 - V_2}{R_3} = I_1$$

$$\frac{V_2 - V_1}{R_3} + \frac{V_2}{R_4} + \frac{V_2}{R_5 + R_6} = 0$$



Find V_1 and V_2.

<u>At v_1:</u>

$$\frac{V_1}{10} + \frac{V_1 - V_2}{5} = 2$$

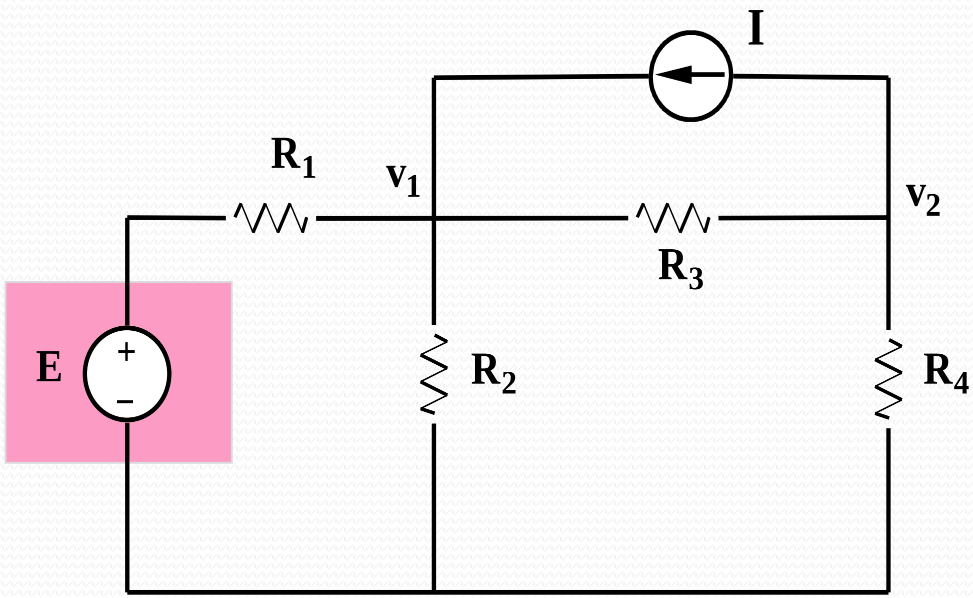
Eq 6.7

<u>At v_2:</u>

$$\frac{V_2 - V_1}{5} + \frac{V_2}{20} = -6$$

Eq 6.8

Basic Circuits



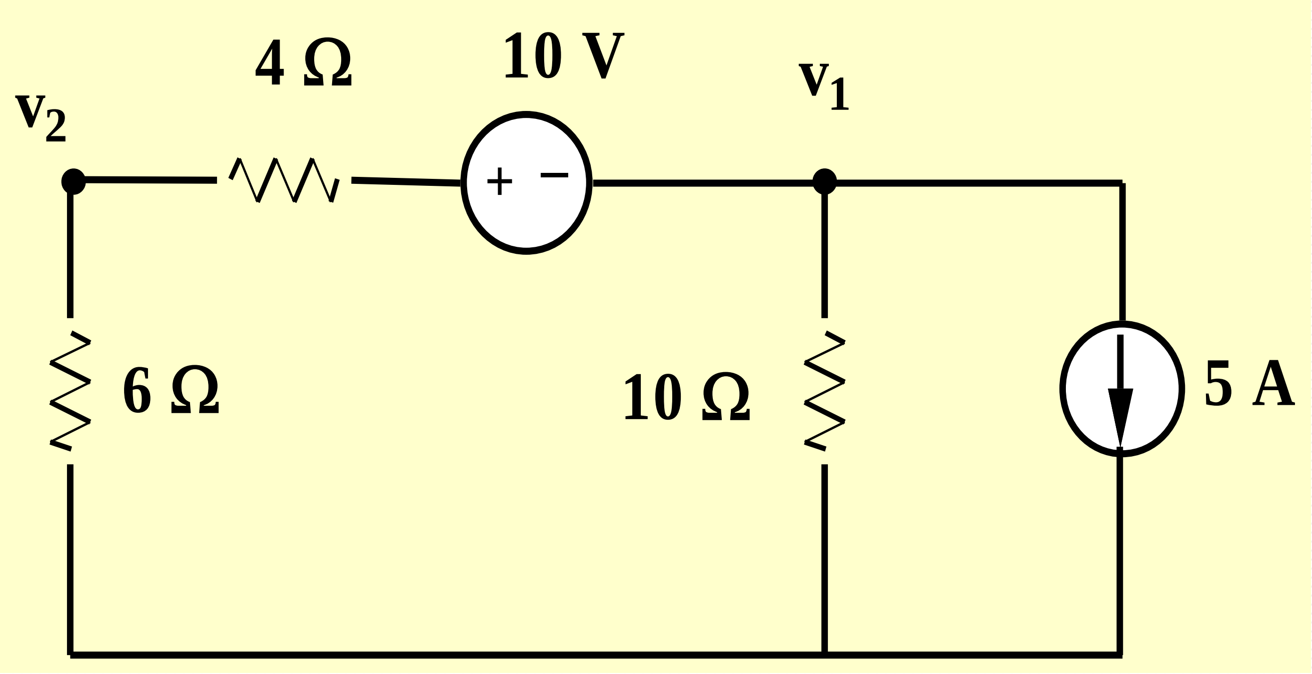
At V_1:

$$\frac{V_1 - E}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - V_2}{R_3} = I$$

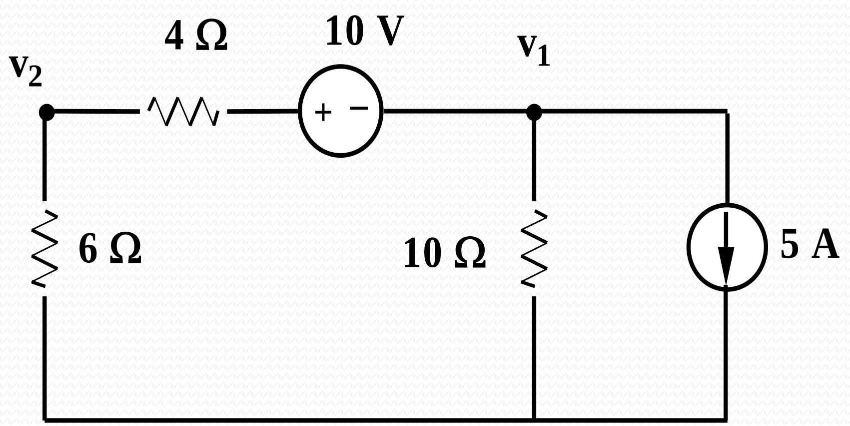
At V_2:

$$\frac{V_2}{R_4} + \frac{V_2 - V_1}{R_3} = -I$$

10



What do we do first?



At v_1:

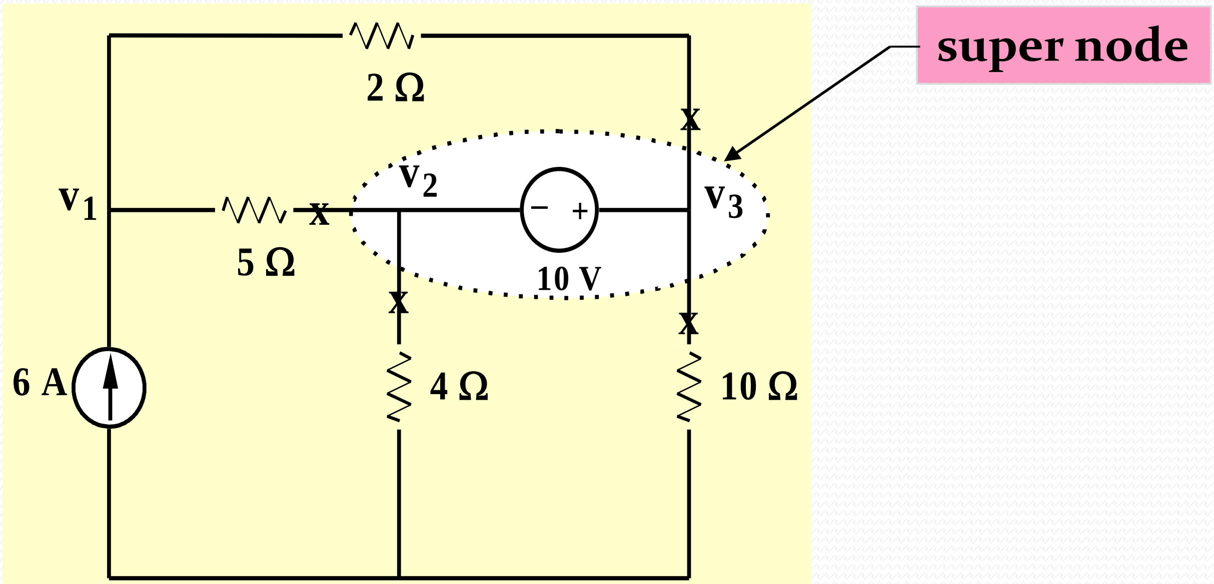
$$\frac{V_1}{10} + \frac{V_1 + 10 - V_2}{4} = -5$$

$V_1 = -30$ V, $V_2 = -12$ V, $I_1 = -2$ A

At v_2:

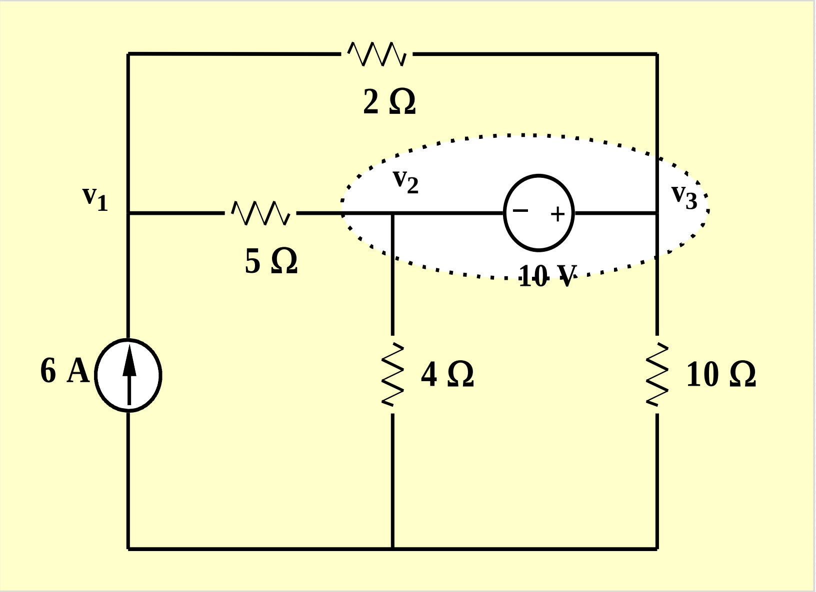
$$\frac{V_2}{6} + \frac{V_2 - 10 - V_1}{4} = 0$$

13



- When a voltage source appears between two nodes, an easy way to handle this is to form a super node.

- The Super Node encircles the voltage source and the tips of the branches connected to the nodes.



Constraint Equation

$$V_2 - V_3 = -10$$

At V_1

$$\frac{V_1 - V_2}{5} + \frac{V_1 - V_3}{2} = 6$$

At super node

$$\frac{V_2 - V_1}{5} + \frac{V_2}{4} + \frac{V_3}{10} + \frac{V_3 - V_1}{2} = 0$$

$$7V_1 - 2V_2 - 5V_3 = 60$$

$$-14V_1 + 9V_2 + 12V_3 = 0$$

$$V_2 - V_3 = -10$$

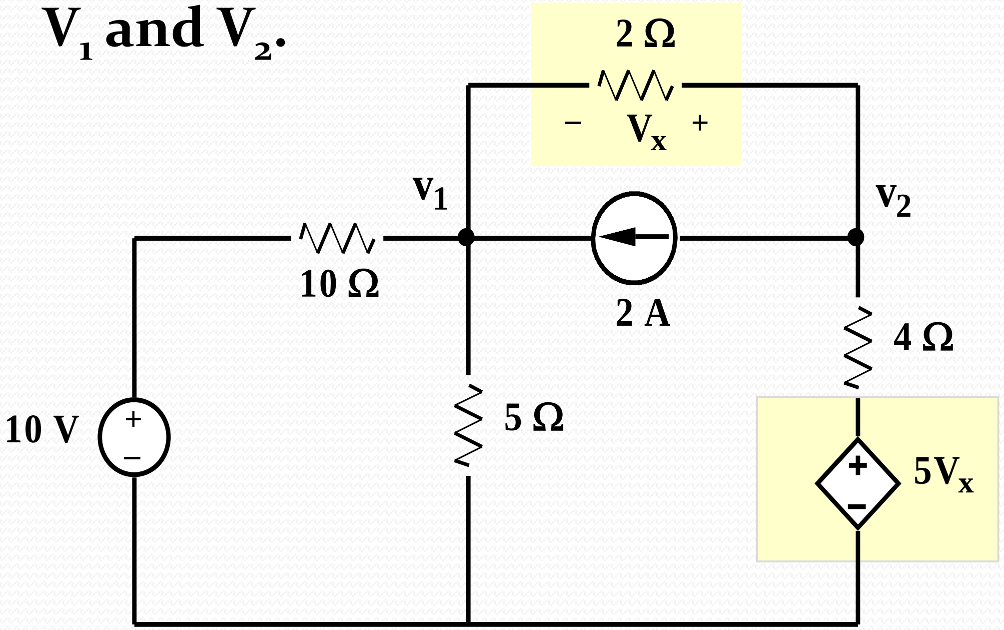
Solving gives:

$$V_1 = 30\text{ V}, \quad V_2 = 14.29\text{ V}, \quad V_3 = 24.29\text{ V}$$

17

Nodal Analysis with Dependent Sources.

Consider the circuit below. We desire to solve for the node voltages V_1 and V_2.



In this case we have a dependent source, $5V_x$, that must be reckoned with. Actually, there is a constraint equation of

$$V_2 - V_x - V_1 = 0$$

UNIT-II
Coupled circuits

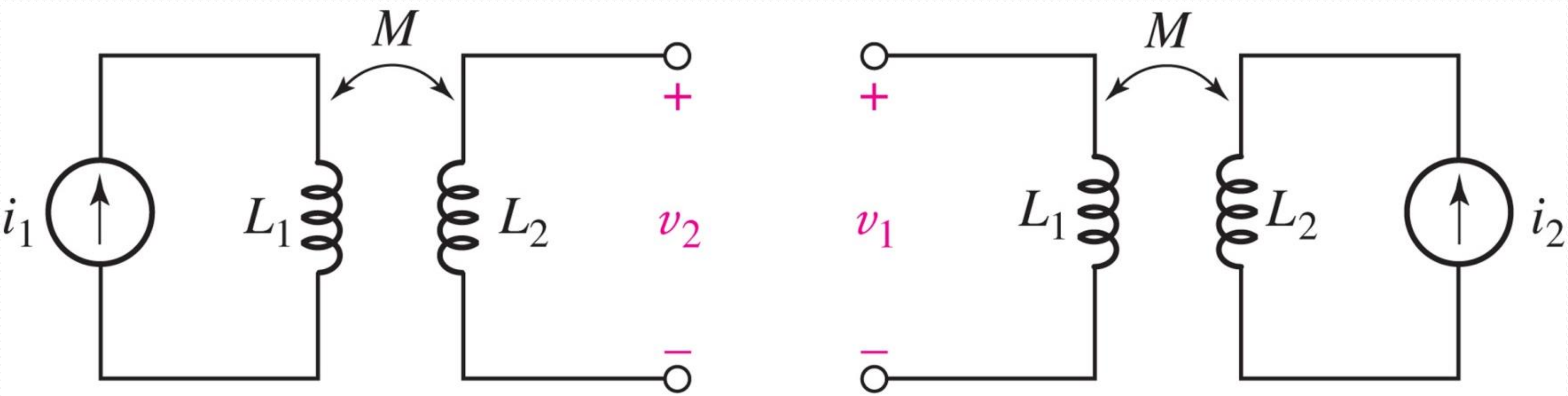
- Whenever current flows through a conductor, whether ac or dc, a magnetic field is generated about that conductor.
- When time varying magnetic field generated by one loop penetrates a second loop, a voltage is induced between the ends of second wire.
- In order to distinguish this phenomenon from the inductance, we defined more properly termed "self inductance", and "Mutual inductance".
- There is no such device as a "Mutual Inductor", but the principle forms the basis for an extremely important device – TRANSFORMER.

- The relation between the terminal voltage and current in a inductance

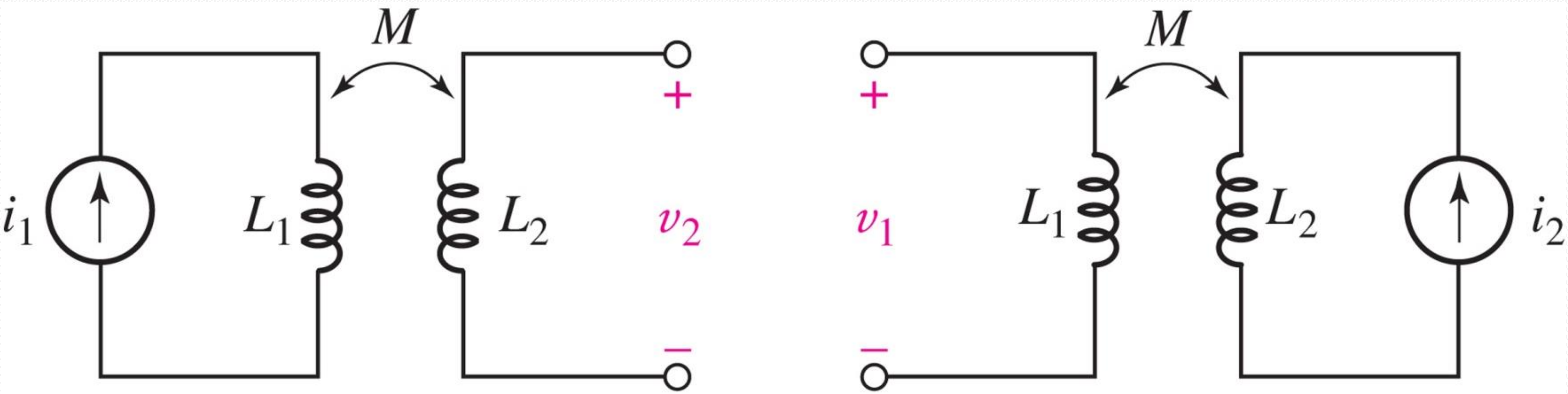
$$v(t) = L\frac{di(t)}{dt}$$

- The physical basis for a such a current-voltage characteristic rests upon two things
- The production of a magnetic flux by a current, the flux being proportional to the current in linear conductors.
- The production of a voltage by the time-varying magnetic field, the voltage being proportional to the time rate of change of the magnetic field or the magnetic flux.

➢ Mutual Inductance



➢Mutual Inductance

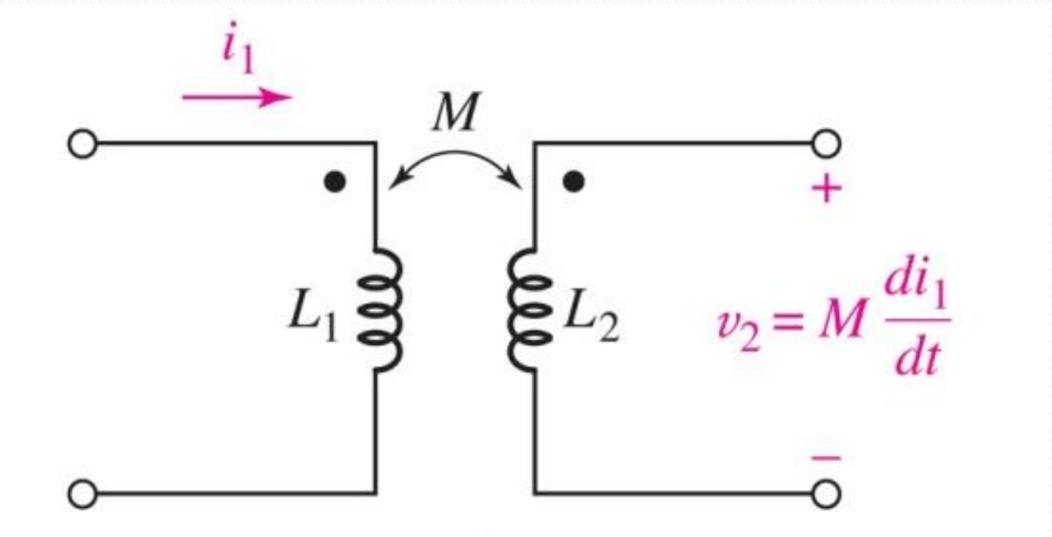


$$v_2(t) = M_{21}\frac{di_1(t)}{dt} \qquad v_1(t) = M_{12}\frac{di_2(t)}{dt}$$

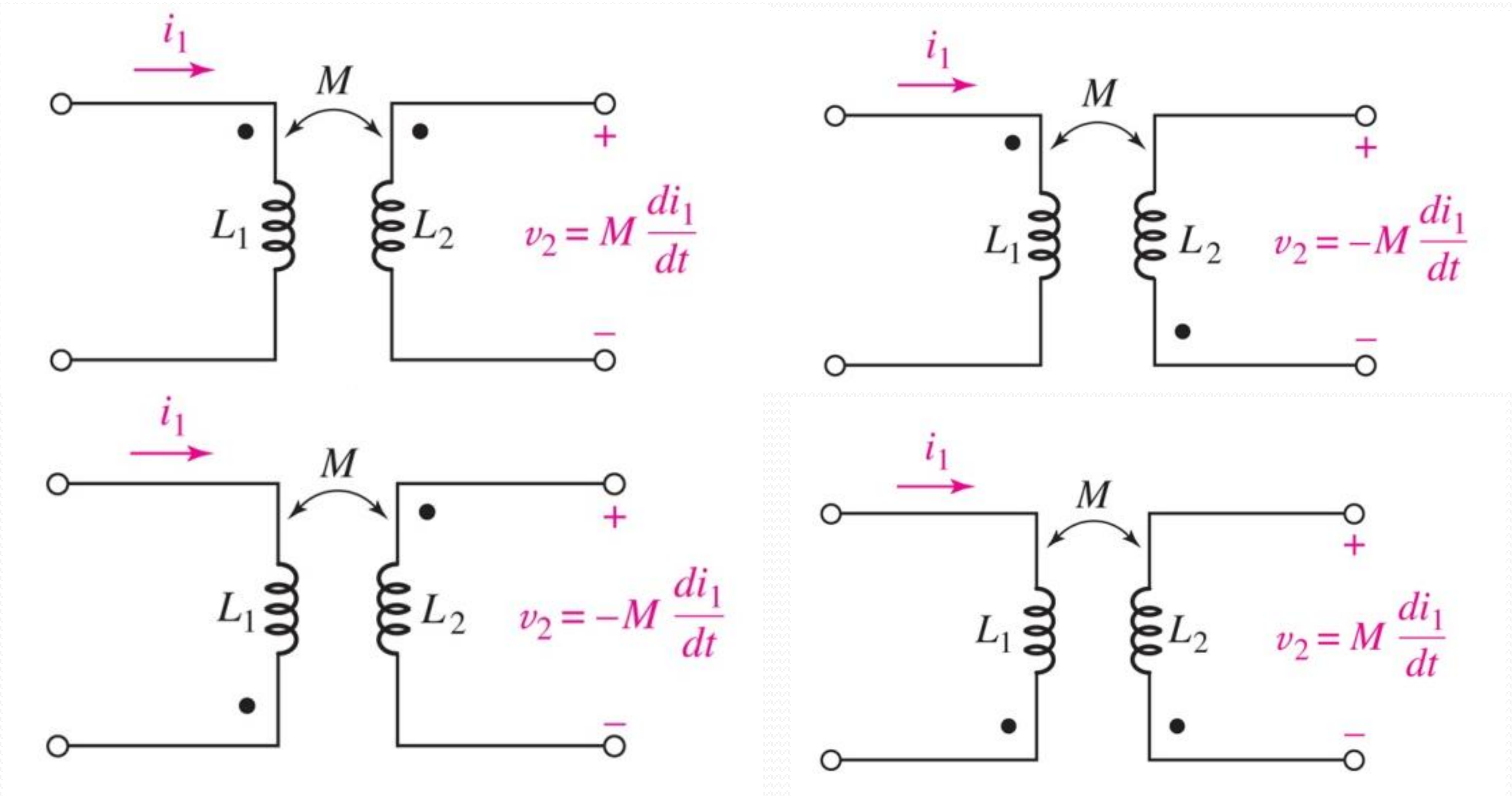
The double headed arrow indicates that these inductors are coupled.

➢ The Dot Convention

❖ A current entering the dotted terminal of one coil produces an open circuit voltage with a positive voltage reference at the dotted terminal of the second coil.



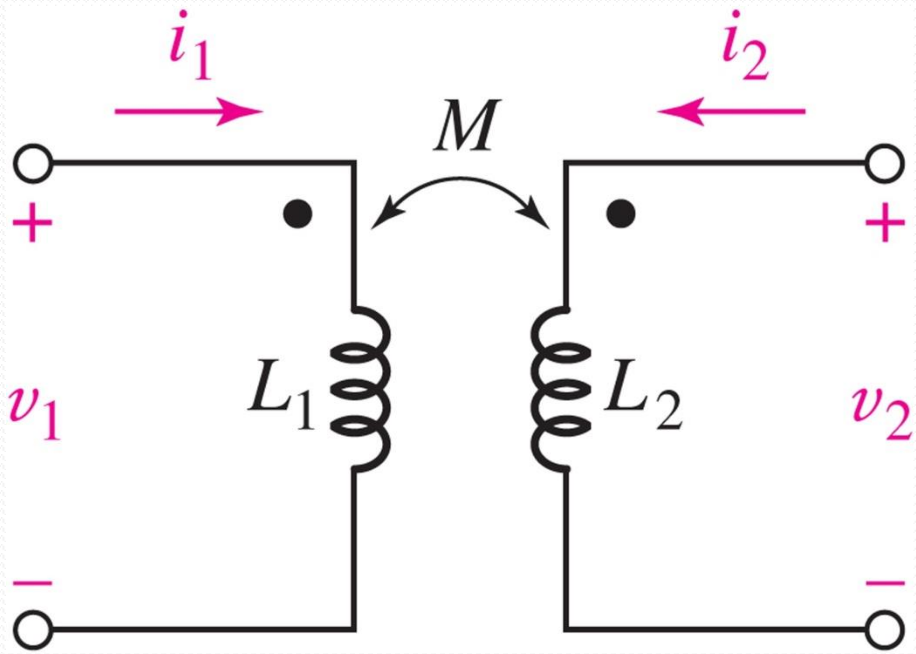
➢ Dot Convention: Four Cases



➢ Combined Self- and Mutual-Induction Voltages

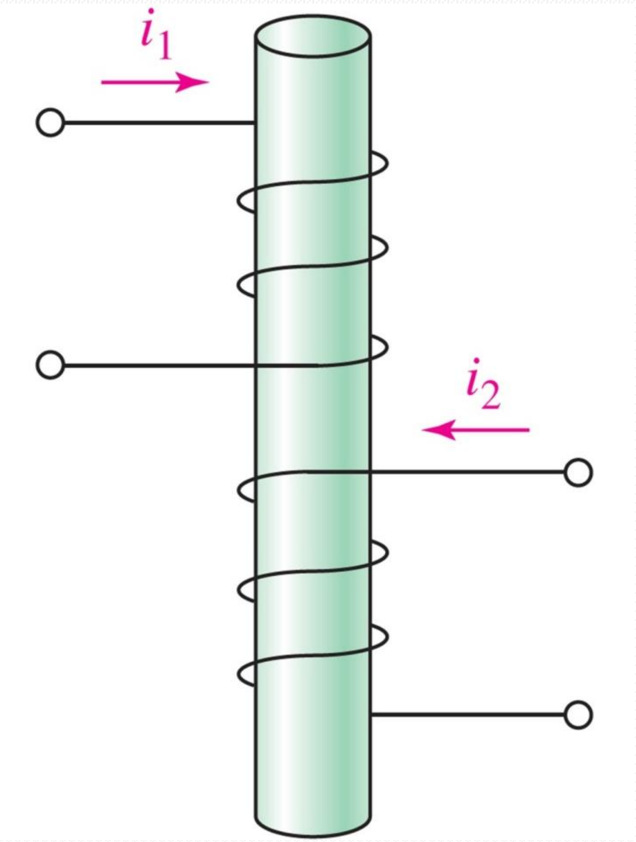
$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

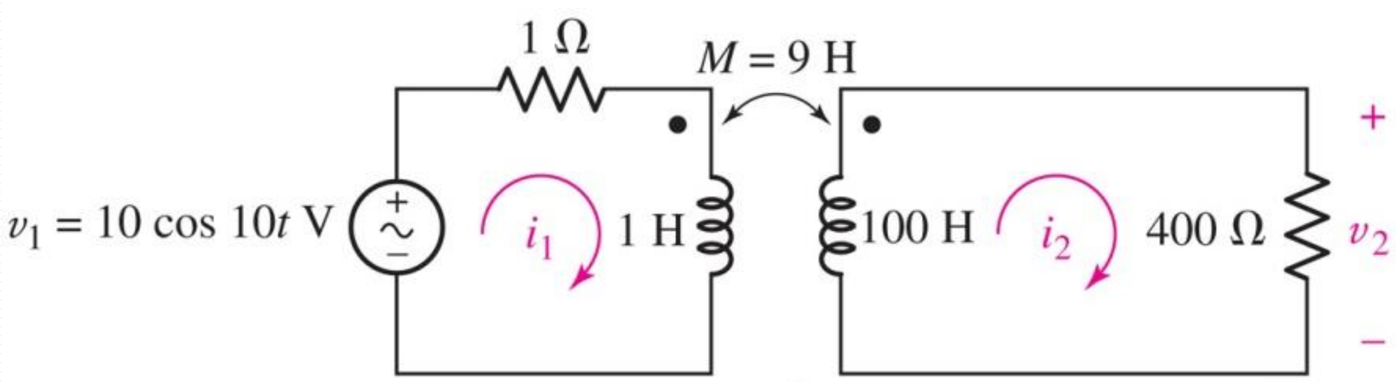


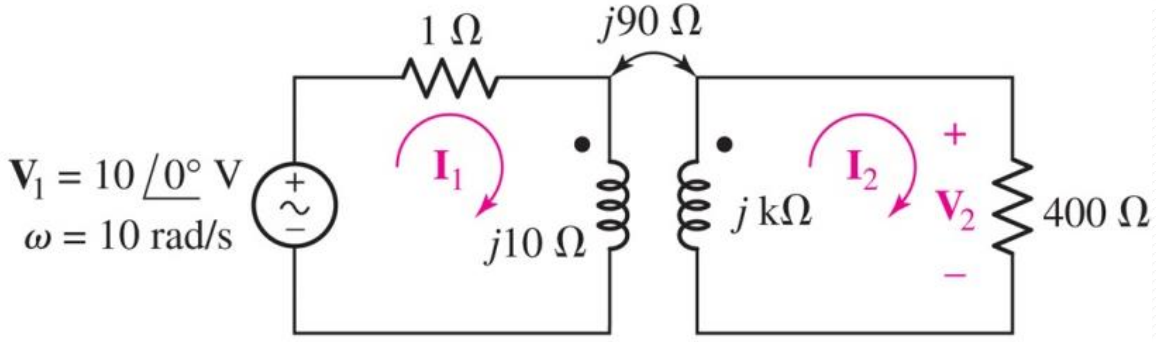
➢ Physical Basis for Dot Convention

- ❖ The assumed currents i_1 and i_2 produce additive fluxes.
- ❖ Dots may be placed either on the upper terminal of each coil or on the lower terminal of each coil.



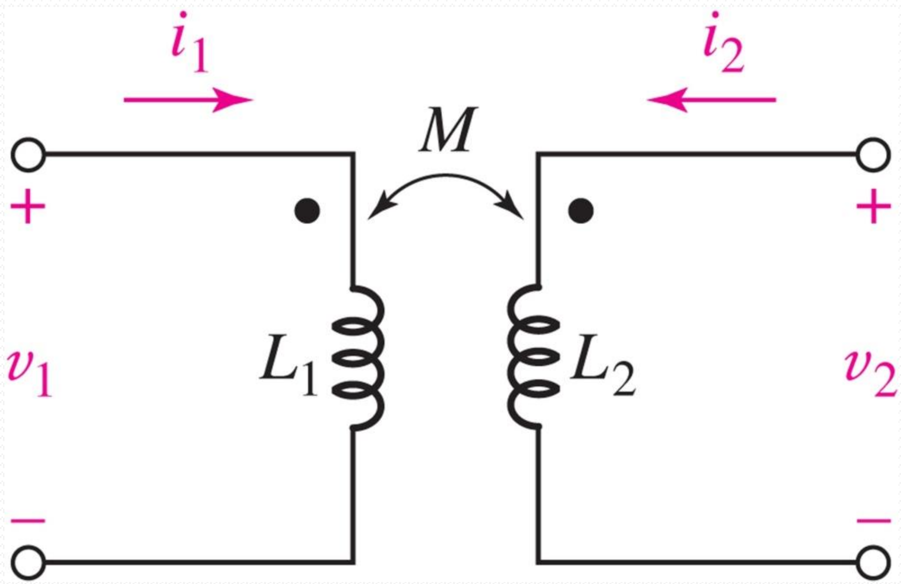
➢Example: Voltage Gain





Show that $V_2/V_1 = 6.88e^{-j16.70°}$

➤ Energy in Coupled Inductors



$$w(t) = \frac{1}{2} L_1 [i_1(t)]^2 + \frac{1}{2} L_2 [i_2(t)]^2 + M\big[i_1(t)\big]\big[i_2(t)\big]$$

This equation implies a limit on M: $M \leq \sqrt{L_1 L_2}$

➢ The Coupling Coefficient

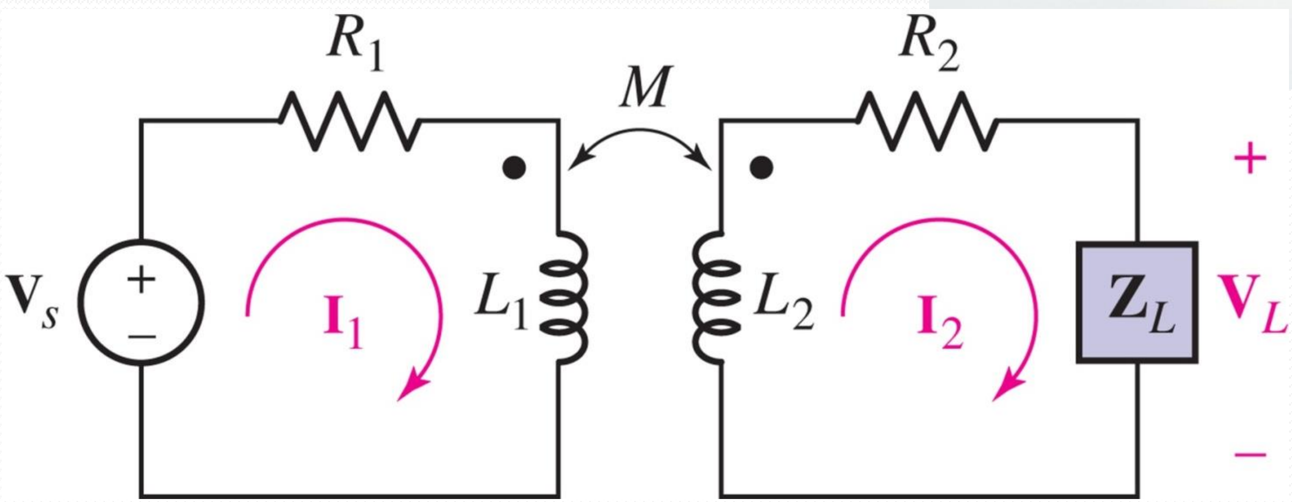
❖ **The coupling coefficient k measures how tightly coupled the two inductors are:**

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

where $0 \le k \le 1$

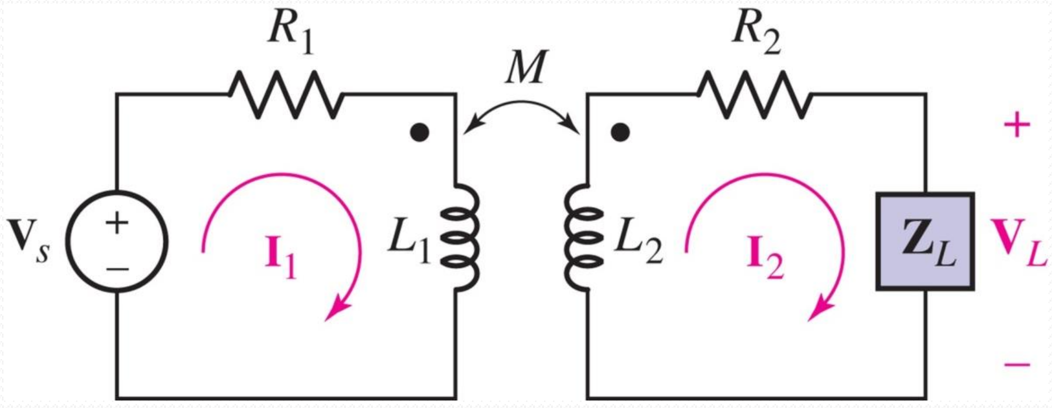
➢ The Linear Transformer

Transformers have a primary (source side) and a secondary (load side).



➢ Transformer: Reflected Impedance

❖The impedance Z_{in} seen by the source is

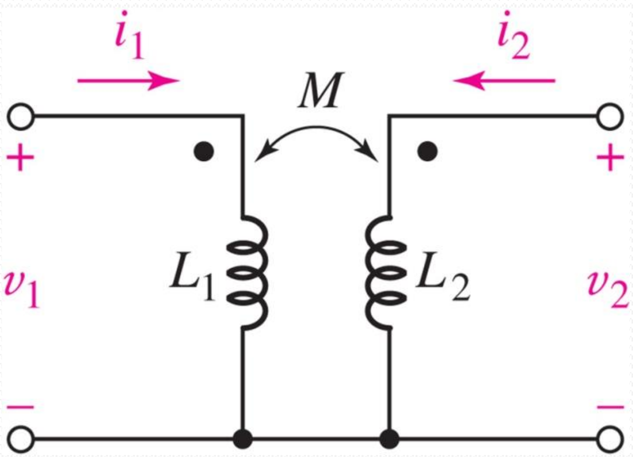


$$Z_{in} = Z_{11} - \frac{(j\omega)^2 M^2}{Z_{22}}$$

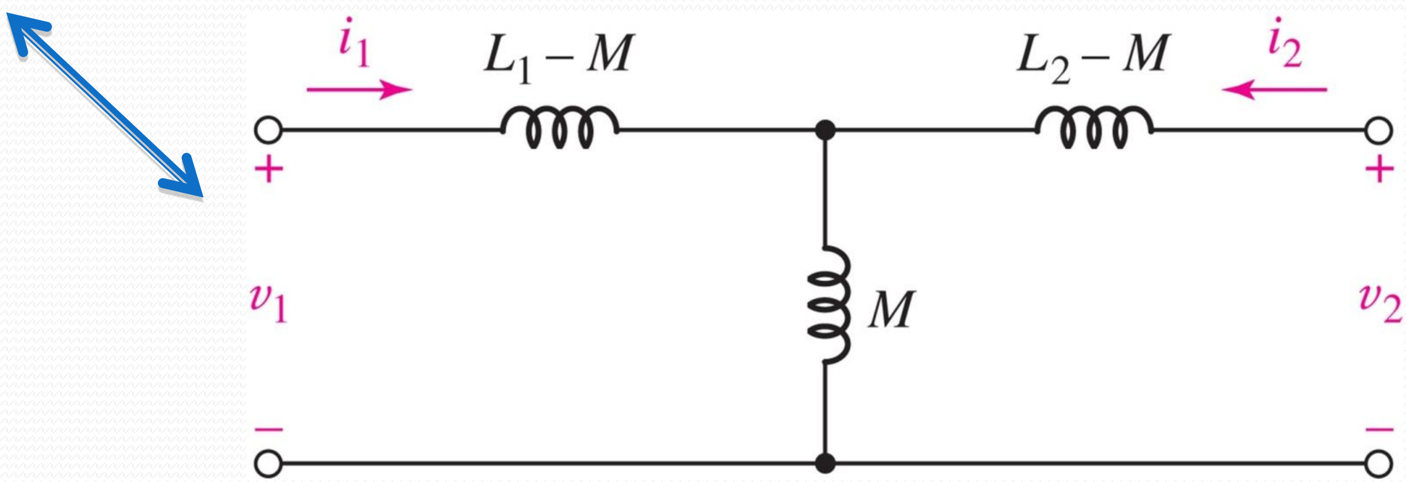
$$Z_{11} = R_1 + j\omega L_1$$

$$Z_{22} = R_2 + j\omega L_2 + Z_L$$

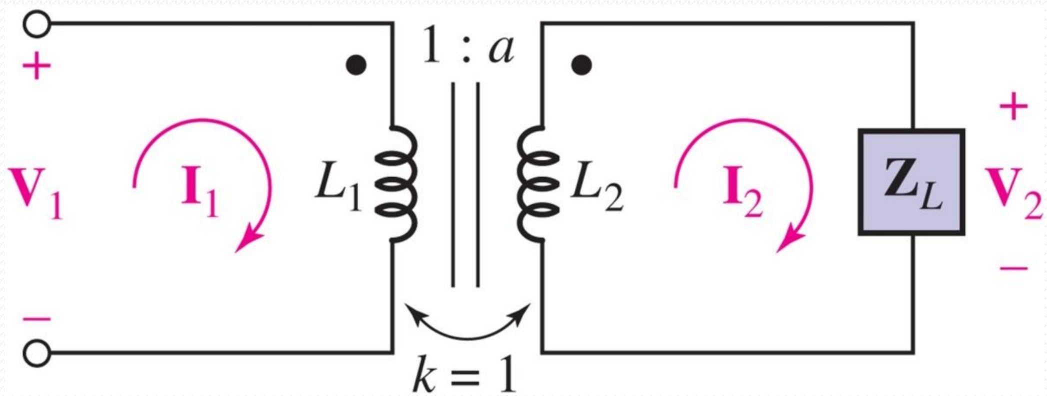
➢ The “T” Equivalent Network



Consider the mesh-current equations to show that these circuits are equivalent.



➢ The Ideal Transformer



- In an ideal transformer, *k=1* and the inductances are assumed large in comparison to the other impedances.

- The turns ratio *a* is defined as

$$a^2 = \frac{L_2}{L_1} = \frac{N_2^2}{N_1^2}$$

Transformer Applications

Step Up

Step Down

Power

Electronics

(c)
(Photos courtesy of Dr. Wade Enright, Te Kura Pukaha Vira O Te Whare Wananga O Waitaha, Aotearoa.)

Step Down to Household

➢ Transformer Applications

Impedance matching:

$$Z_{in} = \frac{Z_L}{a^2}$$

Current adjustment:

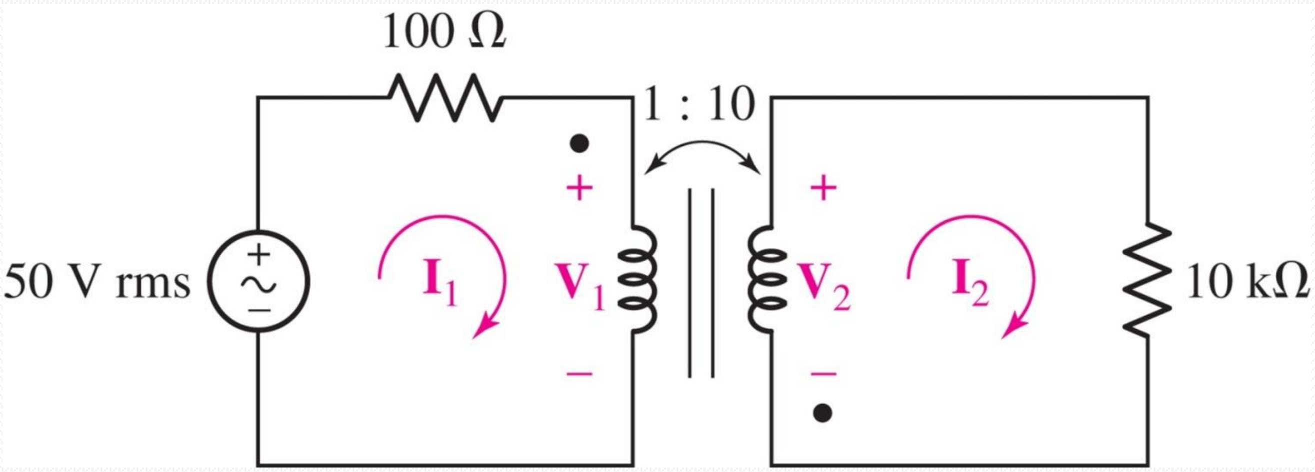
$$\frac{I_2}{I_1} = \frac{1}{a}$$

Voltage adjustment:

$$\frac{V_2}{V_1} = a$$

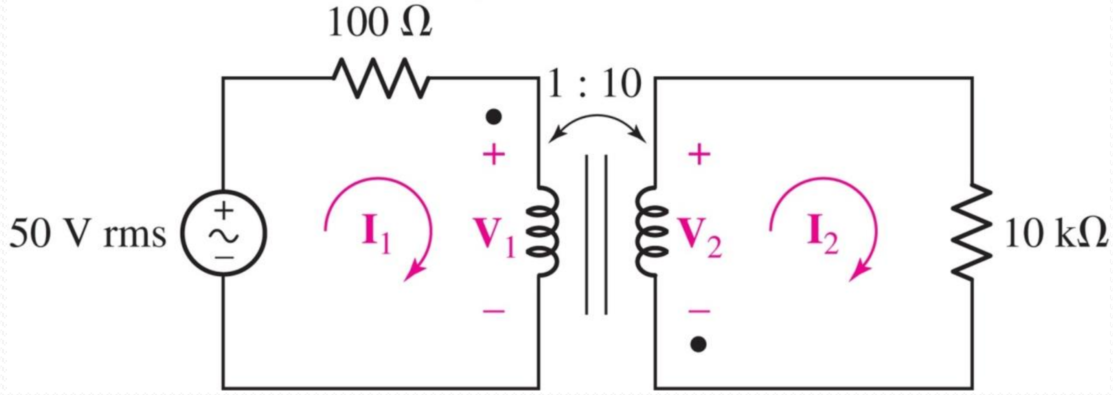
Example: Transformer Calculations

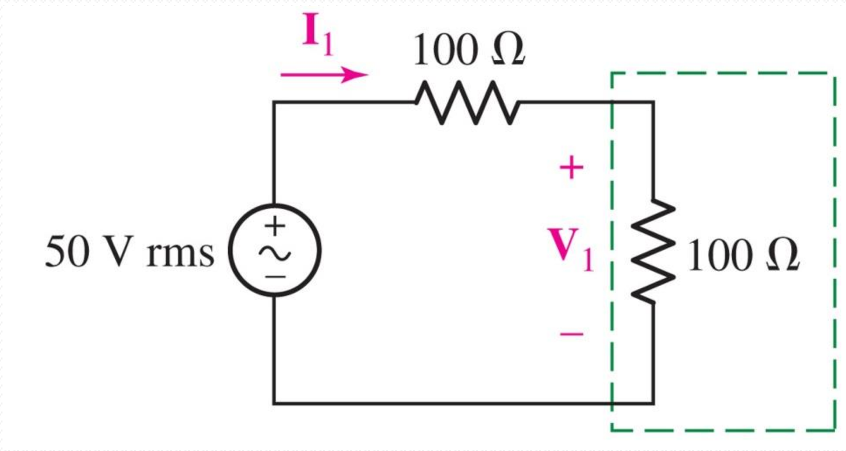
Determine the average power dissipated in the 10 kΩ resistor.

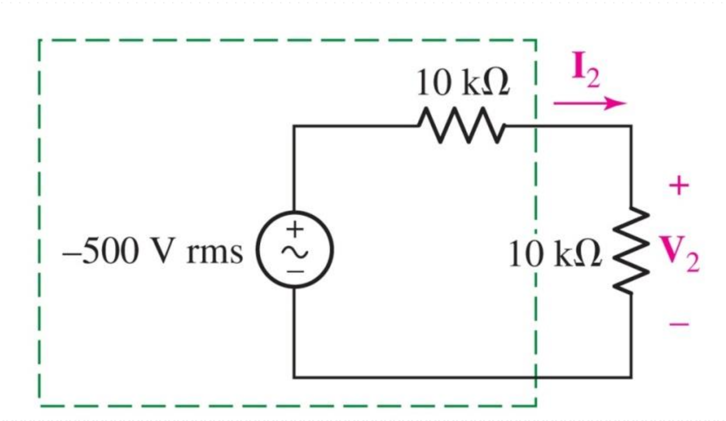


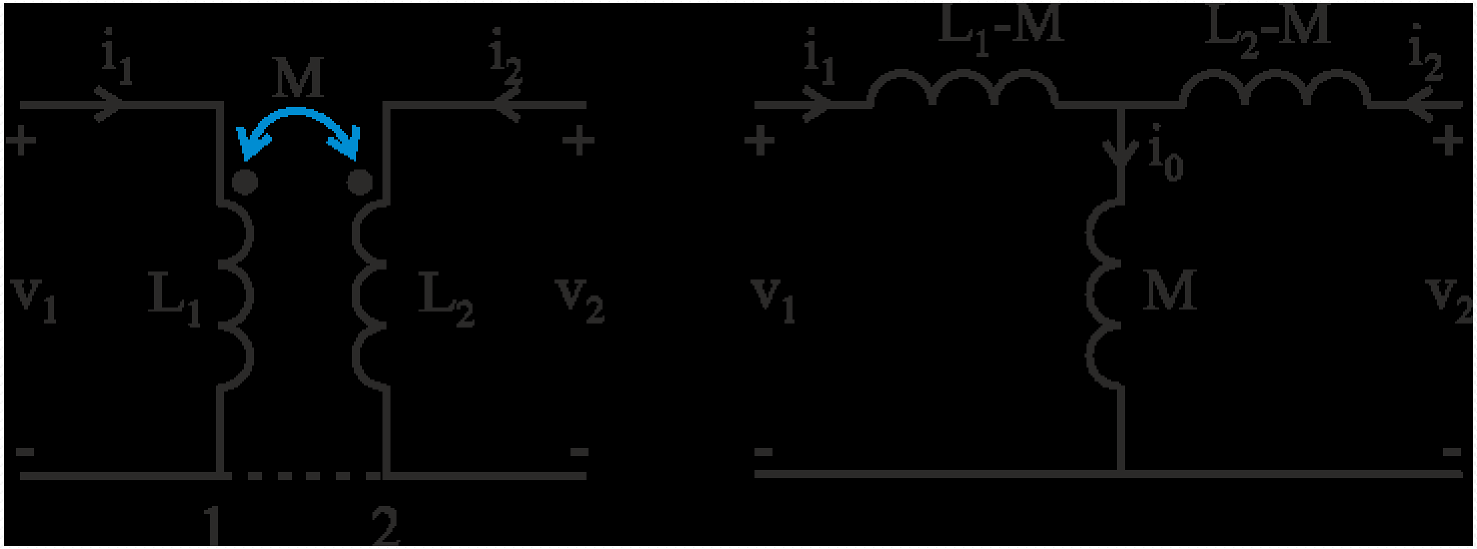
Answer: 6.25 W

Example: Thévenin Equivalent









i1
M
i2
+
+
v1
L1
L2
v2
-
-
1
2
i1
L1-M
L2-M
i2
+
i0
+
v1
M
v2
-
-

UNIT-III
AC Fundamentals

- **The majority of electrical power in the world is generated, distributed and consumed in the form of 50 Hz or 60-Hz sinusoidal alternating current (AC) and voltage.**

- **It is used for household and industrial applications such as television sets, computers, microwave ovens, electric stoves, to the large motors used in the industry.**

- **AC has several advantages over DC. The major advantage of AC is the fact that it can be transformed, however, DC cannot.**

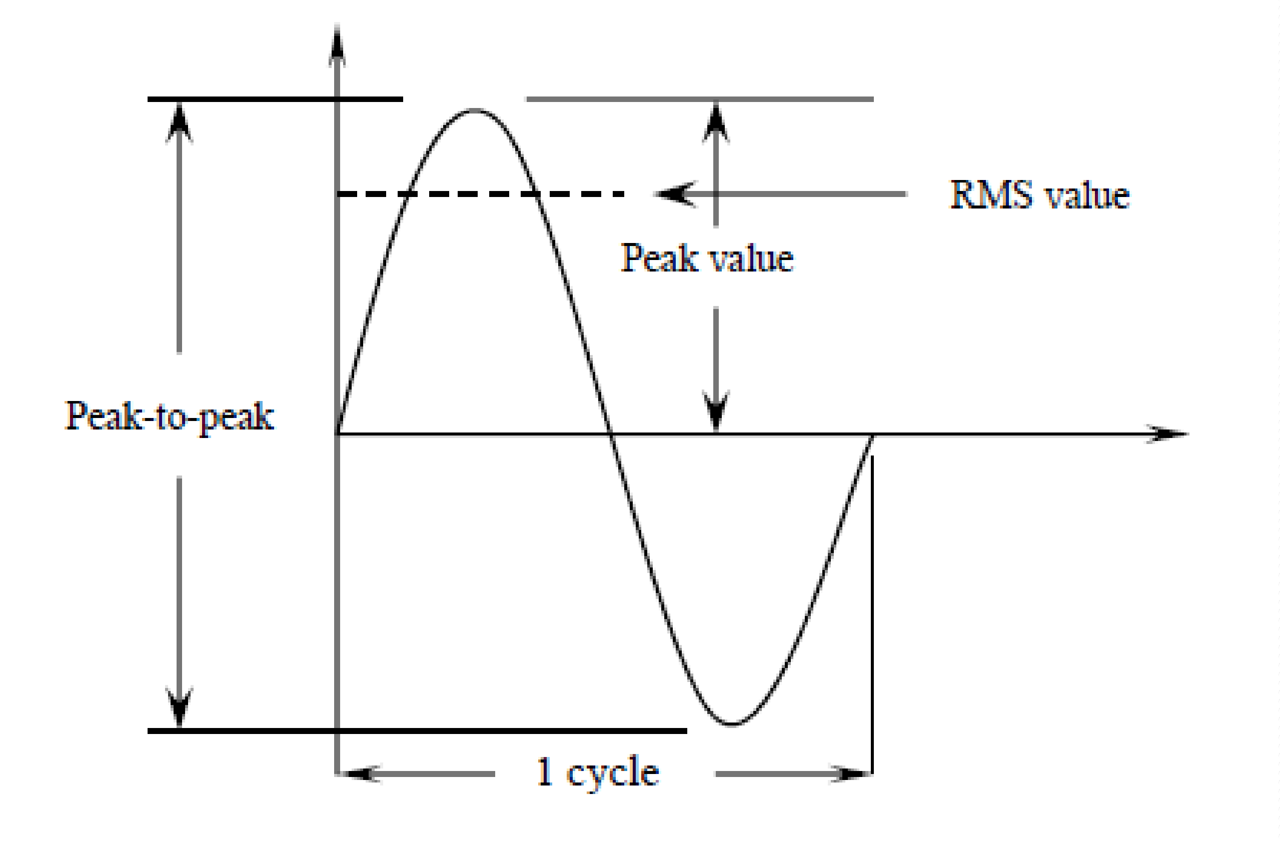
- **A transformer permits voltage to be stepped up or down for the purpose of transmission. Transmission of high voltage (in terms of kV) is that less current is required to produce the same amount of power. Less current permits smaller wires to be used for transmission.**

- **AC unlike DC flows first in one direction then in the opposite direction. The most common AC waveform is a sine (or sinusoidal) waveform. Sine waves are the signal whose shape neither is nor altered by a linear circuit, therefore, it is ideal as a test signal.**

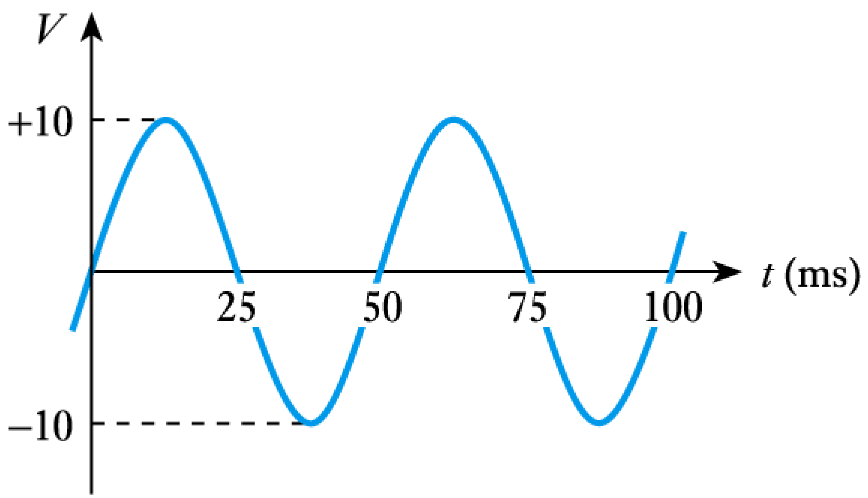
- **In discussing AC signal, it is necessary to express the current and voltage in terms of maximum or peak values, peak-to-peak values, effective values, average values, or instantaneous values. Each of these values has a different meaning and is used to describe a different amount of current or voltage.**

➢ **The correspondence mathematical form of sinusoidal AC signal is**

$$v(t) = V_P \cos(\omega t + \theta)$$



- **Example** – Determine the equation of the following voltage signal.



From diagram:

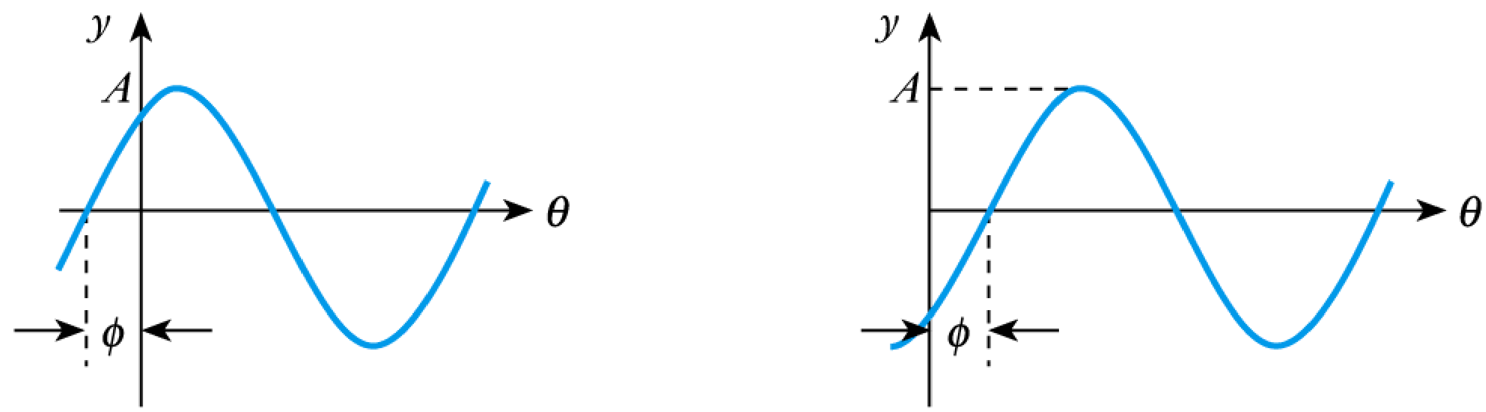
- Period is 50 ms = 0.05 s
- Thus f = 1/T =1/0.05 = 20 Hz
- Peak voltage is 10 V
- Therefore

$$v = V_p \sin 2\pi f t$$

$$= 10 \sin 2\pi 20 t$$

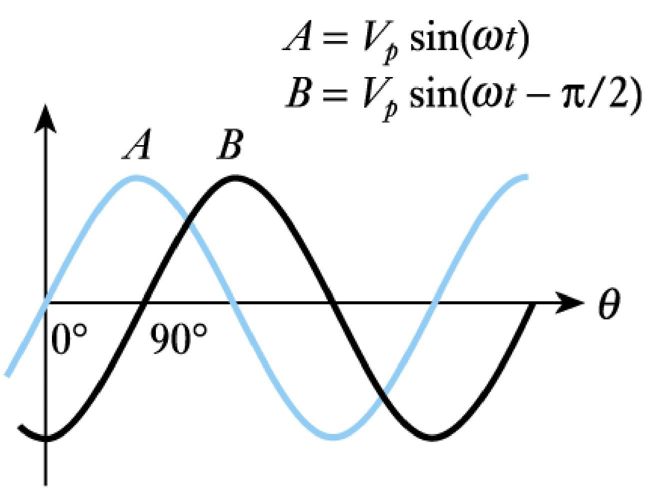
$$= 10 \sin 126 t$$

- **Phase angles**
  - the expressions given above assume the angle of the sine wave is zero at $t = 0$
  - if this is not the case the expression is modified by adding the angle at $t = 0$

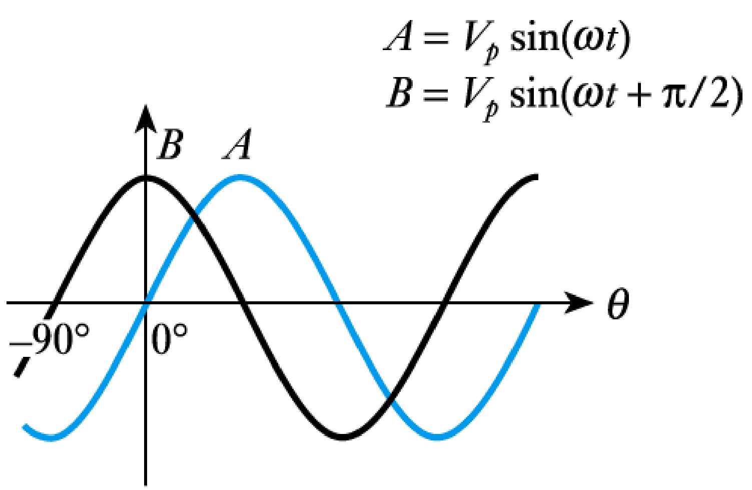


(a) $y = A\sin(\omega t + \phi)$ (b) $y = A\sin(\omega t - \phi)$

- **Phase difference**
  - two waveforms of the same frequency may have a constant phase difference
    - we say that one is **phase-shifted** with respect to the other



(a) B lags A by 90°



(b) B leads A by 90°

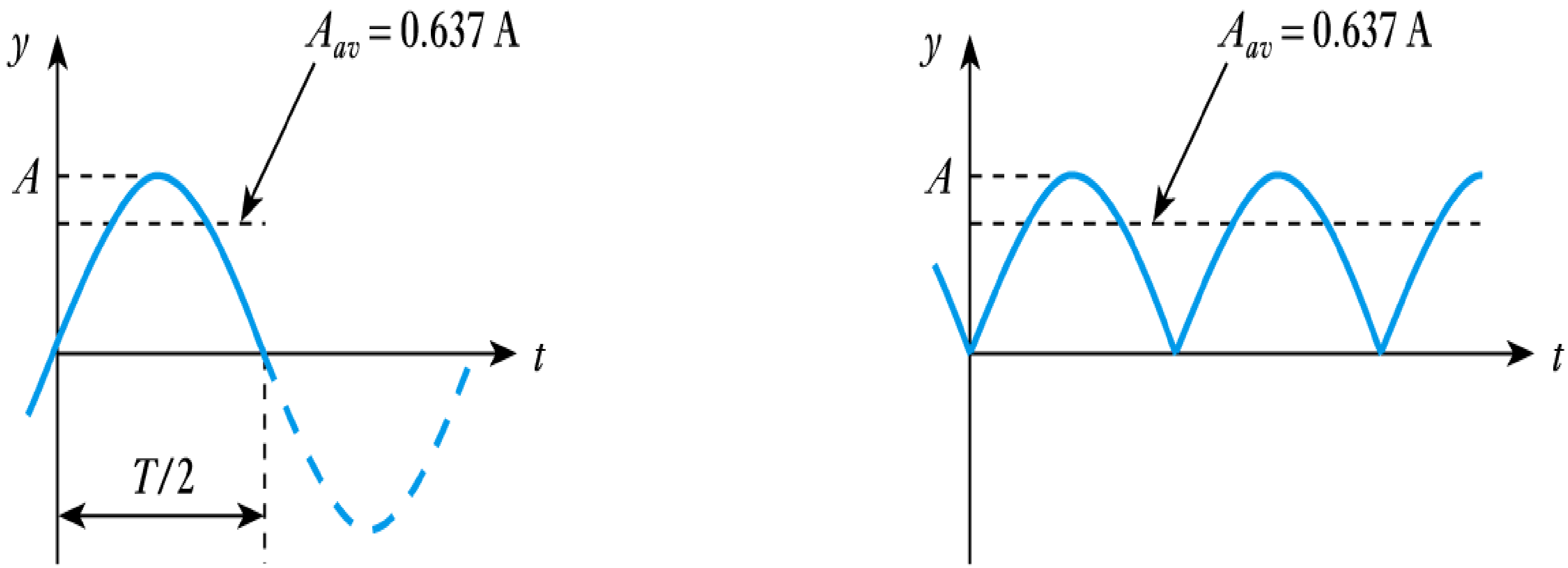
- **Average value of a sine wave**
  - average value over one (or more) cycles is clearly zero
  - however, it is often useful to know the average magnitude of the waveform independent of its polarity
    - we can think of this as the average value over half a cycle…
    - … or as the average value of the rectified signal

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} V_p \sin\theta \, d\theta$$

$$= \frac{V_p}{\pi}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{2V_p}{\pi} = 0.637 \times V_p$$

- **Average value of a sine wave**



(a) Average value over half a cycle of a sine wave

(b) Average value of a rectified sine wave

- **r.m.s. value of a sine wave**
  - the instantaneous power (p) in a resistor is given by

$$p = \frac{v^2}{R}$$

  - therefore the average power is given by

$$P_{av} = \frac{[\text{average (or mean) of } v^2]}{R} = \frac{\overline{v^2}}{R}$$

  - where $\overline{v^2}$ is the **mean-square voltage**

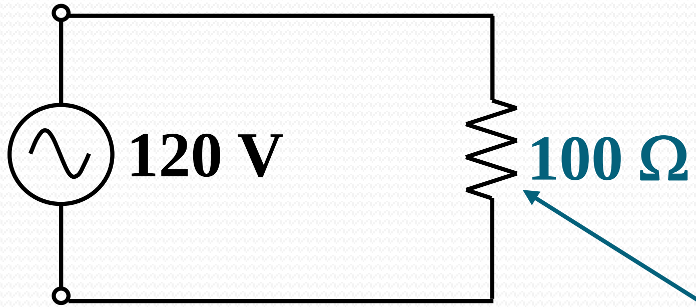
- While the mean-square voltage is useful, more often we use the square root of this quantity, namely the root-mean-square voltage V_{rms}
  - where $V_{rms} = \sqrt{\overline{v^2}}$
  - we can also define $I_{rms} = \sqrt{\overline{i^2}}$
  - it is relatively easy to show that

$$V_{rms} = \frac{1}{\sqrt{2}} \times V_p = 0.707 \times V_p$$

$$I_{rms} = \frac{1}{\sqrt{2}} \times I_p = 0.707 \times I_p$$

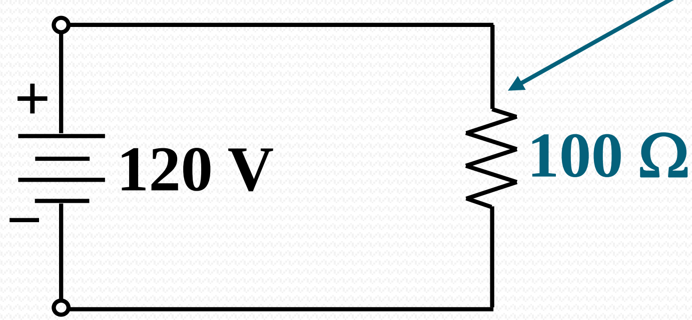
Voltage and Current Values for a Sine Wave

The *default* sine wave ac measurement is V_{rms} .



V_{rms} is the *effective* value.
The heating effect of these two sources is identical.

Same power dissipation



- r.m.s. values are useful because their relationship to average power is similar to the corresponding DC values

$$P_{av} = V_{rms} I_{rms}$$

$$P_{av} = \frac{V_{rms}^{2}}{R}$$

$$P_{av} = I_{rms}^{2} R$$

- **Form factor**
  - for ***any waveform*** the form factor is defined as

$$\text{Form factor} = \frac{\text{r.m.s. value}}{\text{average value}}$$

  - for a ***sine wave*** this gives

$$\text{Form factor} = \frac{0.707\,V_p}{0.637\,V_p} = 1.11$$

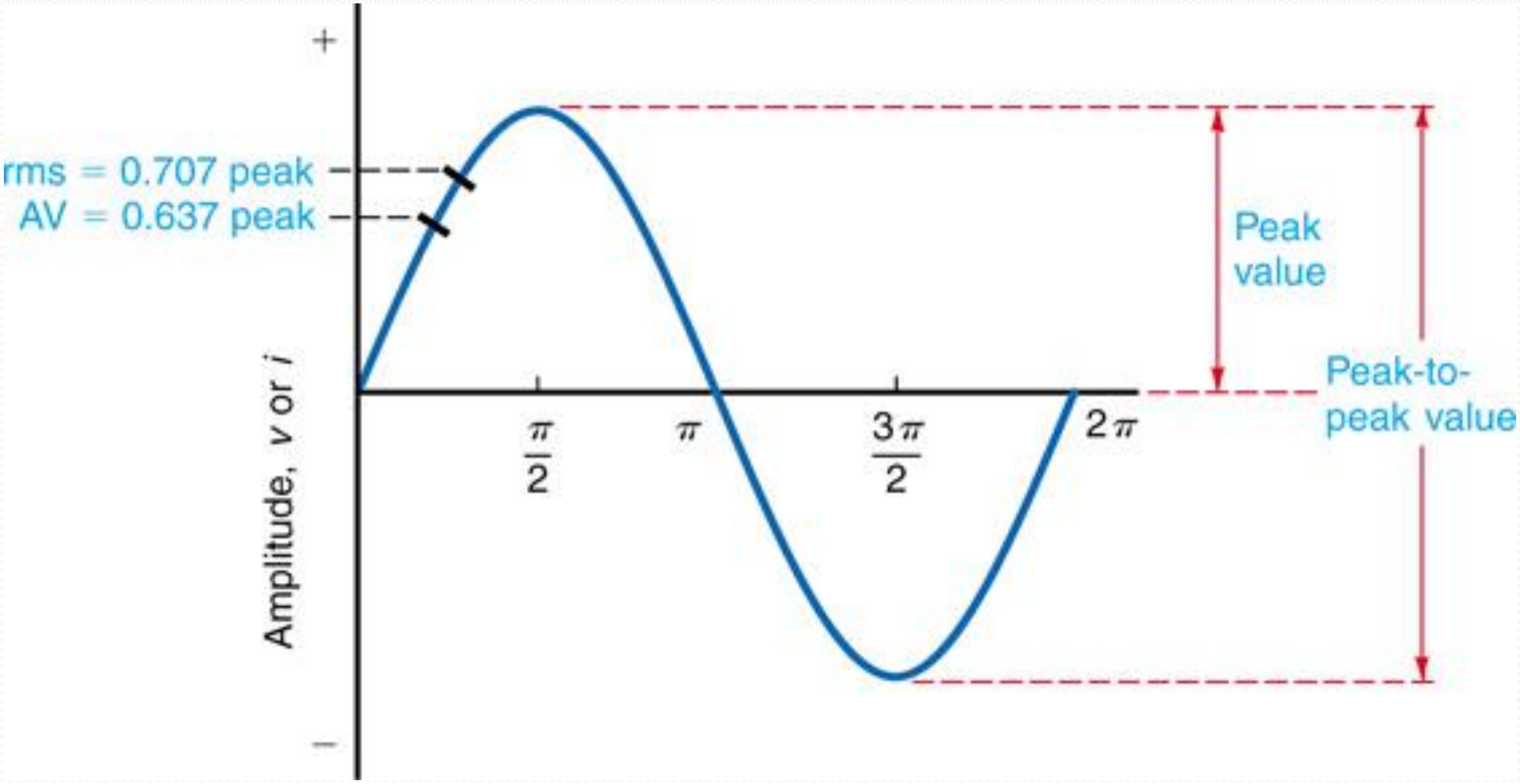
- **Peak factor**
  - for ***any waveform*** the peak factor is defined as

$$\text{Peak factor} = \frac{\text{peak value}}{\text{r.m.s. value}}$$

  - for a ***sine wave*** this gives

$$\text{Peak factor} = \frac{V_p}{0.707\,V_p} = 1.414$$

Voltage and Current Values for a Sine Wave



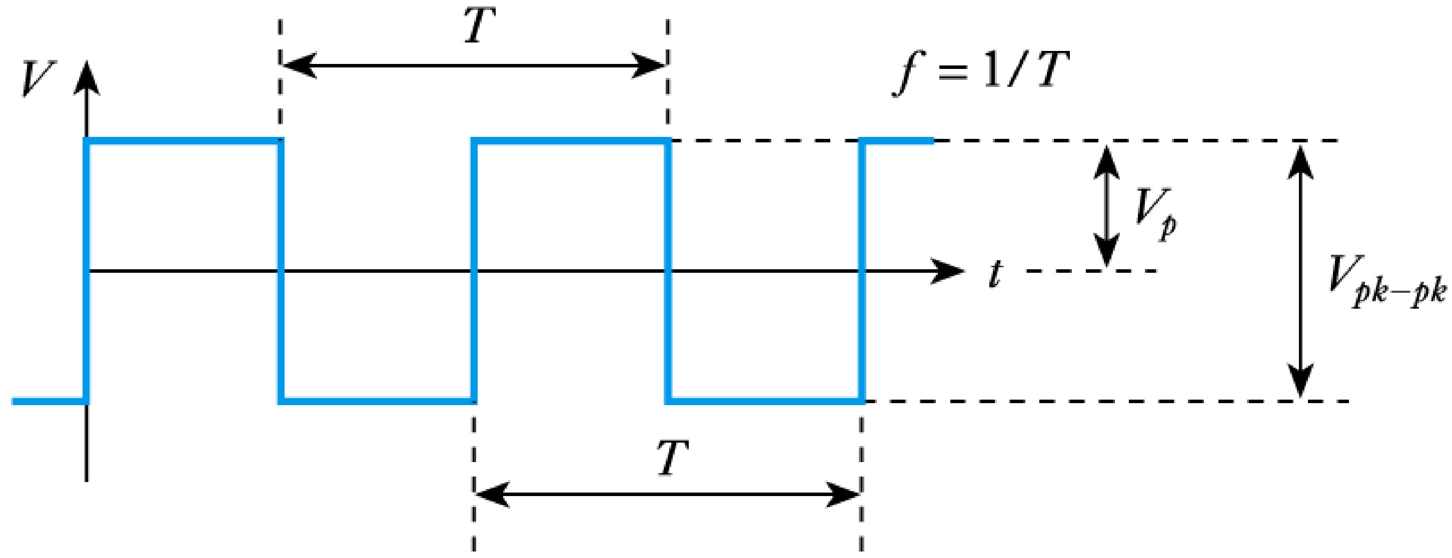
Definitions of important amplitude values for a sine wave of voltage or current.

Voltage and Current Values for a Sine Wave

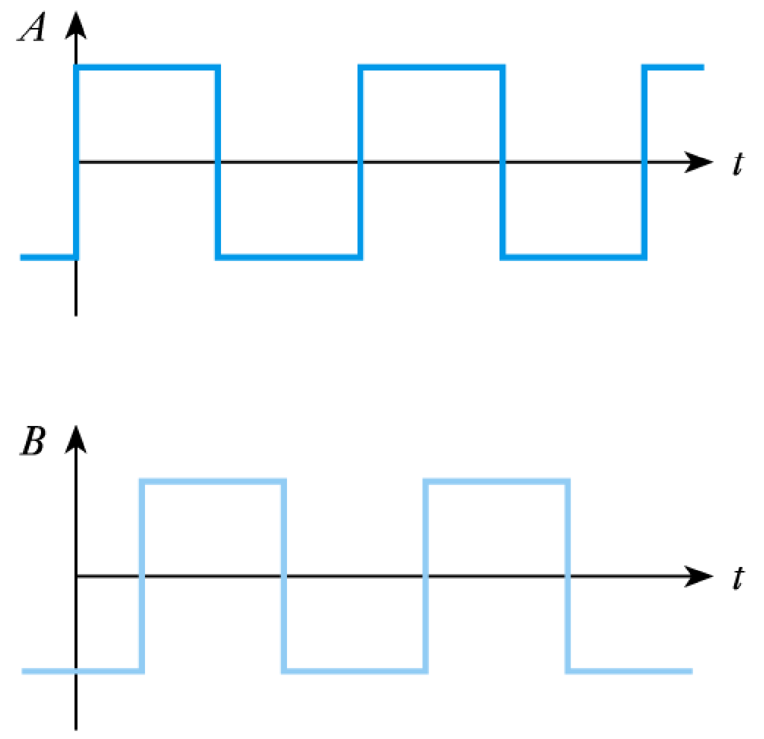
- The average value is 0.637 × peak value.
- The rms value is 0.707 × peak value.
- The peak value is 1.414 × rms value.
- The peak-to-peak value is 2.828 × rms value.

Square Waves

- **Frequency**, **period**, **peak value** and **peak-to-peak value** have the same meaning for all repetitive waveforms



- **Phase angle**
  - we can divide the period into 360° or 2π radians
  - useful in defining phase relationship between signals
  - in the waveforms shown here, *B* lags *A* by 90°
  - we could alternatively give the time delay of one with respect to the other



- **Average and r.m.s. values**
  - the average value of a symmetrical waveform is its average value over the positive half-cycle
  - thus the average value of a symmetrical square wave is equal to its peak value

$$V_{av} = V_p$$

  - similarly, since the instantaneous value of a square wave is either its peak positive or peak negative value, the square of this is the peak value squared, and

$$V_{rms} = V_p$$

- **Form factor** and **peak factor**
  - from the earlier definitions, for a square wave

$$\text{Form factor} = \frac{\text{r.m.s. value}}{\text{average value}} = \frac{V_p}{V_p} = 1.0$$

$$\text{Peak factor} = \frac{\text{peak value}}{\text{r.m.s. value}} = \frac{V_p}{V_p} = 1.0$$

Root-mean squared value of a periodic waveform with period T

$$V_{rms}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} v^2(t)dt$$

Compare to the average power expression

$$P_{avg} = \frac{1}{T}\int_{t_o}^{t_o+T} p(t)dt$$

The average value of the squared voltage

compare

Apply v(t) to a resistor

$$P_{avg} = \frac{1}{T}\int_{t_o}^{t_o+T} p(t)dt = \frac{1}{T}\int_{t_o}^{t_o+T}\left[\frac{v^2(t)}{R}\right]dt = \frac{1}{RT}\int_{t_o}^{t_o+T} v^2(t)dt$$

$$P_{avg} = \frac{V_{rms}^2}{R}$$

rms is based on a power concept, describing the equivalent voltage that will produce a given average power to a resistor

Root-mean squared value of a periodic waveform with period T

$$V_{rms}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} v^2(t)dt$$

For the sinusoidal case $v(t) = V\sin(\omega_o t + \delta),$

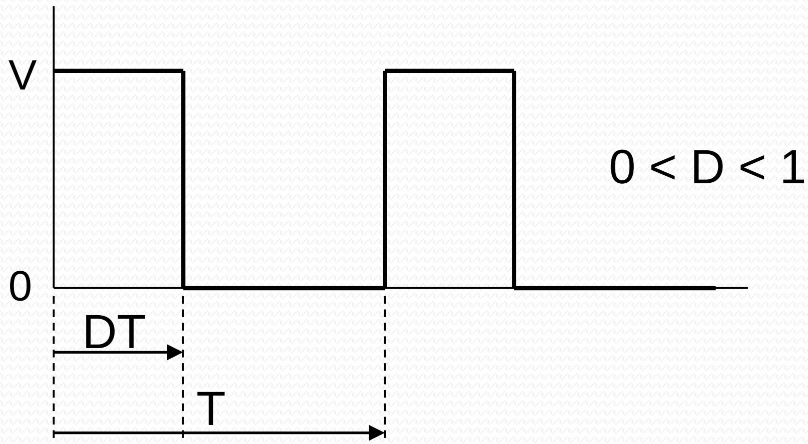
$$V_{rms}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} V^2 \sin^2(\omega_o t + \delta)dt$$

$$V_{rms}^2 = \frac{V^2}{2T}\int_{t_o}^{t_o+T} [1 - \cos 2(\omega_o t + \delta)]dt = \frac{V^2}{2T}\left[t - \frac{\sin 2(\omega_o t + \delta)}{2\omega_o}\right]_{t_o}^{t_o+T}$$

$$V_{rms}^2 = \frac{V^2}{2}, \quad \boxed{V_{rms} = \frac{V}{\sqrt{2}}}$$

RMS of some common periodic waveforms

Duty cycle controller



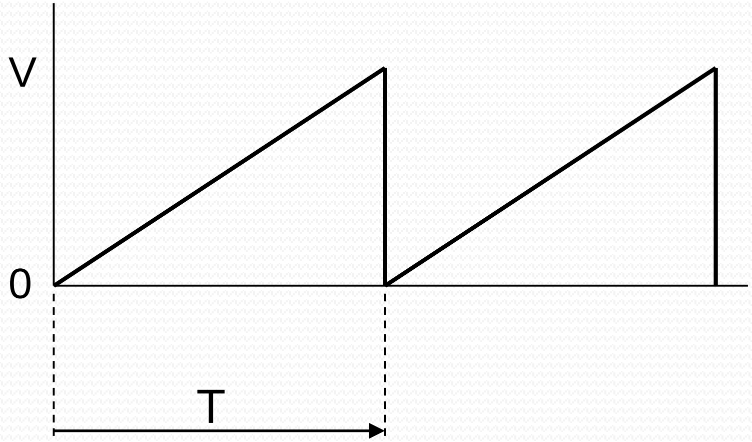
By inspection, this is the average value of the squared waveform

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2(t)dt = \frac{1}{T}\int_0^{DT} V^2 dt = \frac{V^2}{T} \bullet DT = DV^2$$

$$V_{rms} = V\sqrt{D}$$

RMS of common periodic waveforms, cont.

Sawtooth

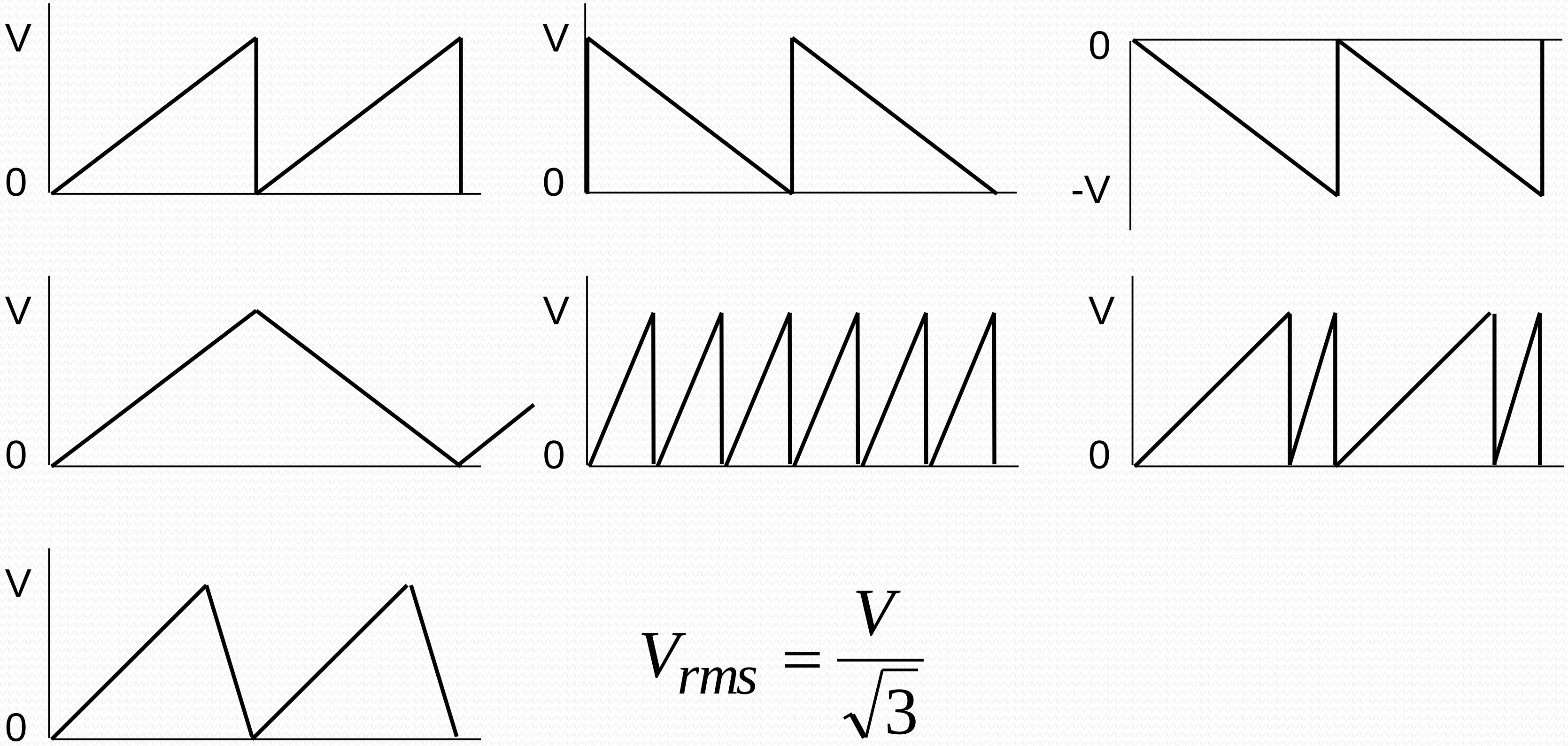


$$V_{rms}^2 = \frac{1}{T}\int_0^T \left[\frac{V}{T}t\right]^2 dt = \frac{V^2}{T^3}\int_0^T t^2 dt = \frac{V^2}{3T^3}t^3\Big|_0^T$$

$$V_{rms} = \frac{V}{\sqrt{3}}$$

RMS of common periodic waveforms, cont.

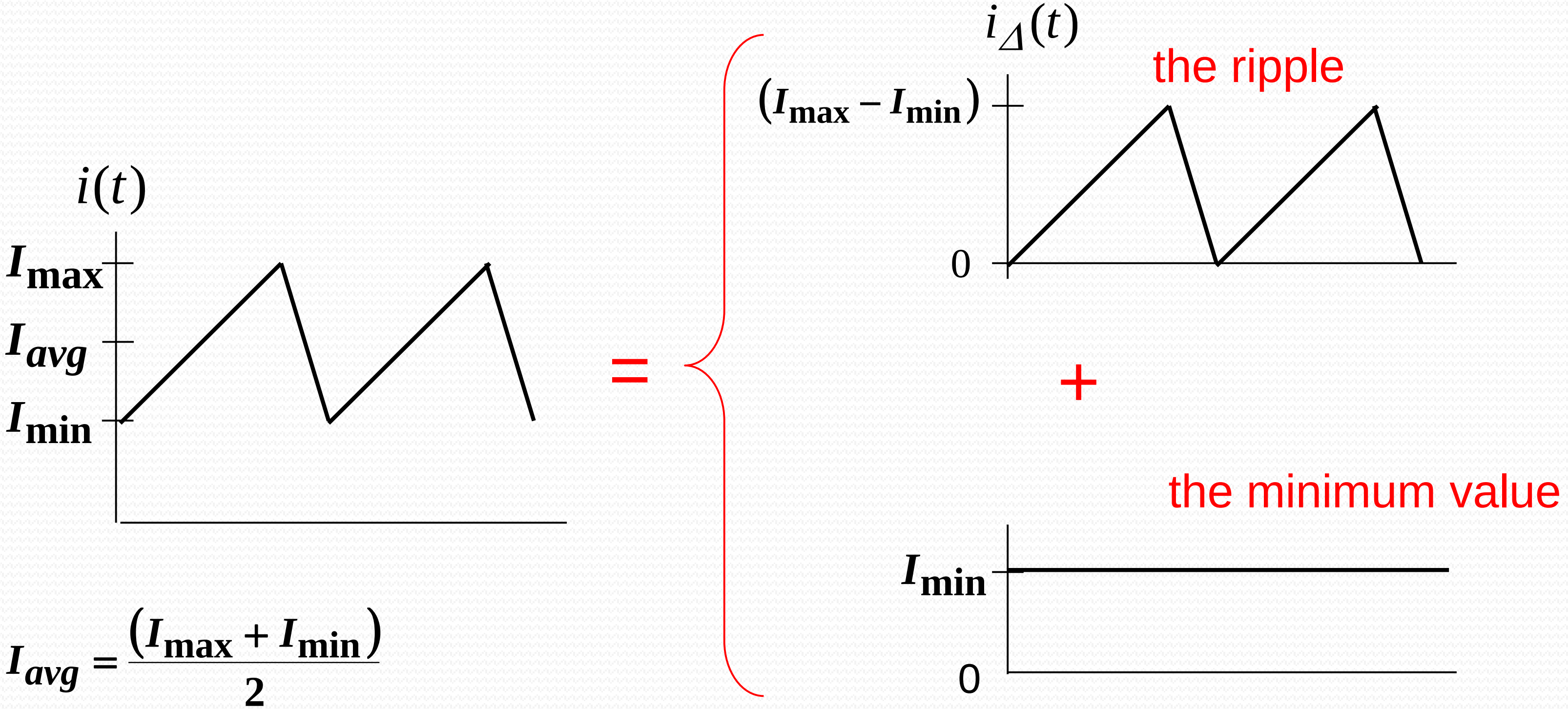
Using the power concept, it is easy to reason that the following waveforms would all produce the same average power to a resistor, and thus their rms values are identical and equal to the previous example



$$V_{rms} = \frac{V}{\sqrt{3}}$$

RMS of common periodic waveforms, cont.

Now, consider a useful example, based upon a waveform that is often seen in DC-DC converter currents. Decompose the waveform into its ripple, plus its minimum value.



$$I_{avg} = \frac{(I_{max} + I_{min})}{2}$$

RMS of common periodic waveforms, cont.

$$I_{rms}^2 = Avg\left\{\left(i_\Delta(t) + I_{\min}\right)^2\right\}$$

$$I_{rms}^2 = Avg\left\{i_\Delta^2(t) + 2i_\Delta(t) \bullet I_{\min} + I_{\min}^2\right\}$$

$$I_{rms}^2 = Avg\left\{i_\Delta^2(t)\right\} + 2I_{\min} \bullet Avg\left\{i_\Delta(t)\right\} + I_{\min}^2$$

$$I_{rms}^2 = \frac{\left(I_{\max} - I_{\min}\right)^2}{3} + 2I_{\min} \bullet \frac{\left(I_{\max} - I_{\min}\right)}{2} + I_{\min}^2$$

Define $I_{PP} = I_{\max} - I_{\min}$

$$I_{rms}^2 = \frac{I_{PP}^2}{3} + I_{\min} I_{PP} + I_{\min}^2$$

RMS of common periodic waveforms, cont.

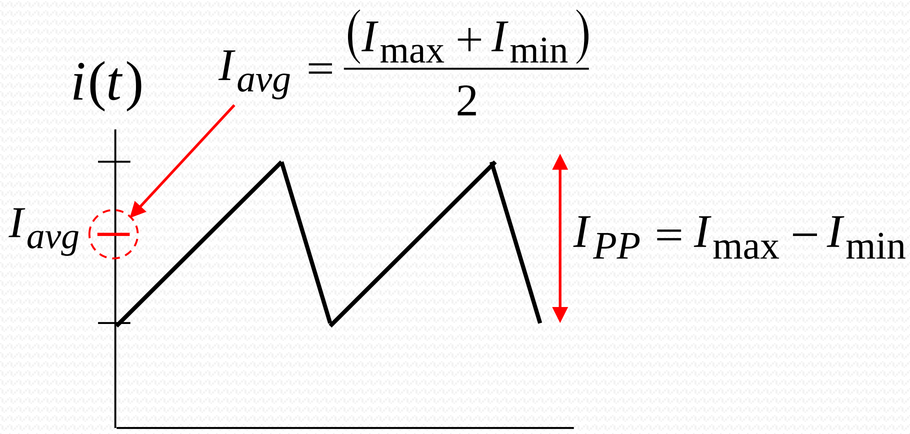
Recognize that $I_{\min} = I_{avg} - \dfrac{I_{PP}}{2}$

$$I_{rms}^2 = \frac{I_{PP}^2}{3} + \left(I_{avg} - \frac{I_{PP}}{2}\right)I_{PP} + \left(I_{avg} - \frac{I_{PP}}{2}\right)^2$$

$$I_{rms}^2 = \frac{I_{PP}^2}{3} + I_{avg}I_{PP} - \frac{I_{PP}^2}{2} + I_{avg}^2 - I_{avg}I_{PP} + \frac{I_{PP}^2}{4}$$

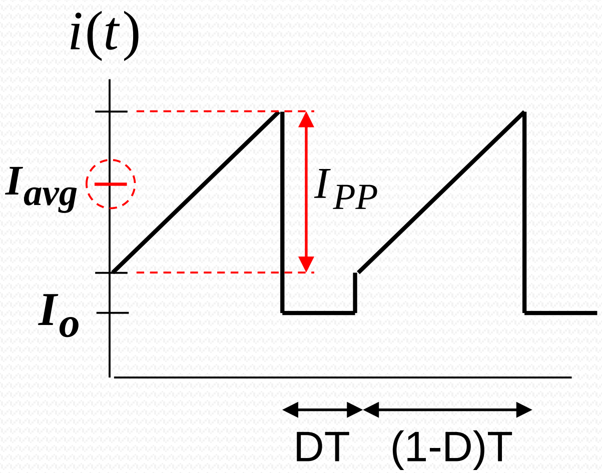
$$I_{rms}^2 = \frac{I_{PP}^2}{3} - \frac{I_{PP}^2}{4} + I_{avg}^2$$

$$I_{rms}^2 = I_{avg}^2 + \frac{I_{PP}^2}{12}$$



RMS of segmented waveforms

Consider a modification of the previous example. A constant value exists during D of the cycle, and a sawtooth exists during (1-D) of the cycle.



In this example, I_{avg} is defined as the average value of the sawtooth portion

$$I_{rms}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} i^2(t)dt = \frac{1}{T}\left[\int_{t_o}^{t_o+DT} i^2(t)dt + \int_{t_o+DT}^{t_o+T} i^2(t)dt\right]$$

$$I_{rms}^2 = \frac{1}{T}\left[DT \bullet \frac{1}{DT}\int_{t_o}^{t_o+DT} i^2(t)dt + (1-D)T \bullet \frac{1}{(1-D)T}\int_{t_o+DT}^{t_o+T} i^2(t)dt\right]$$

RMS of segmented waveforms, cont.

$$I_{rms}^2 = \frac{1}{T}\left[DT \bullet \frac{1}{DT}\int_{t_o}^{t_o+DT} i^2(t)dt + (1-D)T \bullet \frac{1}{(1-D)T}\int_{t_o+DT}^{t_o+T} i^2(t)dt \right]$$

$$I_{rms}^2 = \frac{1}{T}\left[DT \bullet Avg\left\{i^2(t)\right\}_{over\ DT} + (1-D)T \bullet Avg\left\{i^2(t)\right\}_{over\ (1\text{-}D)T} \right]$$

$$I_{rms}^2 = D \bullet Avg\left\{i^2(t)\right\}_{over\ DT} + (1-D) \bullet Avg\left\{i^2(t)\right\}_{over\ (1\text{-}D)T}$$

$$I_{rms}^2 = D \bullet I_o^2 + (1-D) \bullet \left[I_{avg}^2 + \frac{I_{PP}^2}{12} \right] \Longleftrightarrow$$

a weighted average

So, the squared rms value of a segmented waveform can be computed by finding the squared rms values of each segment, weighting each by its fraction of T, and adding

RMS in terms of Fourier Coefficients

$$\left(V_{rms}\right)^2 = V_{avg}^2 + \sum_{k=1}^{\infty} \frac{V_k^2}{2}$$

which means that $V_{rms} \geq \left|V_{avg}\right|$

and that

$V_{rms} \geq \dfrac{\left|V_k\right|}{\sqrt{2}}$ for any k

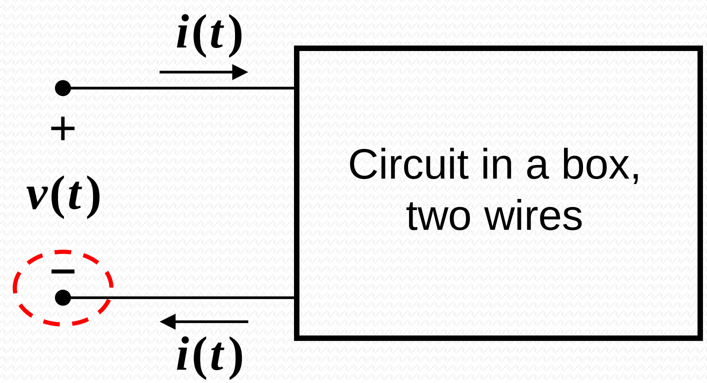
Bounds on RMS

From the power concept, it is obvious that the rms voltage or current can never be greater than the maximum absolute value of the corresponding v(t) or i(t)

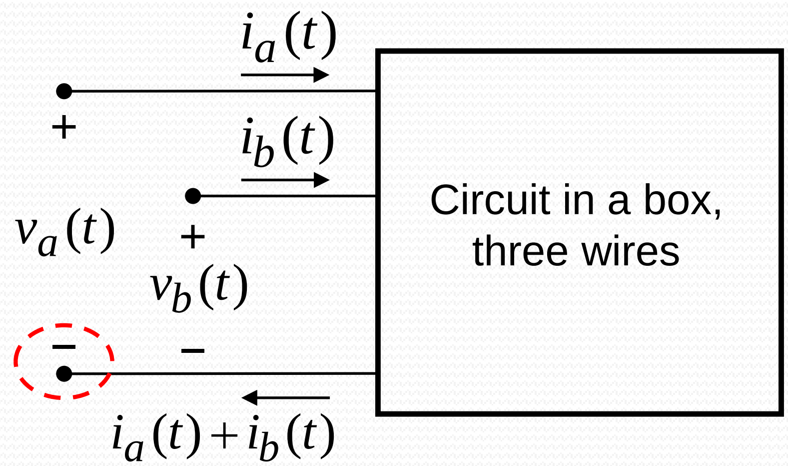
From the Fourier concept, it is obvious that the rms voltage or current can never be less than the absolute value of the average of the corresponding v(t) or i(t)

Instantaneous power p(t) flowing into the box

$$p(t) = v(t) \bullet i(t)$$

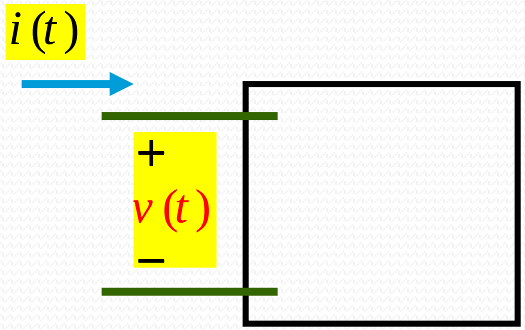


$$p(t) = v_a(t) \bullet i_a(t) + v_b(t) \bullet i_b(t)$$



Any wire can be the voltage reference

Works for any circuit, as long as all N wires are accounted for. There must be (N – 1) voltage measurements, and (N – 1) current measurements.



$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$\Rightarrow \quad i(t) = I_m \cos(\omega t)$$

$$v(t) = V_m \cos(\omega t + \theta_v - \theta_i)$$

$$p(t) = v(t)\,i(t) = \{V_m \cos(\omega t + \theta_v - \theta_i)\}\{I_m \cos(\omega t)\}$$

$$= V_m I_m \cos(\omega t + \theta_v - \theta_i)\cos(\omega t)$$

Since

$$\cos\alpha \cos\beta = \frac{1}{2}\cos(\alpha - \beta) + \frac{1}{2}\cos(\alpha + \beta)$$

Therefore

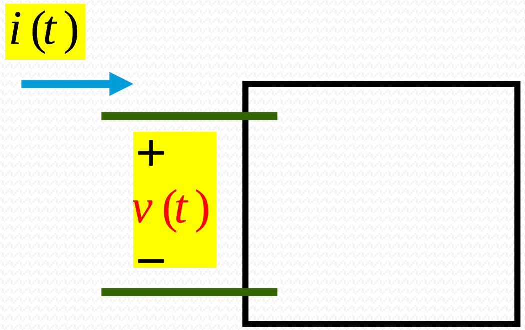
$$p(t) = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(2\omega t + \theta_v - \theta_i)$$

Since

$$\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$$

$$\Rightarrow \quad \cos(2\omega t + \theta_v - \theta_i) = \cos(\theta_v - \theta_i)\cos(2\omega t) - \sin(\theta_v - \theta_i)\sin(2\omega t)$$

$$\Rightarrow \quad p(t) = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos(2\omega t) - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin(2\omega t)$$



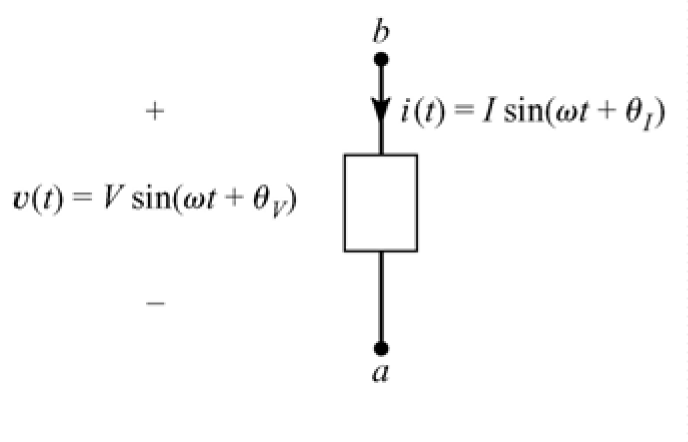
$$i(t) = I_m \cos(\omega t)$$

$$v(t) = V_m \cos(\omega t + \theta_v - \theta_i)$$

You can see that that the frequency of the Instantaneous
power is twice the frequency of the voltage or current

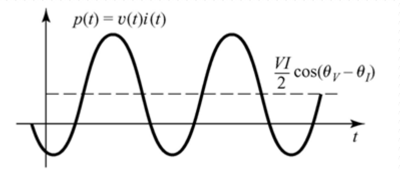
$$p(t) = \underbrace{\frac{V_m I_m}{2}\cos(\theta_v - \theta_i)}_{P} + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos(2\omega t) - \underbrace{\frac{V_m I_m}{2}\sin(\theta_v - \theta_i)}_{Q}\sin(2\omega t)$$

$$p(t) = P - P\cos(2\omega t) + Q\sin(2\omega t)$$



The instantaneous power

$$p(t) = P - P\cos(2\omega t) + Q\sin(2\omega t)$$



$$P = \frac{VI}{2}\cos(\theta_V - \theta_I)$$ **real (or average) power (Watts)**

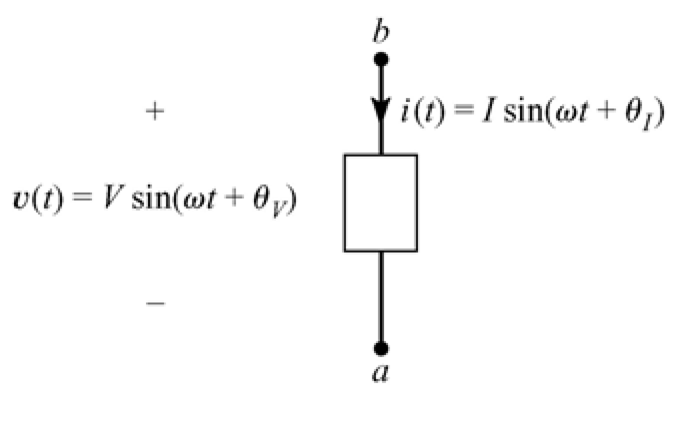
Which is the actual power absorb by the element

Examples Electric Heater , Electric Stove , oven Toasters, Iron ...etc

$$Q = \frac{VI}{2}\sin(\theta_V - \theta_I)$$ **reactive power**

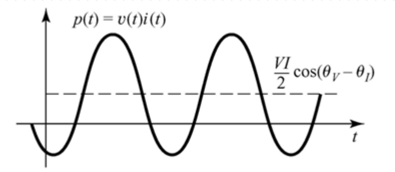
Which is the reactive power absorb or deliver by the element

Reactive power represents energy stored in reactive elements (inductors and capacitors). Its unit is **V**olt **A**mpere **R**eactive (VAR)



The instantaneous power

$$p(t) = P - P\cos(2\omega t) + Q\sin(2\omega t)$$



$$P = \frac{VI}{2}\cos(\theta_V - \theta_I)$$ **real (or average) power (Watts)**

$$Q = \frac{VI}{2}\sin(\theta_V - \theta_I)$$ **reactive power (<u>V</u>olt <u>A</u>mpere <u>R</u>eactive (VAR))**

Complex Power

Previously, we found it convenient to introduce sinusoidal voltage and current in terms of the complex number the **phasor**

Definition

Let the **complex power** be the complex sum of real power and reactive power

$$\hat{\mathrm{P}} = P + jQ$$

were

$\hat{\mathrm{P}}$ is the complex power

P is the average power

Q is the reactive power

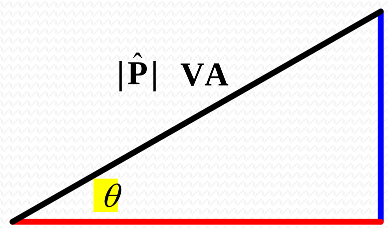
Advantages of using complex power $\hat{P} = P + jQ$

– We can compute the average and reactive power from the complex power ***S***

$$P = \mathrm{Real}\{\hat{P}\} = \frac{VI}{2}\cos(\theta_v - \theta_i) \qquad Q = \mathrm{Imag}\{\hat{P}\} = \frac{VI}{2}\sin(\theta_v - \theta_i)$$

– complex power $\hat{P}$ provide a geometric interpretation

$$\hat{P} = P + jQ = |\hat{P}|\, e^{j\theta}$$



were

$$|\hat{P}| = \sqrt{P^2 + Q^2}$$ Is called **apparent power (VA)**

$$\theta = \tan^{-1}\left(\frac{Q}{P}\right) = \tan^{-1}\left(\frac{VI\sin(\theta_v - \theta_i)}{VI\cos(\theta_v - \theta_i)}\right) = \tan^{-1}\left(\frac{\sin(\theta_v - \theta_i)}{\cos(\theta_v - \theta_i)}\right) = \tan^{-1}\left(\tan(\theta_v - \theta_i)\right) = \underbrace{\theta_v - \theta_i}_{\text{power factor angle}}$$

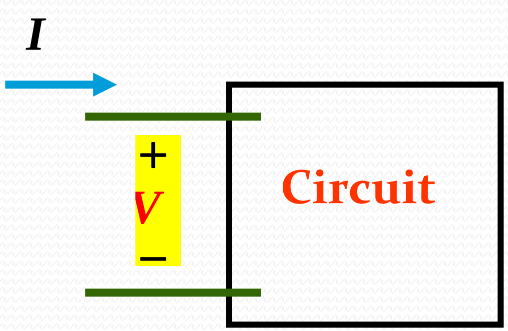
The geometric relations for a right triangle mean the four power triangle dimensions ($|\hat{P}|$, P, Q, θ) can be determined if **any two** of the four are known

Power Calculations

$$\hat{P} = P + jQ \quad = \frac{VI}{2}\cos(\theta_v - \theta_i) + j\,\frac{VI}{2}\sin(\theta_v - \theta_i)$$

$$= \frac{VI}{2}\left[\cos(\theta_v - \theta_i) + j\,\sin(\theta_v - \theta_i)\right] \quad = \frac{VI}{2}e^{j(\theta_v - \theta_i)} \quad = \frac{1}{2}Ve^{j(\theta_v)}Ie^{-j(\theta_i)} \quad \boxed{= \frac{1}{2}\boldsymbol{V}\boldsymbol{I}^*}$$

were $\boldsymbol{I}^*$ Is the conjugate of the current phasor $\boldsymbol{I}$

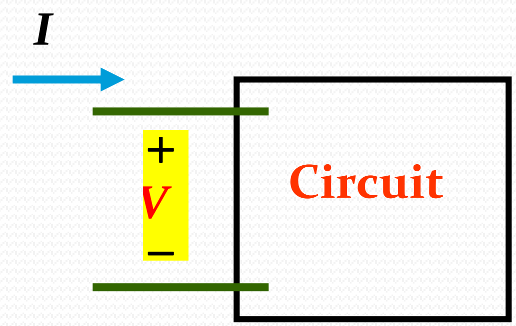


$$\hat{\mathbf{P}} = \frac{1}{2}\boldsymbol{V}\boldsymbol{I}^*$$

Since $\hat{P} = \frac{1}{2}\boldsymbol{V}\boldsymbol{I}^*$ $\Rightarrow$ $\hat{P} = \frac{1}{2}(\boldsymbol{Z}\boldsymbol{I})\boldsymbol{I}^* \quad = \frac{1}{2}\boldsymbol{Z}\boldsymbol{I}\boldsymbol{I}^* \quad \boxed{= \frac{1}{2}\boldsymbol{Z}\left|\boldsymbol{I}\right|^2}$

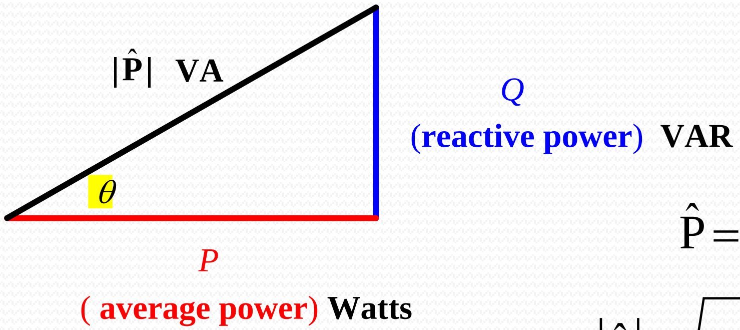
Similarly $\hat{P} = \frac{1}{2}\boldsymbol{V}\boldsymbol{I}^*$ $\Rightarrow$ $\hat{P} = \frac{1}{2}\boldsymbol{V}\left(\frac{\boldsymbol{V}}{\boldsymbol{Z}}\right)^* \quad = \frac{1}{2}\frac{\boldsymbol{V}\boldsymbol{V}^*}{\boldsymbol{Z}^*} \quad \boxed{= \frac{1}{2}\frac{\left|\boldsymbol{V}\right|^2}{\boldsymbol{Z}^*}}$

Power Calculations Summery



$$\hat{\mathbf{P}} = P + jQ \quad = \frac{VI}{2}\cos(\theta_v - \theta_i) + j\,\frac{VI}{2}\sin(\theta_v - \theta_i)$$

$$\hat{\mathbf{P}} = \frac{1}{2}\,\boldsymbol{VI}^* \quad \boxed{= \frac{1}{2}\boldsymbol{Z}\left|\boldsymbol{I}\right|^2} \quad \boxed{= \frac{1}{2}\frac{\left|\boldsymbol{V}\right|^2}{\boldsymbol{Z}^*}}$$



$$\hat{\mathrm{P}} = P + jQ \quad = \left|\hat{\mathrm{P}}\right| e^{j\theta}$$

$\left|\hat{\mathrm{P}}\right| = \sqrt{P^2 + Q^2}$ Is called **apparent power (VA)**

$$\boxed{\theta = \underbrace{\theta_v - \theta_i}_{\text{power factor angle}}}$$

In any circuit, conservation of complex power is achieved

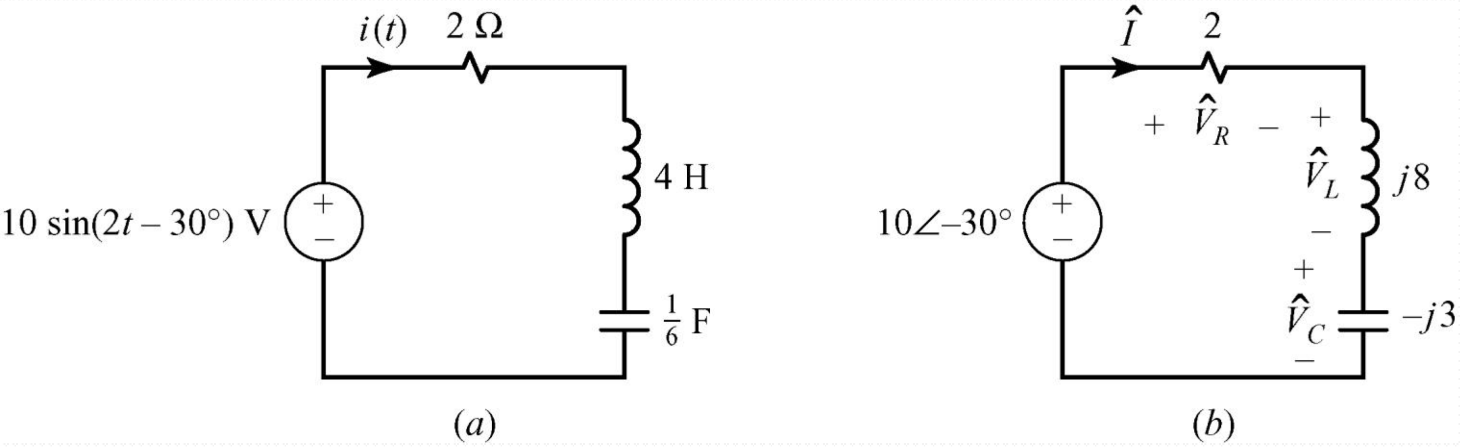
$$\sum_{\text{all circuit elements}} \hat{P}_i = 0$$

This implies that in any circuit, conservation of average power and Conservation of reactive power are achieved

$$\sum_{\text{all circuit elements}} Q_i = 0 \qquad \sum_{\text{all circuit elements}} P_{i,AV} = 0$$

However, the apparent power (the magnitude of the complex power) is not conserved

Ex: Determine the average and reactive power delivered by the source.



The phasor current leaving the source is $\hat{I} = \dfrac{10\angle -30^\circ}{2 + j8 - j3} = 1.86\angle -98.2^\circ$

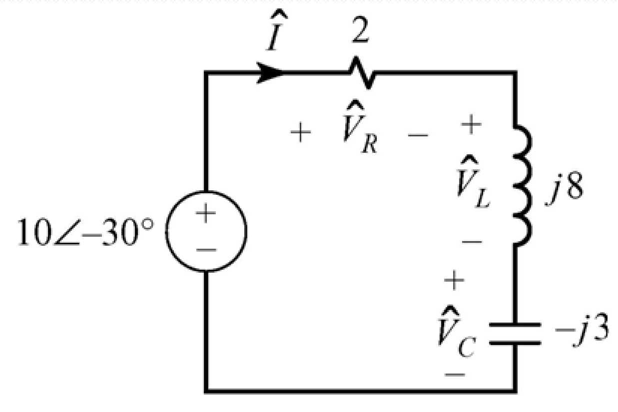
The average power delivered by the source is:

$$P_{AV,source} = \frac{1}{2}\text{Re}\left[(10\angle -30)(1.86\angle -98.2)^*\right]$$

$$= \frac{1}{2}\text{Re}\left[(10\angle -30)(1.86\angle 98.2)\right]$$

$$= 9.28\cos(-30 + 98.2) = 3.45\text{W}$$

The reactive power delivered by the source is:



$$Q_{\text{source}} = \frac{1}{2}\operatorname{Im}\left[(10\angle -30)(1.86\angle -98.2)^*\right]$$

$$= \frac{1}{2}\operatorname{Im}\left[(10\angle -30)(1.86\angle 98.2)\right]$$

$$= 9.28\sin(-30+98.2) = 8.62 \ \ \textbf{VAR}$$

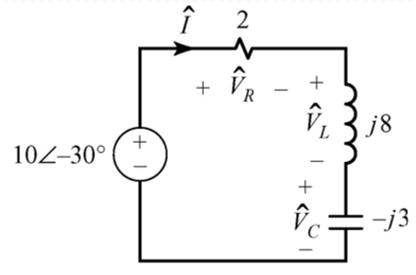
And the complex power delivered by the source is

$$\hat{P}_{\text{source}} = P_{\text{AV,source}} + jQ_{\text{source}} \quad = 3.45 + j8.62\text{VA}$$

Determine the average power and reactive power delivered to each element

The voltage across the elements are:

$\hat{I} = 1.86\angle -98.2^\circ$



$$\hat{V}_R = 2\hat{I} = 2(1.86\angle -98.2^\circ) = 3.71\angle -98.2 \ \ \text{V}$$

$$\hat{V}_L = j8\hat{I} = (j8)(1.86\angle -98.2^\circ) = (8\angle 90.0^\circ)(1.86\angle -98.2^\circ)$$

$$= 14.86\angle -8.2 \ \ \text{V}$$

$$\hat{V}_C = -j3\hat{I} = (-j3)(1.86\angle -98.2^\circ) = (3\angle -90.0^\circ)(1.86\angle -98.2^\circ) = 5.57\angle -188.2 \ \ \text{V}$$

Thus the complex power delivered to each element is

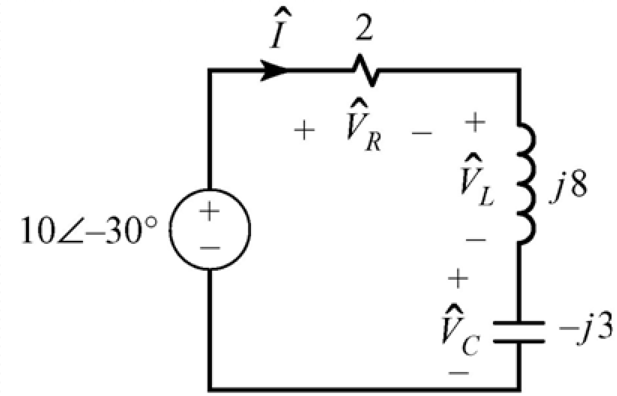
$$\hat{P}_R = \frac{1}{2}\hat{V}_R \hat{I}* = \frac{1}{2}(3.71\angle -98.2)(1.86\angle -98.2^\circ)^* = \frac{1}{2}(3.71\angle -98.2)(1.86\angle 98.2^\circ)$$

$$= 3.45\angle 0^\circ \qquad = 3.45 + j0 \ \ \text{VA}$$

$$\hat{P}_L = \frac{1}{2}\hat{V}_L \hat{I}* = \frac{1}{2}(14.86\angle -8.2)(1.86\angle -98.2^\circ)^* = 13.79\angle 90^\circ \qquad = 0 + j13.79 \ \ \text{VA}$$

$$\hat{P}_C = \frac{1}{2}\hat{V}_C \hat{I}* = \frac{1}{2}(5.57\angle -188.2)(1.86\angle -98.2^\circ)^* = 5.17\angle -90^\circ \qquad = 0 - j5.17 \ \ \text{VA}$$

show that conservation of complex power, average power, and reactive power is achieved.

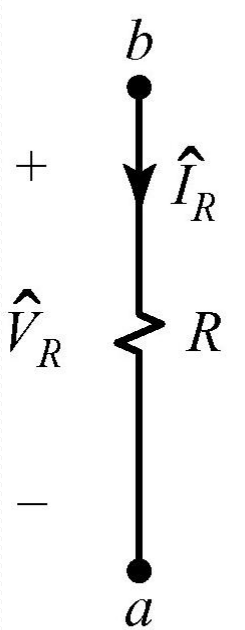


$$\hat{P}_{source} = \hat{P}_R + \hat{P}_L + \hat{P}_C$$
$$3.45 + j8.62 = 3.45 + j0 \quad + 0 + j13.79 \quad + 0 - j5.17$$

$$P_{AV,source} = P_{AV,R} + P_{AV,L} + P_{AV,C}$$
$$3.45 = 3.45 + 0 + 0$$

$$Q_{source} = Q_R + Q_L + Q_C$$
$$8.62 = 0 + 13.79 - 5.17$$

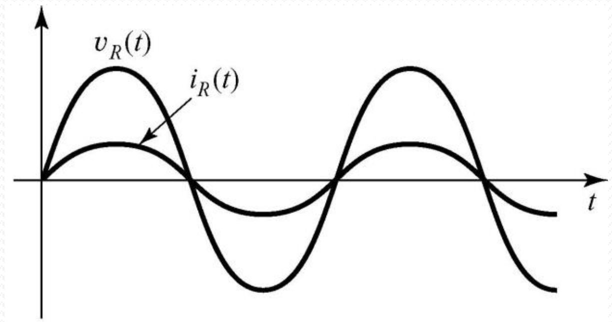
Power Relations for the Resistor



$$\hat{V}_R = R\hat{I}_R$$

$$\hat{P} = P_{AV} + j0$$

$$P_{AV} = \frac{V_R^2}{2R} = \frac{1}{2} I_R^2 R$$

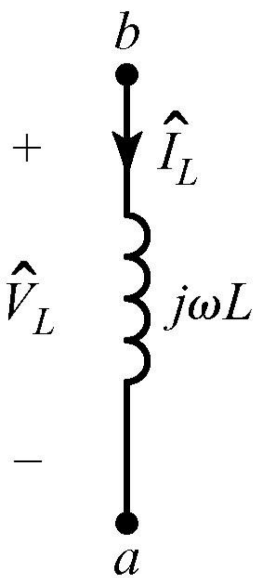


The voltage and current are in phase so $\theta_V - \theta_I = 0^\circ$

Average power is: $$P_{AV,R} = \frac{1}{2}\frac{V_R^2}{R} = \frac{1}{2} I_R^2 R$$

Reactive power is zero for resistor $$Q_R = 0$$

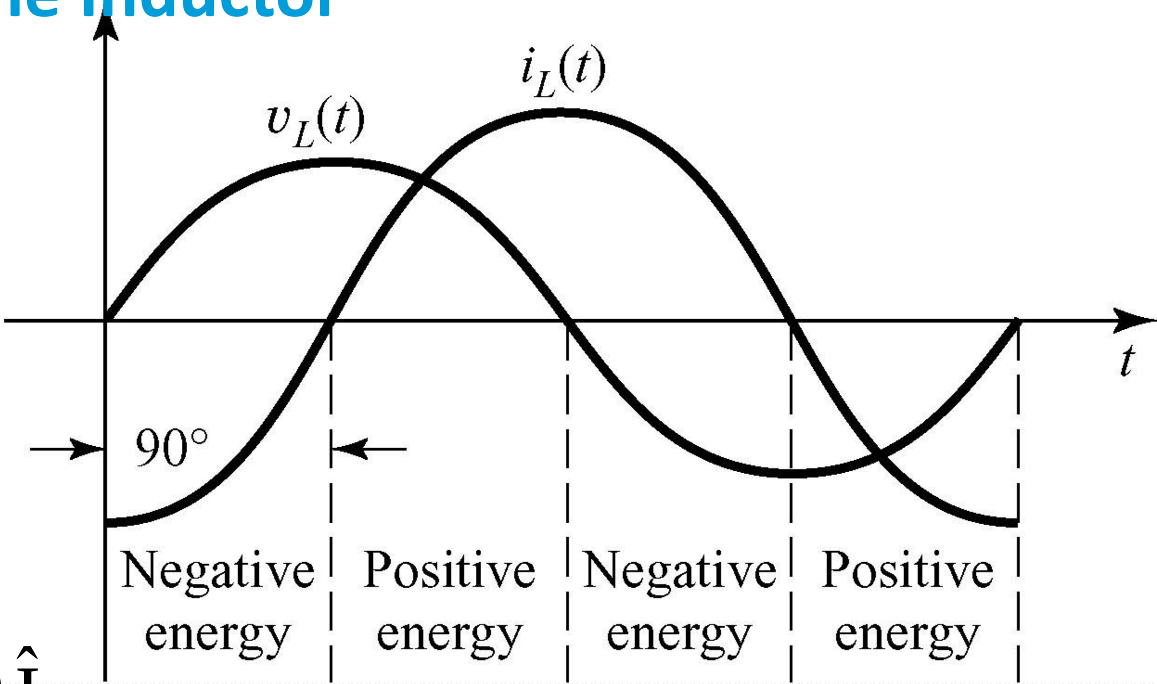
Power Relations for the Inductor



$$\hat{V}_L = j\omega L \hat{I}_L$$

$$\hat{P} = 0 + jQ$$

$$Q = \frac{V_L I_L}{2}$$



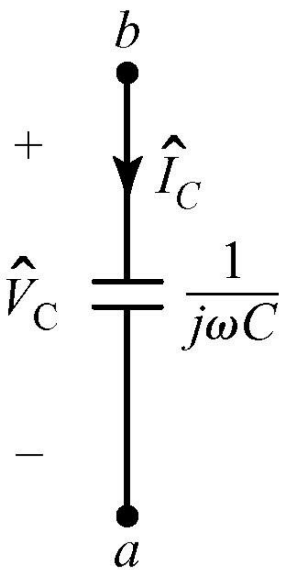
$$\hat{V}_L = j\omega L \hat{I}_L = \left(\omega L \angle 90^\circ\right)\hat{I}_L$$

The voltage leads the current by 90 so that $\theta_V - \theta_I = 90^\circ$

$$P_{AV,L} = \frac{1}{2} V_L I_L \cos 90^\circ = 0$$

$$Q_L = \frac{1}{2} V_L I_L \sin 90^\circ = \frac{1}{2} V_L I_L$$

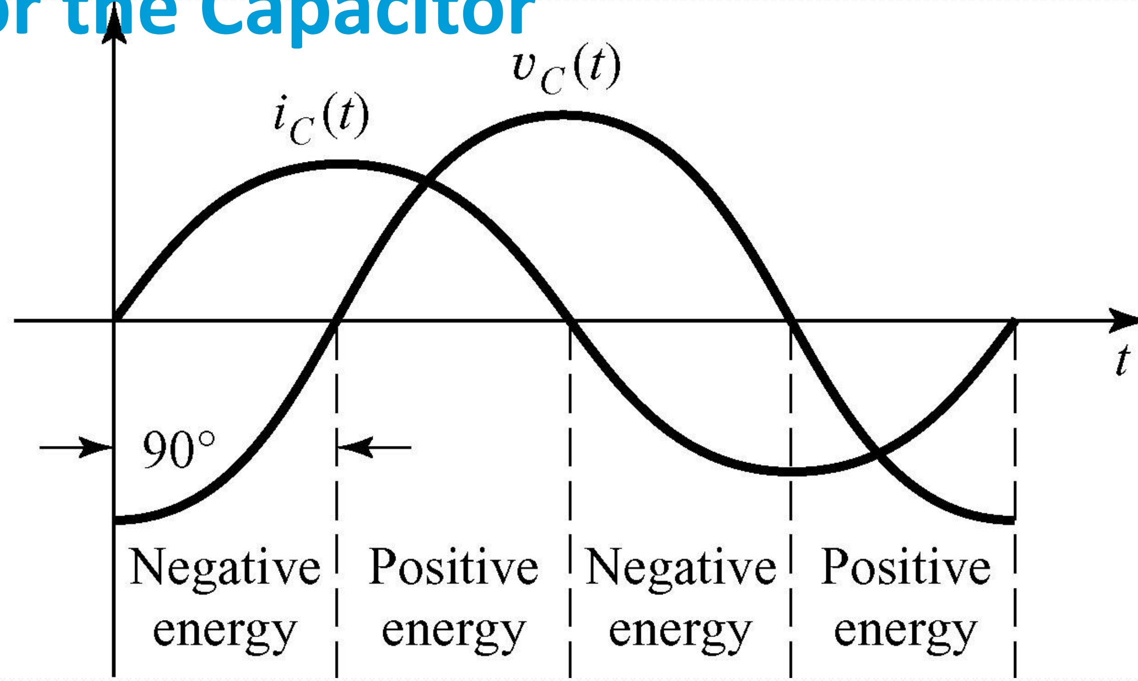
Power Relations for the Capacitor



$$\hat{I}_C = j\omega C \hat{V}_C$$

$$\hat{P} = 0 + jQ$$

$$Q = -\frac{V_C I_C}{2}$$



The current leads the voltage by 90 so that $\theta_V - \theta_I = -90^\circ$

$$\hat{I}_C = j\omega C \hat{V}_C = \left(\omega C \angle 90^\circ\right)\hat{V}_C$$

$$\hat{V}_C = \frac{1}{j\omega C}\hat{I}_C = -j\frac{1}{\omega C}\hat{I}_C = \left(\frac{1}{\omega C}\angle -90^\circ\right)\hat{I}_C$$

$$P_{AV,C} = \frac{1}{2}V_C I_C \cos\left(-90^\circ\right) = 0$$

$$Q_C = \frac{1}{2}V_C I_C \sin\left(-90^\circ\right) = -\frac{1}{2}V_C I_C$$

The power factor

Recall the Instantaneous power $p(t)$

$$p(t) = \underbrace{\frac{V_m I_m}{2}\cos(\theta_v - \theta_i)}_{P \text{ average power}} + \underbrace{\frac{V_m I_m}{2}\cos(\theta_v - \theta_i)}_{P \text{ average power}}\cos(2\omega t) - \underbrace{\frac{V_m I_m}{2}\sin(\theta_v - \theta_i)}_{Q \text{ reactive power}}\sin(2\omega t)$$

$$= P + P\cos(2\omega t) - Q\sin(2\omega t)$$

The angle $\theta_v - \theta_i$ plays a role in the computation of both **average** and **reactive** power

The angle $\theta_v - \theta_i$ is referred to as the **power factor angle**

We now define the following :

The **power factor** $\quad \mathbf{pf} = \cos(\theta_v - \theta_i)$

The **power factor** $\mathbf{pf} = \cos(\theta_v - \theta_i)$

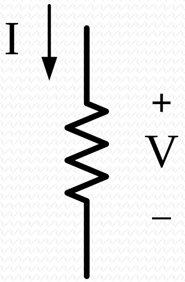
Knowing the power factor **pf** does not tell you the power factor angle , because

$$\cos(\theta_v - \theta_i) = \cos(\theta_i - \theta_v)$$

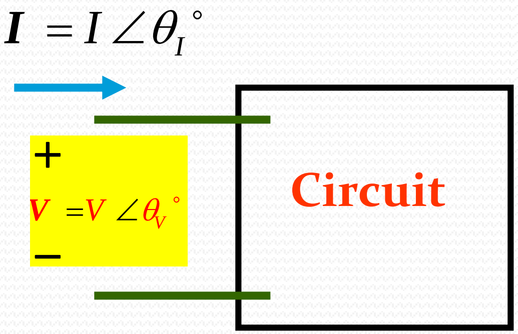
To completely describe this angle, we use the descriptive phrases **lagging power factor** and **leading power factor**

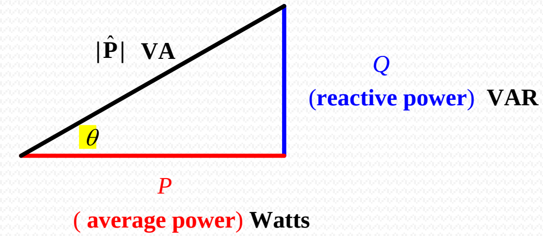
Lagging power factor implies that current **lags** voltage hence an inductive load

Leading power factor implies that current **leads** voltage hence a capacitive load

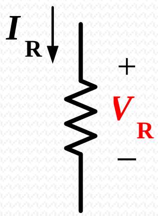


$$P = VI = \frac{V^2}{R} = RI^2$$

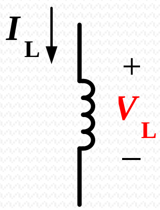




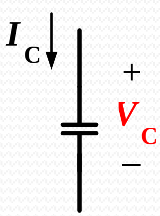
$$\hat{\mathbf{P}} = \frac{1}{2}\boldsymbol{V}\boldsymbol{I}^* = P + jQ = \frac{VI}{2}\cos(\theta_v - \theta_i) + j\,\frac{VI}{2}\sin(\theta_v - \theta_i) = \frac{1}{2}\frac{|\boldsymbol{V}|^2}{Z^*} = \frac{1}{2}Z|\boldsymbol{I}|^2$$



$$\hat{\mathbf{P}}_R = \frac{1}{2}\boldsymbol{V}_R\boldsymbol{I}_R^* \boxed{= \frac{V_R I_R}{2}} \boxed{= \frac{1}{2}RI_R^2}$$

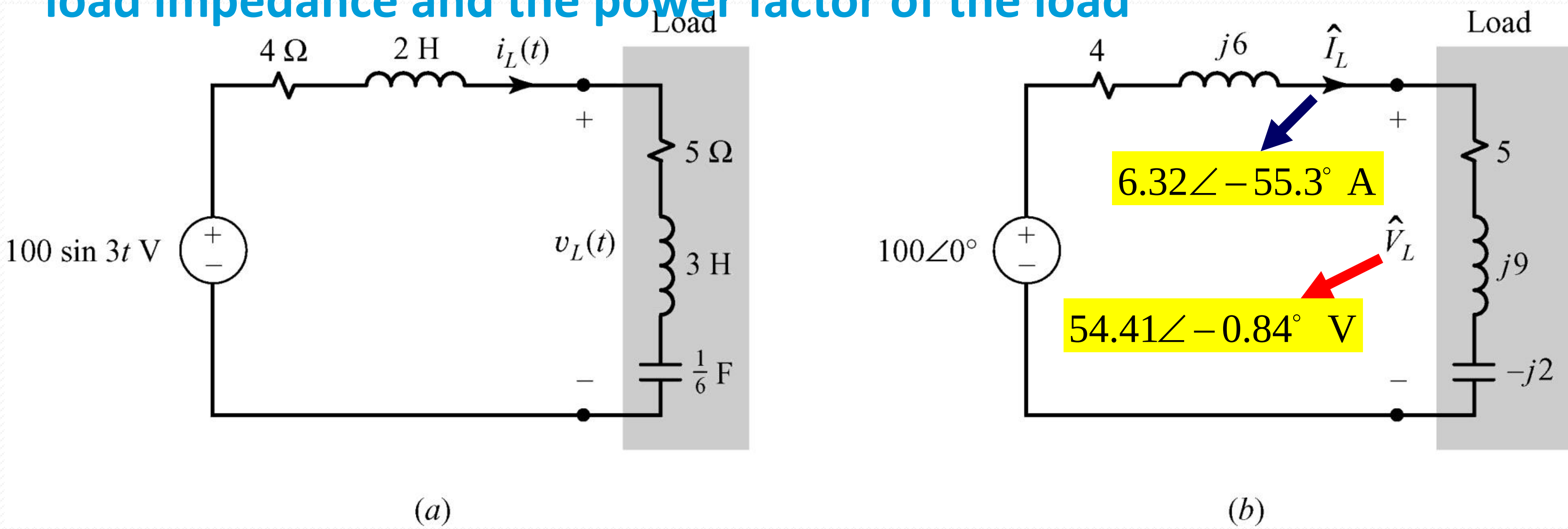


$$\hat{\mathbf{P}}_L = \frac{1}{2}\boldsymbol{V}_L\boldsymbol{I}_L^* \boxed{= \frac{V_L I_L}{2}}$$



$$\hat{\mathbf{P}}_C = \frac{1}{2}\boldsymbol{V}_C\boldsymbol{I}_C^* \boxed{= \frac{V_C I_C}{2}}$$

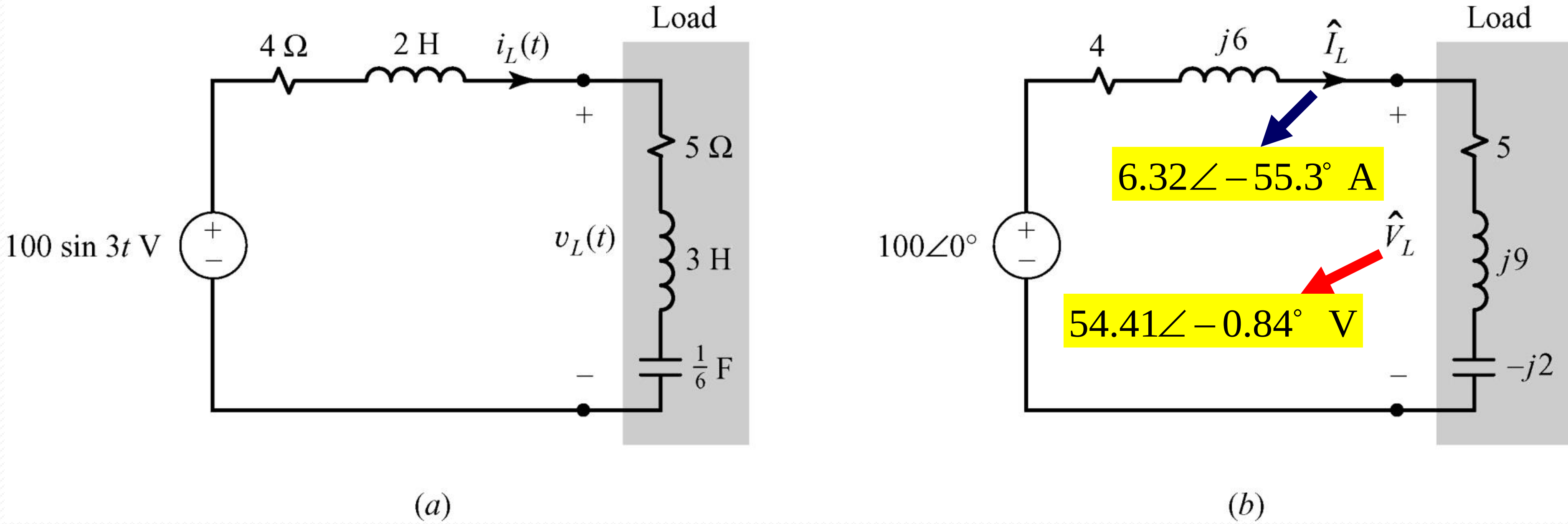
EX: Determine the average and reactive powers delivered to the load impedance and the power factor of the load



$$\hat{V}_L = \frac{5 + j9 - j2}{4 + j6 + 5 + j9 - j2} 100\angle 0^\circ \quad = 54.41\angle -0.84^\circ \quad V$$

$$\hat{I}_L = \frac{100\angle 0^\circ}{4 + j6 + 5 + j9 - j2} \quad = 6.32\angle -55.3^\circ \quad A$$

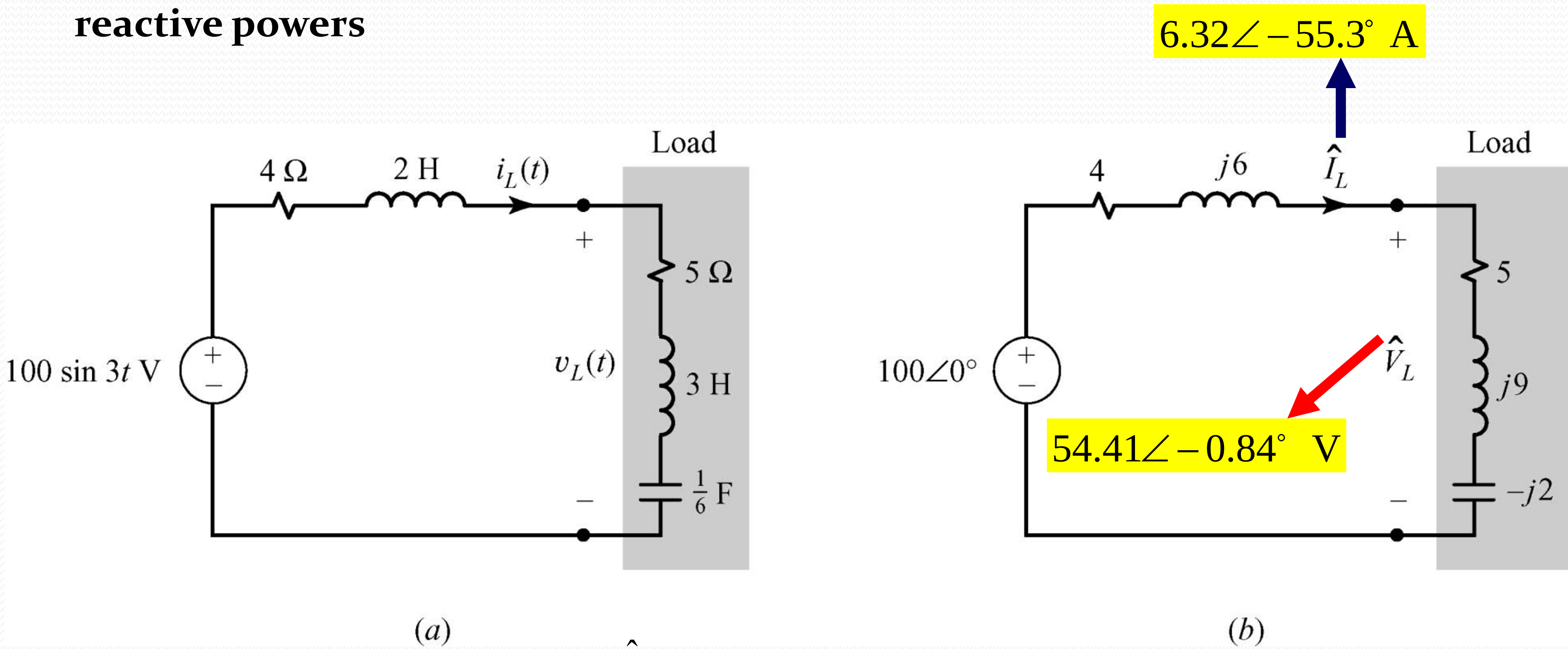
Average power $\quad P = \frac{V_{load} I_{load}}{2} \cos(\theta_v - \theta_i) = \frac{(54.41)(6.32)}{2} \cos(-0.84^\circ + 55.3^\circ) = 100\ W$



Average power $P = \frac{V_{load} I_{load}}{2}\cos(\theta_v - \theta_i) = \frac{(54.41)(6.32)}{2}\cos(-0.84^\circ + 55.3^\circ)$ $= 100$ W

OR $P = \frac{1}{2} I_R^2 \times R = \frac{1}{2}\left|\hat{I}_L\right|^2 \times 5 = \frac{1}{2}(6.32)^2 \times 5$ $= 100$W

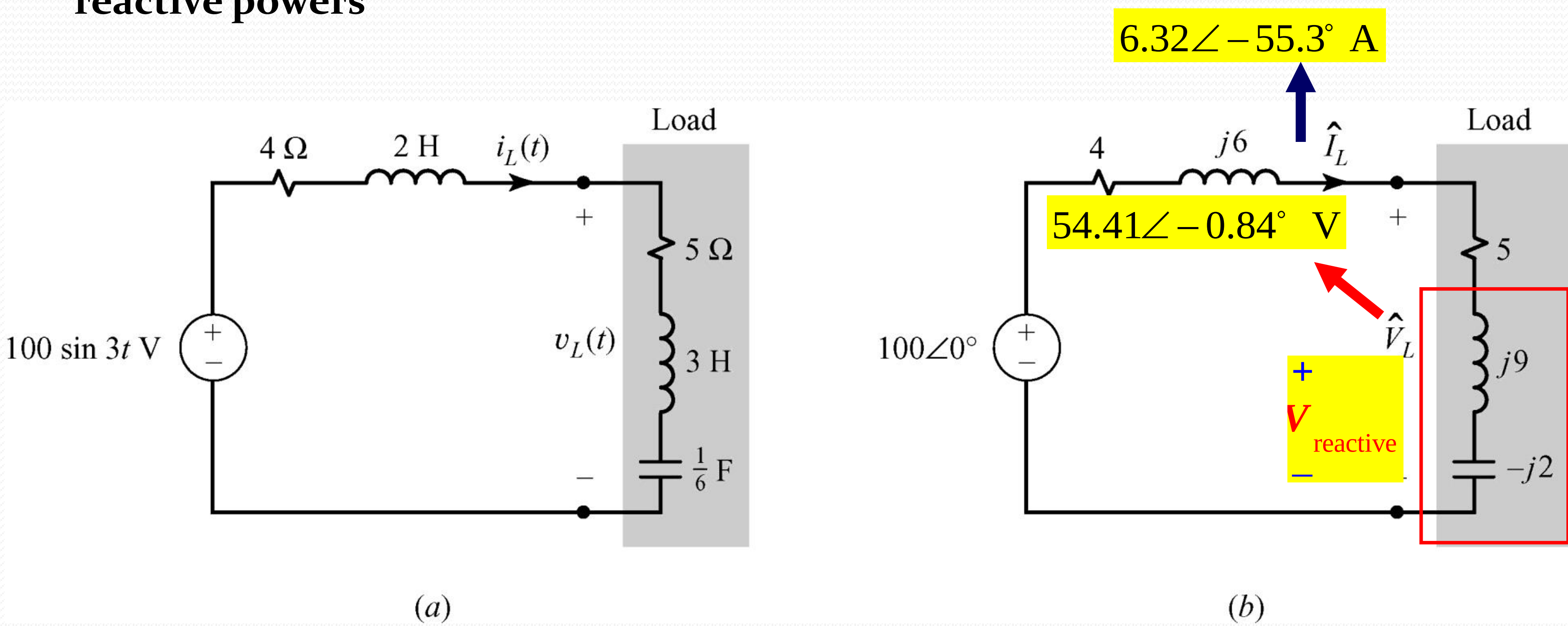
reactive powers



$\hat{V}_L = 54.41\angle -0.84^{\circ}$ V $\quad\quad \hat{I}_L = 6.32\angle -55.3^{\circ}$ A

$$Q_{load} = \frac{V_L I_L}{2}\sin(\theta_v - \theta_i) = \frac{(54.41)(6.32)}{2}\sin(-0.84^{\circ} + 55.3^{\circ}) = 140 \text{ VAR}$$

reactive powers

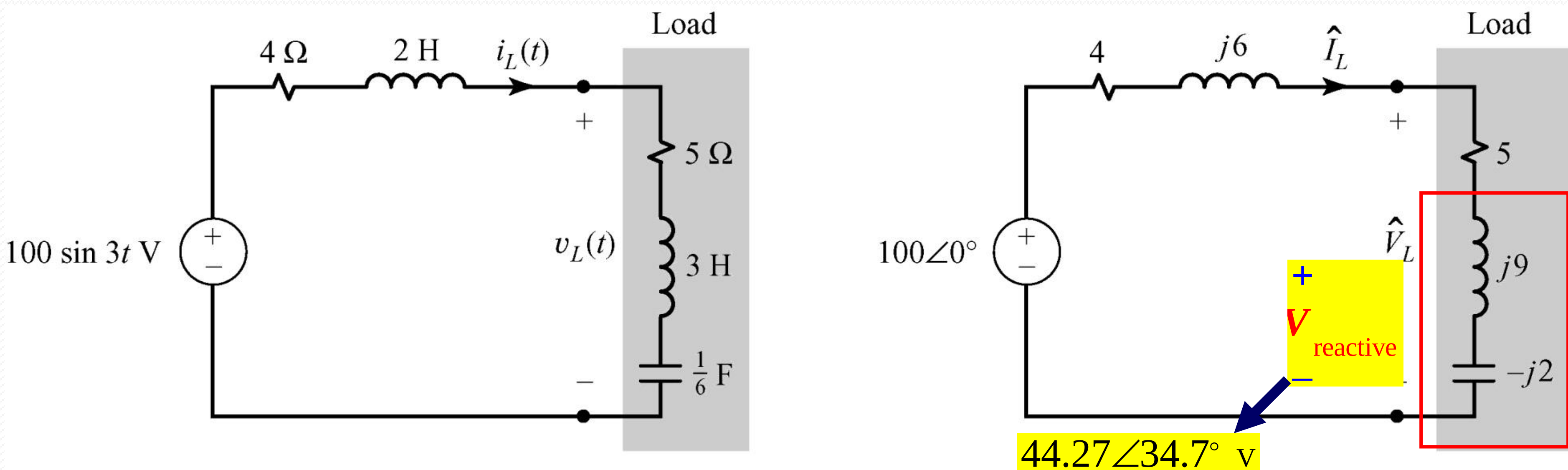


$$Q_{load} = \frac{V_{load} I_{load}}{2}\sin(\theta_v - \theta_i) = \frac{(54.41)(6.32)}{2}\sin(-0.84^\circ + 55.3^\circ) = 140 \text{ VAR}$$

OR $\quad Q_{load} = \frac{V_{reactive} I_{reactive}}{2} \qquad I_{reactive} = \hat{I}_L \Rightarrow I_{reactive} = |\hat{I}_L| = 6.32$

$$V_{reactive} = \frac{(j9 - j2)}{5 + (j9 - j2)} 54.41\angle -0.84^\circ = \frac{j7}{5 + j7} 54.41\angle -0.84^\circ = \frac{7\angle 90^\circ}{8.6\angle 54.46^\circ} 54.41\angle -0.84^\circ = 44.27\angle 34.7^\circ \text{ V}$$

EX: Determine the average and reactive powers delivered to the load impedance and the power factor of the load



(a) (b)

$\hat{V}_{load} = 54.41\angle -0.84^\circ \ \ V$ $\hat{I}_L = 6.32\angle -55.3^\circ \ \ A$

$$Q_{load} = \frac{V_L I_L}{2}\sin(\theta_v - \theta_i) \quad =140 \ \ VAR$$

OR $Q_{load} = \dfrac{V_{reactive} I_{reactive}}{2}$ $I_{reactive} = |\hat{I}_L| = 6.32$ $V_{reactive} = 44.27 \ \ V$

$$Q_{load} = \frac{(44.27)(6.32)}{2} = 139.89 \approx 140 \ VAR$$

This could also be calculated from the complex power delivered to the load

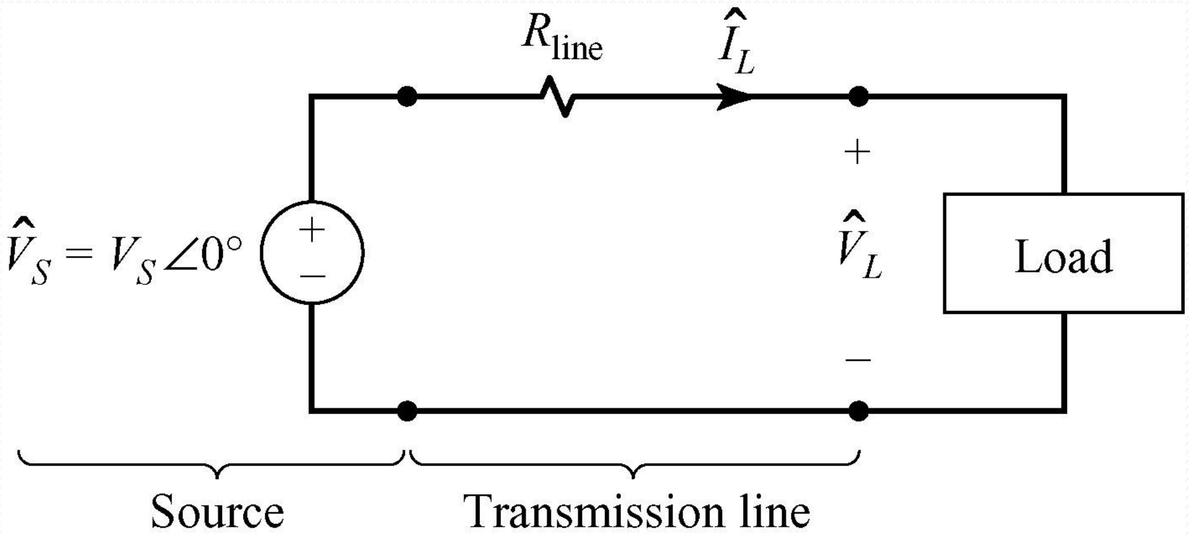
$$\hat{P}_{load} = \frac{1}{2}\hat{V}_{load}\hat{I}^{*}_{load} = \frac{1}{2}\left(54.41\angle -0.84^{\circ}\right)\left(6.32\angle -55.3^{\circ}\right)^{*}$$
$$= 100 + j140 VA$$

The power factor of the load is:

$$pf = \cos\left(\theta_{v} - \theta_{I}\right) = \cos\left(-0.84 + 55.3\right) = 0.581$$

The load is lagging because the current lags the voltage

A typical power distribution circuit

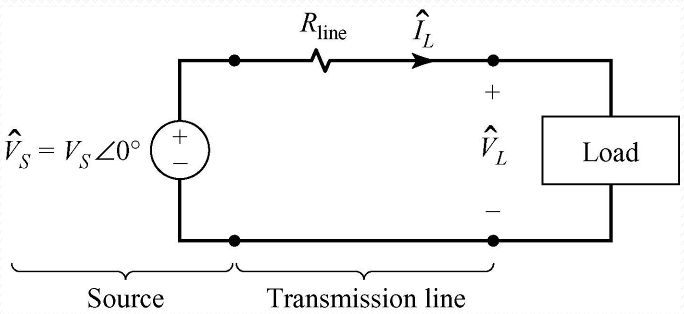


The consumer is charged for the average power consumed by the load

$$\frac{VI}{2}\cos\left(\theta_V - \theta_I\right) = \frac{VI}{2}\times \text{pf}$$

The load requires a certain total apparent power $\frac{VI}{2}$

Ex : Suppose that the load voltage figure is 170V



The line resistance is 0.1 ohm

The load requires 10KW of average power.

Examine the line losses for a load power factor of unity and for a power factor of 0.7 **lagging**.

The load current is obtained from $P_{AV} = \frac{1}{2} V_L I_L \left(pf\right)$

For unity power factor this is $I_L = \dfrac{2\left(10KW\right)}{\left(170V\right)\left(1\right)} = 117.65A$

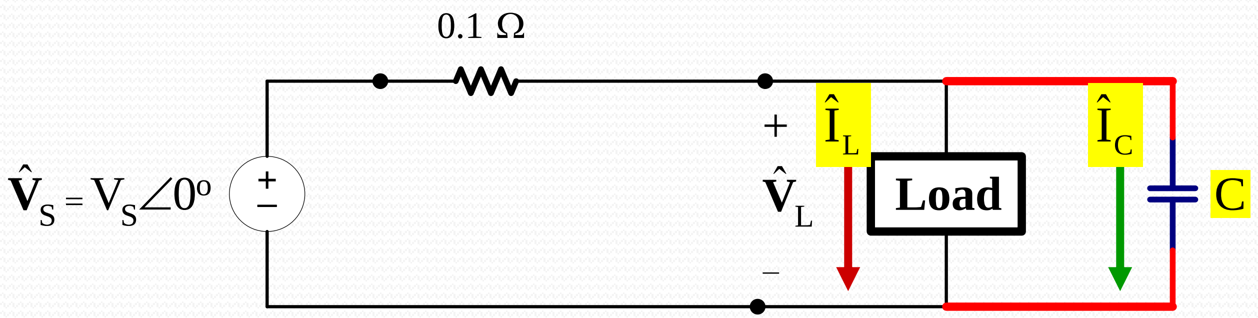
For power factor of 0.7 $I_L = \dfrac{2\left(10KW\right)}{\left(170V\right)\left(0.7\right)} = 168.07A$ $P_{AV,line} = \dfrac{1}{2} I_L^2 R_{line} = \begin{cases} 692.04W & \left(unity \quad pf\right) \\ 1412.33W & \left(pf = 0.7\right) \end{cases}$

The powers consumed in the line losses $P_{AV,line} = \dfrac{1}{2} I_L^2 R_{line} = \begin{cases} 692.04W & \left(unity \quad pf\right) \\ 1412.33W & \left(pf = 0.7\right) \end{cases}$

720 W extra power to be generated if pf is 0.7 to supply the load

Power Factor Correction

Ex: : for previous determine the value of capacitor across the load to correct the power factor from 0.7 to unity if power frequency is 60Hz.



From previous example

For power factor of 0.7

$$I_L = \frac{2(10KW)}{(170V)(0.7)} = 168.07A$$

power factor 0.7 lagging $\Rightarrow$ $\theta_I = -\cos^{-1}(0.7) = -45.57^\circ$ $\Rightarrow$ $\hat{I}_L = 168.07\angle -45.57^\circ$

The current through the added capacitor is: $\hat{I}_C = \dfrac{\hat{V}_L}{Z_C} = \dfrac{170\angle 0^\circ}{(1/j\omega C)} = j\omega C \times 170\angle 0^\circ$

Hence the total current $\hat{I}_{line} = \hat{I}_L + \hat{I}_C \quad = 168.07\angle -45.57^\circ + j\omega C \times 170\angle 0^\circ$

$$= 117.66 - j120.02 + j(2\pi \times 60)C \times 170$$

Unity power factor $\Rightarrow$ $\cos(\theta_v - \theta_i) = 1$ $\Rightarrow$ $\theta_v - \theta_i = 0$ $\Rightarrow$ $\theta_i = \theta_v = 0$

$\Rightarrow$ Imaginary component of the line current is zero $-j120.02 + j(2\pi \times 60)C \times 170 = 0$

$\Rightarrow$ $C = \dfrac{120.02}{(2\pi \times 60) \times 170}$ $= 1873\ uF$

Averaging the instantaneous over the common period $T = 2\pi / \omega$

$$P_{AV} = \frac{1}{T}\int_0^T p(t)dt$$

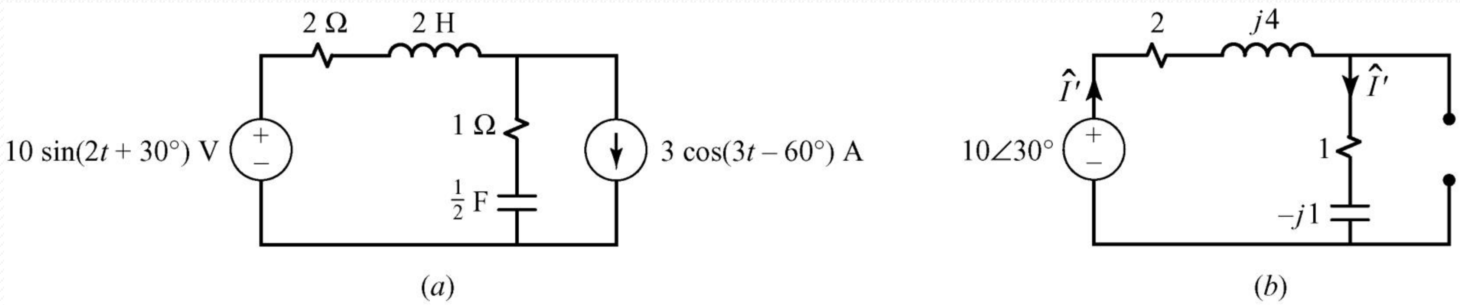
$$= \begin{cases} P_{AV1} + P_{AV2} & \text{if } n \neq m \\ P_{AV1} + P_{AV2} + \frac{V_1 I_2}{2}\cos(\theta_{V1} - \theta_{I2}) + \frac{V_2 I_1}{2}\cos(\theta_{V2} - \theta_{I1}) & \text{if } n = m \end{cases}$$

THUS: we may superimpose the average powers delivered by sources of different frequencies, but we may not, in general, apply superposition to average power if the sources are of the same frequency.

where
$$P_{AV1} = \frac{V_1 I_1}{2}\cos(\theta_{V1} - \theta_{I1}) = \frac{1}{2}\mathrm{Re}(\hat{V}_1 \hat{I}_1^*)$$

$$P_{AV2} = \frac{V_2 I_2}{2}\cos(\theta_{V2} - \theta_{I2}) = \frac{1}{2}\mathrm{Re}(\hat{V}_2 \hat{I}_2^*)$$

Ex : Determine the average power delivered by the two sources of the circuit



$$\hat{I}' = \frac{10\angle 30^\circ}{2 + j4 + 1 - j1} = 2.357\angle -15^\circ$$

Hence the average power delivered by the voltage source is

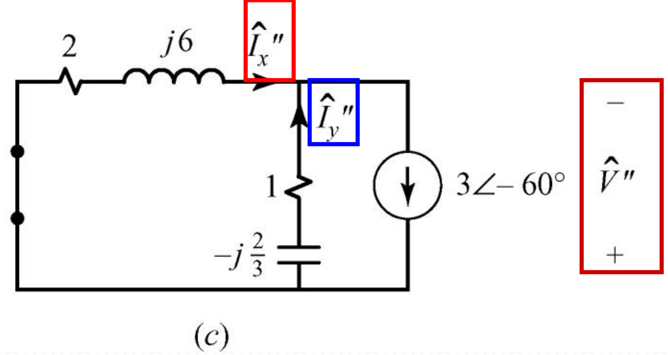
$$P'_{AV} = \frac{1}{2}\mathrm{Re}(10\angle 30^\circ \hat{I}'^{*}) = \frac{1}{2}\times 10\times 2.357\times \cos(30^\circ + 15^\circ) \quad = 8.333\text{ W}$$

This can be confirmed from average powers delivered to the two resistors

$$P'_{AV} = P'_{AV,2\Omega} + P'_{AV,1\Omega}$$

$$= \frac{1}{2}\left|\hat{I}'\right|^2 2 + \frac{1}{2}\left|\hat{I}'\right|^2 1 = 8.333\text{ W}$$

By current division:



(c)

$$\hat{I}_x'' = \frac{1-j\frac{2}{3}}{2+j6+1-j\frac{2}{3}} 3\angle -60^\circ = 0.589\angle -154.33^\circ$$

$$\hat{I}_y'' = \frac{2+j6}{2+j6+1-j\frac{2}{3}} 3\angle -60^\circ = 3.101\angle -49.08^\circ$$

The voltage across the current source is $\hat{V}'' = (2+j6)\hat{I}_x'' = 3.727\angle -82.77^\circ$

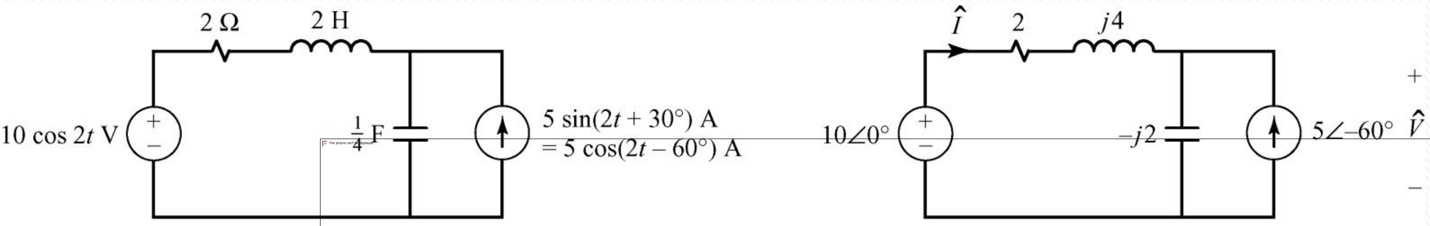
Hence the average power delivered by the current source is

$$P_{AV}'' = \frac{1}{2}\mathrm{Re}(\hat{V}'' 3\angle 60^\circ) = \frac{1}{2}\times 3.727\times 3\times \cos(-82.77^\circ + 60^\circ) = 5.154 \text{ W}$$

This may be again confirmed by computing the average power delivered to the Two resistors: $P_{AV}'' = P_{AV,2\Omega}'' + P_{AV,1\Omega}'' = \frac{1}{2}\left|\hat{I}_x''\right|^2 2 + \frac{1}{2}\left|\hat{I}_y''\right|^2 1 = 5.154 \text{ W}$

Since frequencies are not the same, total average power delivered is the sum of average powers delivered individually by each source

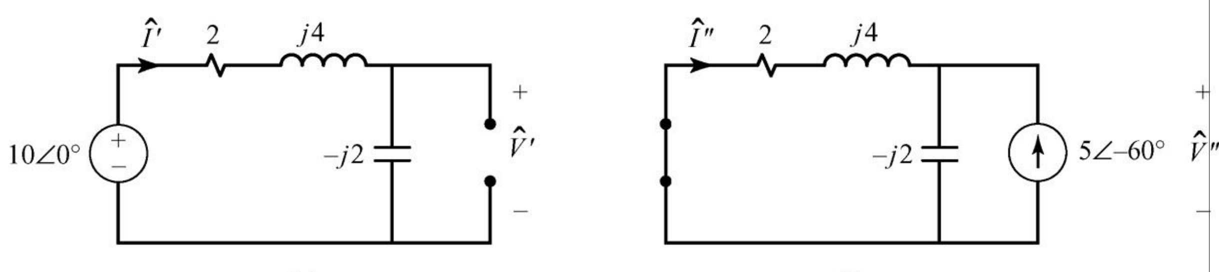
EX 6.19: Determine the average power delivered by the two sources



(a)

Since both sources have the same frequency, we can't use superposition. So we include both sources in one phasor circuit. The total average power delivered by the sources is equal to the average power delivered to the resistor

We use superposition on the phasor circuit to find the current across the resistor



(c) (d)

$$\hat{I}' = \frac{10\angle 0^\circ}{2 + j4 - j2} = 3.536\angle -45^\circ \qquad \hat{I}'' = -\frac{-j2}{2 + j4 - j2} 5\angle -60^\circ = 3.536\angle -15^\circ$$

$$\hat{I} = \hat{I}' + \hat{I}'' = 3.536\angle -45^\circ + 3.536\angle -15^\circ = 6.831\angle -30^\circ$$

The phasor current is: $\hat{I} = \hat{I}^{'} + \hat{I}^{''} = 3.536\angle -45^\circ + 3.536\angle -15^\circ = 6.831\angle -30^\circ$

Hence the average power delivered to the resistor is $P_{AV} = \frac{1}{2}\left|\hat{I}\right|^2 \times 2 = 46.66$ W

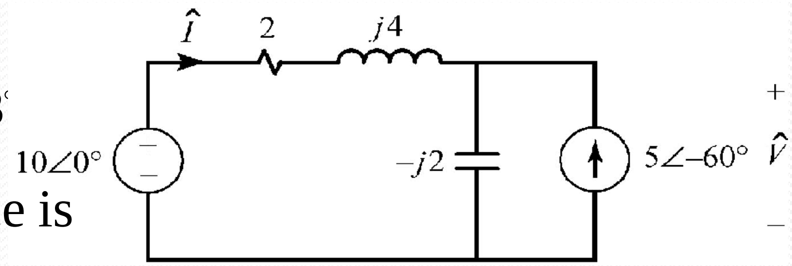
Note that we may not superimpose average powers delivered to the resistors by the individual sources

$$\frac{1}{2}\left|\hat{I}^{'}\right|^2 \times 2 + \frac{1}{2}\left|\hat{I}^{''}\right|^2 \times 2 = 25 \neq 46.66$$

We can compute this total average power by directly computing the average power delivered by the sources from the phasor circuit

The voltage across the current source is

$$\hat{V} = 10\angle 0^\circ - (2 + j\,4)\hat{I} = 22.88\angle -132.63^\circ$$



The average power delivered by voltage source is

$$P_{AV\text{ , voltage source}} = \frac{1}{2}\mathrm{Re}[(10\angle 0^\circ)\hat{I}^*] = 29.58 \text{ W}$$

The average power delivered by the current source is

$$P_{AV\text{ , current source}} = \frac{1}{2}\mathrm{Re}[\hat{V}(5\angle 60^\circ)] = 17.08 \text{ W}$$

The total average power delivered by the sources is

$$P_{AV\text{ , source}} = 29.58 + 17.08 = 46.66 \text{ W}$$

➢ Phasors

- We have learnt from the previous section how to define and express in a single equation the magnitude, frequency, and phase shift of a sinusoidal signal.

- Any linear circuit that contains resistors, capacitors, and inductors do not alter the shape of this signal, nor its frequency.

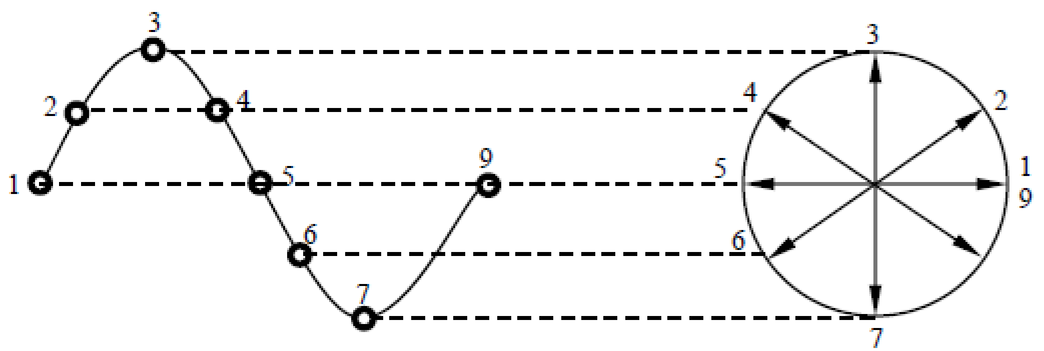
- However, the linear circuit does change the amplitude of the signal (amplification or attenuation) and shift its phase (causing the output signal to lead or lag the input).

- The amplitude and phase are the two important quantities that determine the way the circuit affects the signal.

❖ Accordingly, signal can be expressed as a linear combination of complex sinusoids.

❖ Phase and magnitude defines a phasor (vector) or complex number. The phasor is similar to vector that has been studied in mathematics.

❖ Figure shows how AC sinusoidal quantities are represented by the position of a rotating vector. As the vector rotates it generates an angle. The location of the vector on the plane surface is determined by the magnitude (length) of the vecto

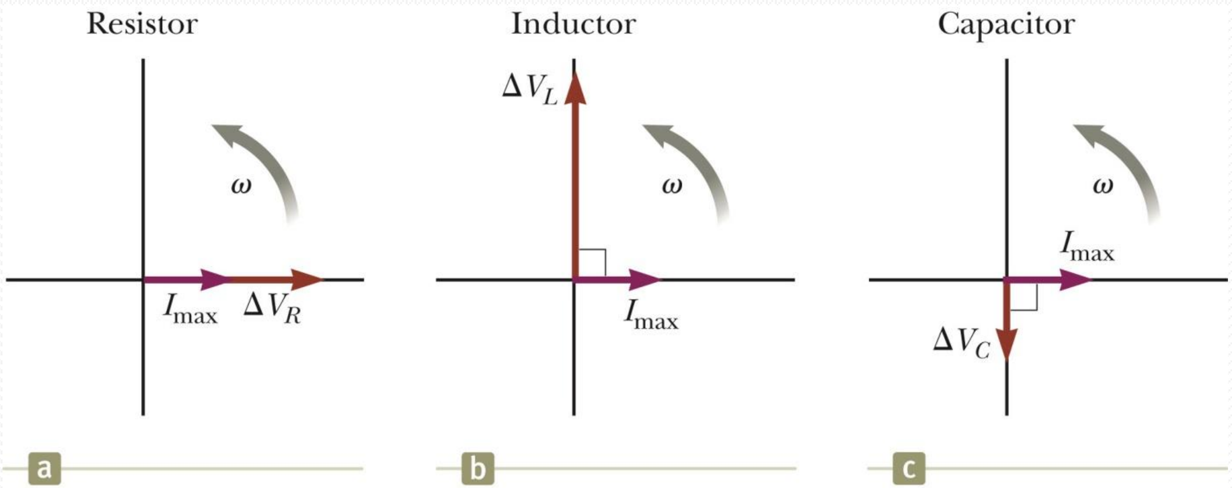


(a) (b)

- ❖ **Representing sinusoidal signals by phasors is useful since circuit analysis laws such as KVL and KCL and familiar algebraic circuit analysis tools, such as series and parallel equivalence, voltage and current division are applicable in the phasor domain, which have been studied in DC circuits can be applied.**

- ❖ **We do not need new analysis techniques to handle circuits in the phasor domain. The only difference is that circuit responses are phasors (complex numbers) rather than DC signals (real numbers).**

Phasor Diagrams

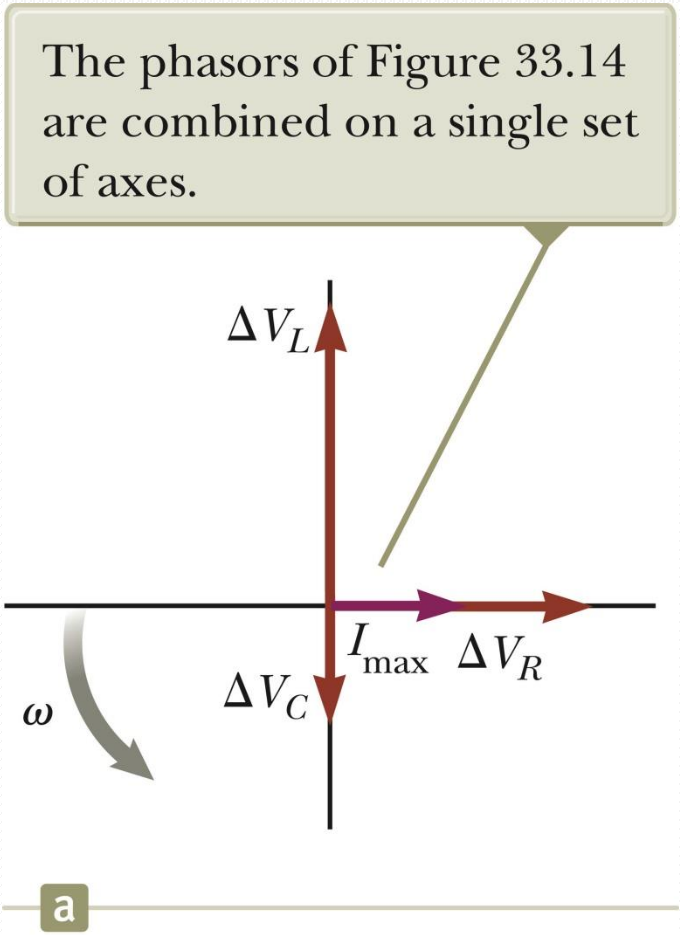


- To account for the different phases of the voltage drops, vector techniques are used.
- Remember the phasors are rotating vectors
- The phasors for the individual elements are shown.

Section 33.5

Resulting Phasor Diagram

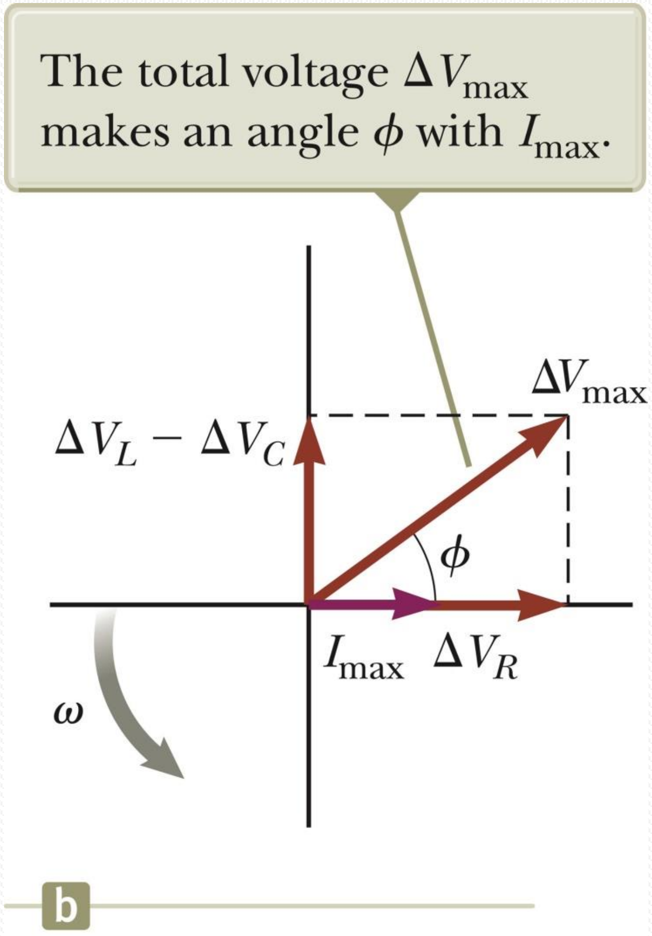
- The individual phasor diagrams can be combined.
- Here a single phasor I_{max} is used to represent the current in each element.
  - In series, the current is the same in each element.



Section 33.5

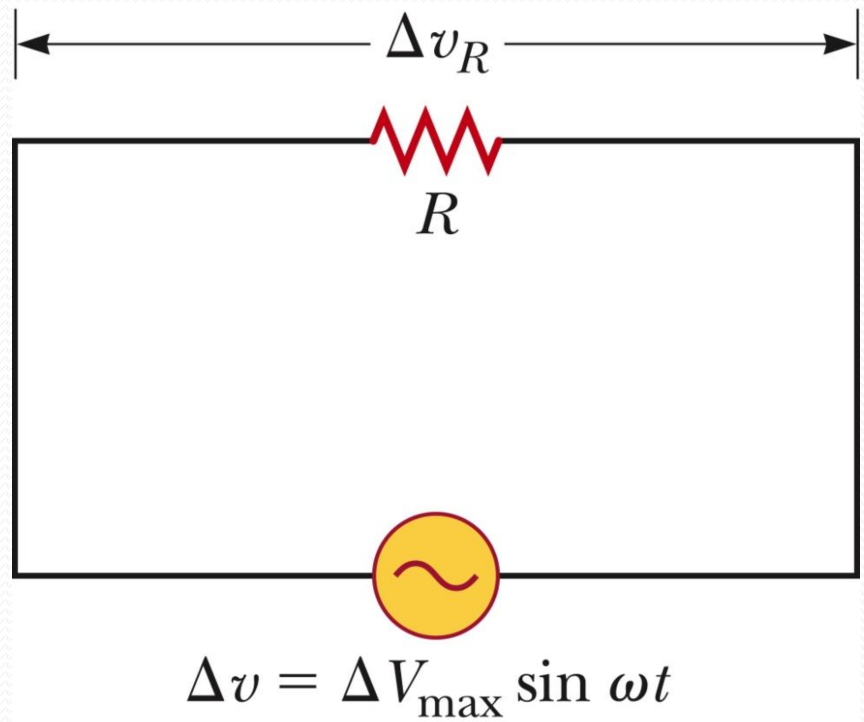
➢ Vector Addition of the Phasor Diagram

- Vector addition is used to combine the voltage phasors.
- ΔV_L and ΔV_C are in opposite directions, so they can be combined.
- Their resultant is perpendicular to $\Delta V_{R.}$
- The resultant of all the individual voltages across the individual elements is $\Delta v_{max.}$
  - This resultant makes an angle of φ with the current phasor $I_{max.}$



➢ Resistors in an AC Circuit

- ❖ Consider a circuit consisting of an AC source and a resistor.
- ❖ The AC source is symbolized by 
- ❖ Δv_R = D V_{max}= V_{max} sin wt
- ❖ Δv_R is the instantaneous voltage across the resistor.



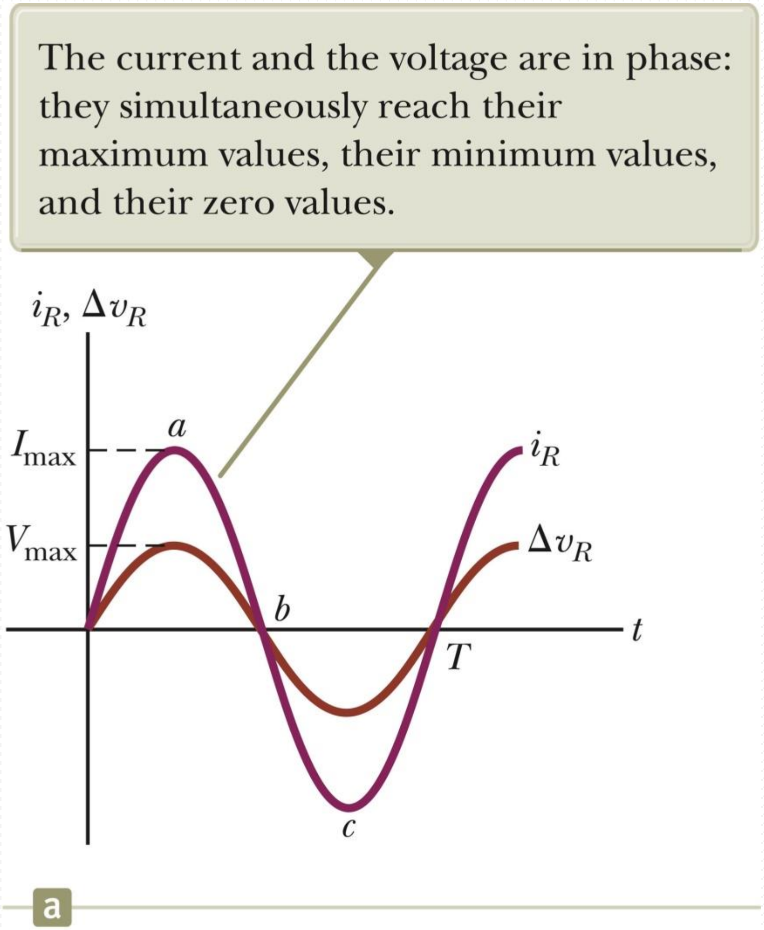
❖**The instantaneous current in the resistor is**

$$i_R = \frac{\Delta v_R}{R} = \frac{\Delta V_{max}}{R}\sin\omega t = I_{max}\sin\omega t$$

❖**The instantaneous voltage across the resistor is also given as**

$$\boldsymbol{\Delta v_R = I_{max}\, R \sin \omega t}$$

- The graph shows the current through and the voltage across the resistor.
- The current and the voltage reach their maximum values at the same time.
- The current and the voltage are said to be *in phase.*
- For a sinusoidal applied voltage, the current in a resistor is always in phase with the voltage across the resistor.
- The direction of the current has no effect on the behavior of the resistor.
- Resistors behave essentially the same way in both DC and AC circuits.



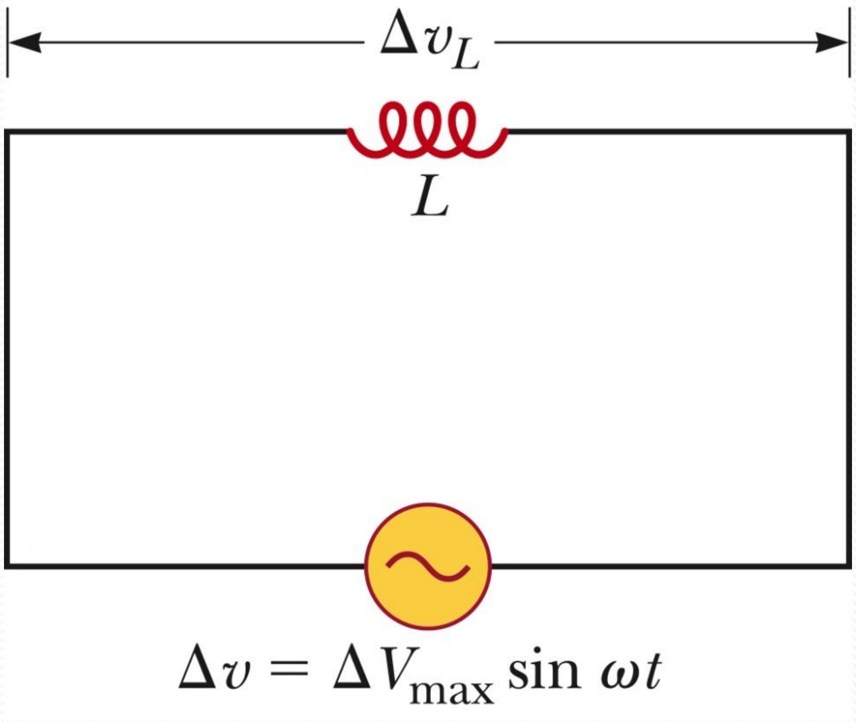
➢ Inductors in an AC Circuit

❖ Kirchhoff's loop rule can be applied and gives:

$$\Delta v + \Delta v_L = 0, \text{ or}$$

$$\Delta v - L\frac{di}{dt} = 0$$

$$\Delta v = L\frac{di}{dt} = \Delta V_{max}\, sin\, \omega t$$



➢ Current in an Inductor

❖ The equation obtained from Kirchhoff's loop rule can be solved for the current

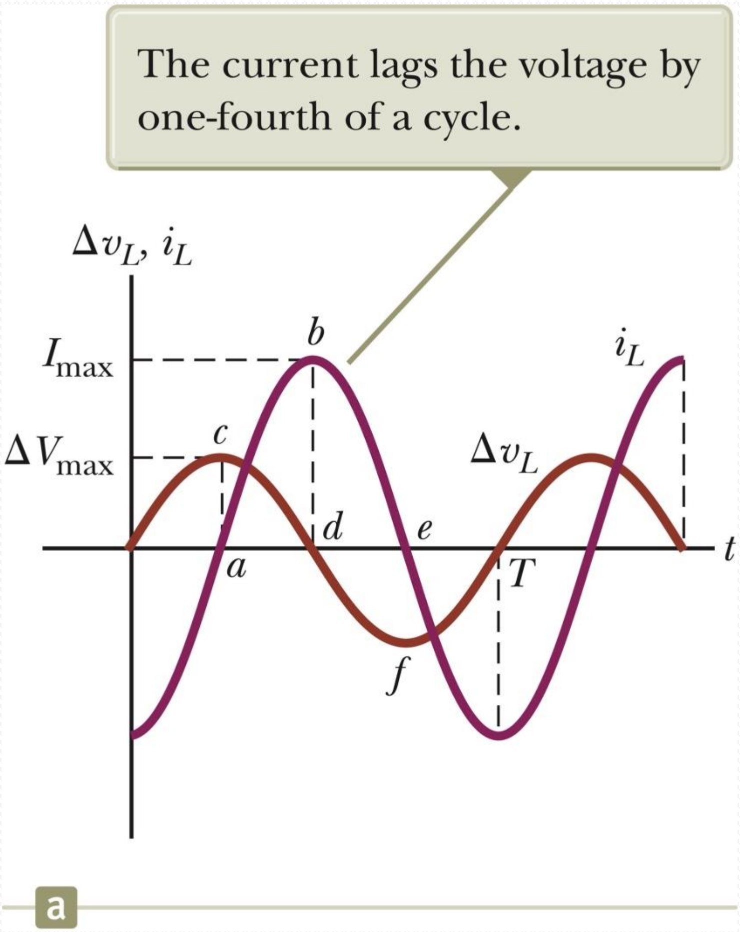
$$i_L = \frac{\Delta V_{\max}}{L}\int \sin \omega t \; dt = -\frac{\Delta V_{max}}{\omega L} cos\,\omega t$$

$$i_L = \frac{\Delta V_{max}}{\omega L} sin\left(\omega t - \frac{\pi}{2}\right) \qquad I_{\ max} = \frac{\Delta V_{max}}{\omega L}$$

❖ This shows that the instantaneous current i_L in the inductor and the instantaneous voltage Δv_L across the inductor are out of phase by (p/2) rad = 90º.

➢ Phase Relationship of Inductors in an AC Circuit

- The current is a maximum when the voltage across the inductor is zero.
  - The current is momentarily not changing
- For a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by 90° ($\pi/2$).



➢ Inductive Reactance

- The factor ωL has the same units as resistance and is related to current and voltage in the same way as resistance.
- Because ωL depends on the frequency, it reacts differently, in terms of offering resistance to current, for different frequencies.
- The factor is the inductive reactance and is given by:

  $X_L = \omega L$

- Current can be expressed in terms of the inductive reactance:

$$I_{\max} = \frac{\Delta V_{\max}}{X_L} \; or \; I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_L}$$

- As the frequency increases, the inductive reactance increases
  - This is consistent with Faraday's Law:
    - The larger the rate of change of the current in the inductor, the larger the back emf, giving an increase in the reactance and a decrease in the current.

➢ Voltage Across the Inductor

❖The instantaneous voltage across the inductor is

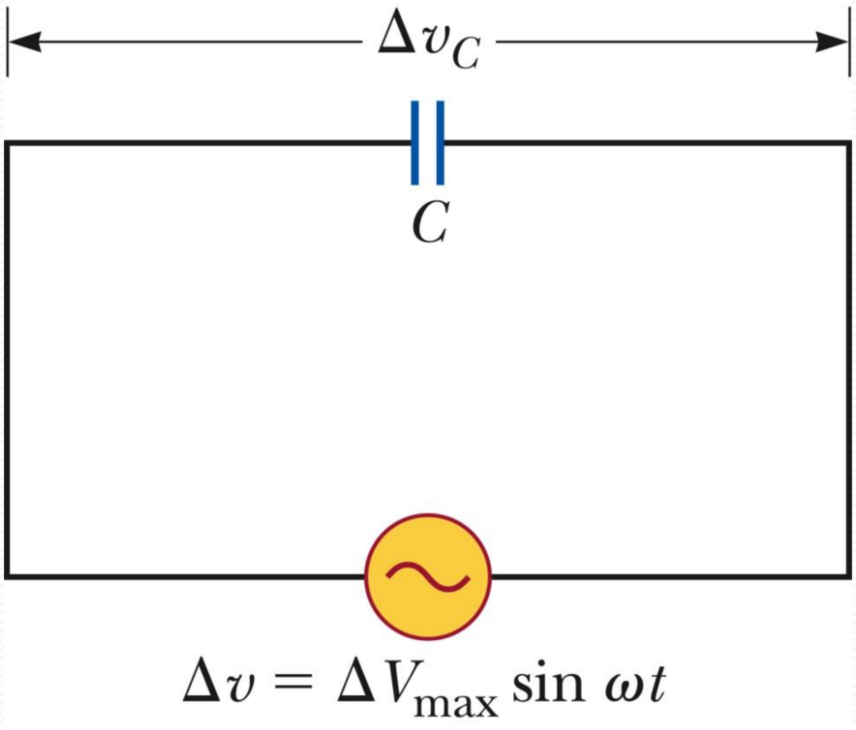
$$\Delta v_L = -L\frac{di}{dt}$$

$$= -\Delta V_{\max}\sin\omega t$$

$$= -I_{\max}X_L\sin\omega t$$

➢ Capacitors in an AC Circuit

- The circuit contains a capacitor and an AC source.
- Kirchhoff's loop rule gives:
- $\Delta v + \Delta v_c = 0$ and so
- $\Delta v = \Delta v_C = \Delta V_{max} \sin \omega t$
  - Δv_c is the instantaneous voltage across the capacitor.



❖The charge is $q = C\Delta V_{max}\ sin\ \omega t$

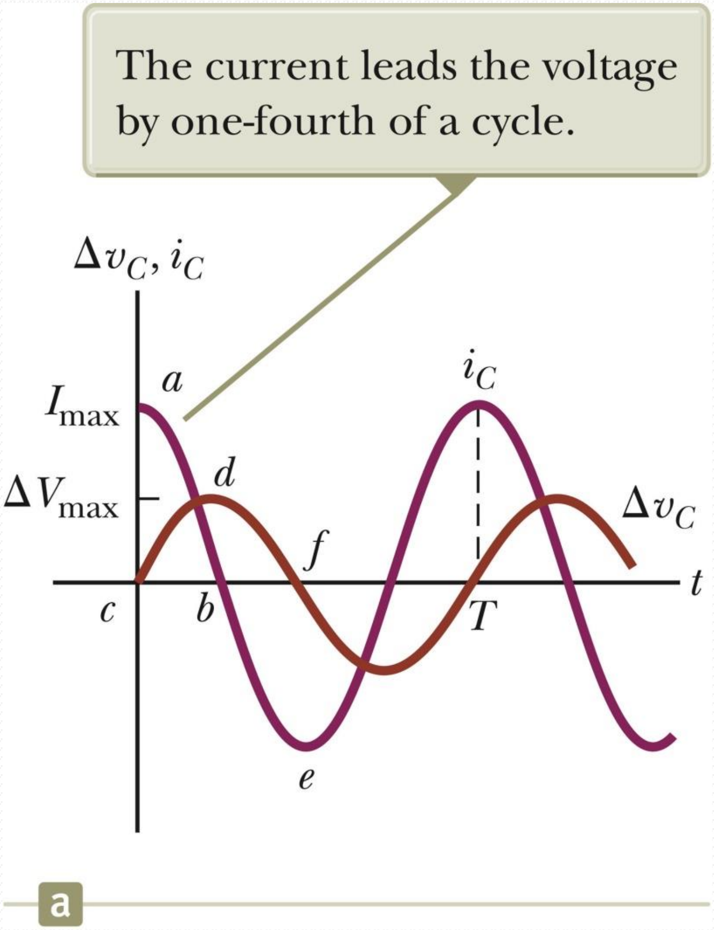
❖The instantaneous current is given by

$$i_C = \frac{dq}{dt} = \omega C \Delta V_{\max} \cos \omega t$$

$$\text{or } i_C = \omega C \Delta V_{\max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

❖The current is p/2 rad = 90° out of phase with the voltage

- The current reaches its maximum value one quarter of a cycle sooner than the voltage reaches its maximum value.
- The current leads the voltage by 90º.



➢ Capacitive Reactance

❖ The maximum current in the circuit occurs at cos ωt = 1 which gives

$$I_{max} = \omega C \Delta V_{max} = \frac{\Delta V_{max}}{(1/\omega C)}$$

❖ The impeding effect of a capacitor on the current in an AC circuit is called the capacitive reactance and is given by

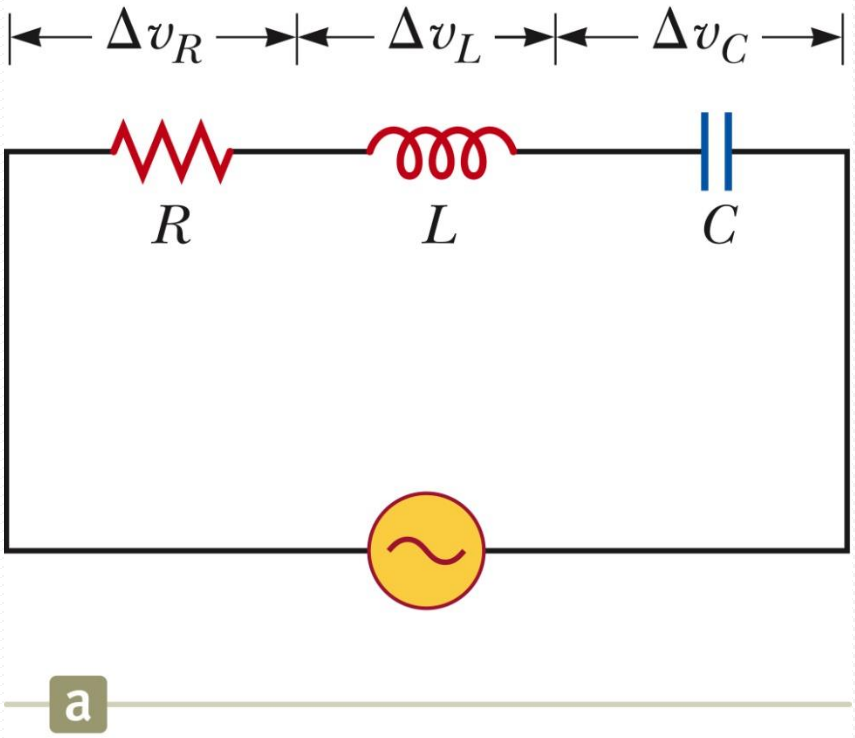
$$X_C \equiv \frac{1}{\omega C} \quad \text{which gives} \quad I_{max} = \frac{\Delta V_{max}}{X_C}$$

➢ Voltage Across a Capacitor

- The instantaneous voltage across the capacitor can be written as $\Delta v_C = \Delta V_{max} \sin \omega t = I_{max} X_C \sin \omega t$.
- As the frequency of the voltage source increases, the capacitive reactance decreases and the maximum current increases.
- As the frequency approaches zero, X_C approaches infinity and the current approaches zero.
  - This would act like a DC voltage and the capacitor would act as an open circuit.

The *RLC* Series Circuit

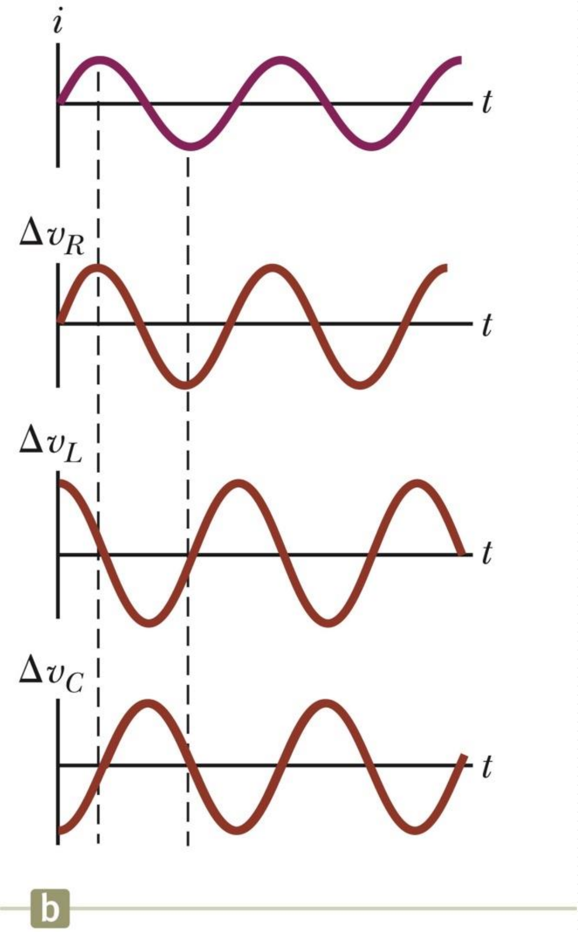
- The resistor, inductor, and capacitor can be combined in a circuit.
- The current and the voltage in the circuit vary sinusoidally with time.



- **The instantaneous voltage would be given by $\Delta v = \Delta V_{max} \sin \omega t$.**
- **The instantaneous current would be given by $i = I_{max} \sin (\omega t - \varphi)$.**
  - **φ is the phase angle between the current and the applied voltage.**
- **Since the elements are in series, the current at all points in the circuit has the same amplitude and phase.**

➢ *I* and *v* Phase Relationships – Graphical View

- ❖ The instantaneous voltage across the resistor is in phase with the current.
- ❖ The instantaneous voltage across the inductor leads the current by 90°.
- ❖ The instantaneous voltage across the capacitor lags the current by 90°.



❖ **The instantaneous voltage across each of the three circuit elements can be expressed as**

$$\Delta v_R = I_{\max} R \sin \omega t = \Delta V_R \sin \omega t$$

$$\Delta v_L = I_{\max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos \omega t$$

$$\Delta v_C = I_{\max} X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos \omega t$$

More About Voltage in RLC Circuits

- ΔV_R is the maximum voltage across the resistor and $\Delta V_R = I_{max}R$.
- ΔV_L is the maximum voltage across the inductor and $\Delta V_L = I_{max}X_L$.
- ΔV_C is the maximum voltage across the capacitor and $\Delta V_C = I_{max}X_C$.
- The sum of these voltages must equal the voltage from the AC source.
- Because of the different phase relationships with the current, they cannot be added directly.

➢ Total Voltage in RLC Circuits

❖ From the vector diagram, ΔV_{max} can be calculated

$$\Delta V_{max} = \sqrt{\Delta V_R^2 + \left(\Delta V_L - \Delta V_C\right)^2}$$

$$= \sqrt{(I_{max} R)^2 + (I_{max} X_L - I_{max} X_C)^2}$$

$$\Delta V_{max} = I_{max} \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

Impedance

❖ The current in an ***RLC*** circuit is

$$I_{max} = \frac{\Delta V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\Delta V_{max}}{Z}$$

❖ Z is called the impedance of the circuit and it plays the role of resistance in the circuit, where

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$

❖ Impedance has units of ohms

➢ Phase Angle

- ❖ The right triangle in the phasor diagram can be used to find the phase angle, φ.

$$\varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

- ❖ The phase angle can be positive or negative and determines the nature of the circuit.

➢ Determining the Nature of the Circuit

- If f is positive
  - $X_L > X_C$ (which occurs at high frequencies)
  - The current lags the applied voltage.
  - The circuit is more inductive than capacitive.
- If f is negative
  - $X_L < X_C$ (which occurs at low frequencies)
  - The current leads the applied voltage.
  - The circuit is more capacitive than inductive.
- If f is zero
  - $X_L = X_C$
  - The circuit is purely resistive.

➢ Power in an AC Circuit

- The average power delivered by the AC source is converted to internal energy in the resistor.
  - $P_{avg} = ½\ I_{max}\ \Delta V_{max}\ \cos f = I_{rms}\Delta V_{rms}\ \cos f$
  - $\cos f$ is called the power factor of the circuit
- We can also find the average power in terms of R.
  - $P_{avg} = I^2_{rms}R$
- When the load is purely resistive, $f = 0$ and $\cos f = 1$
  - $P_{avg} = I_{rms}\ \Delta V_{rms}$

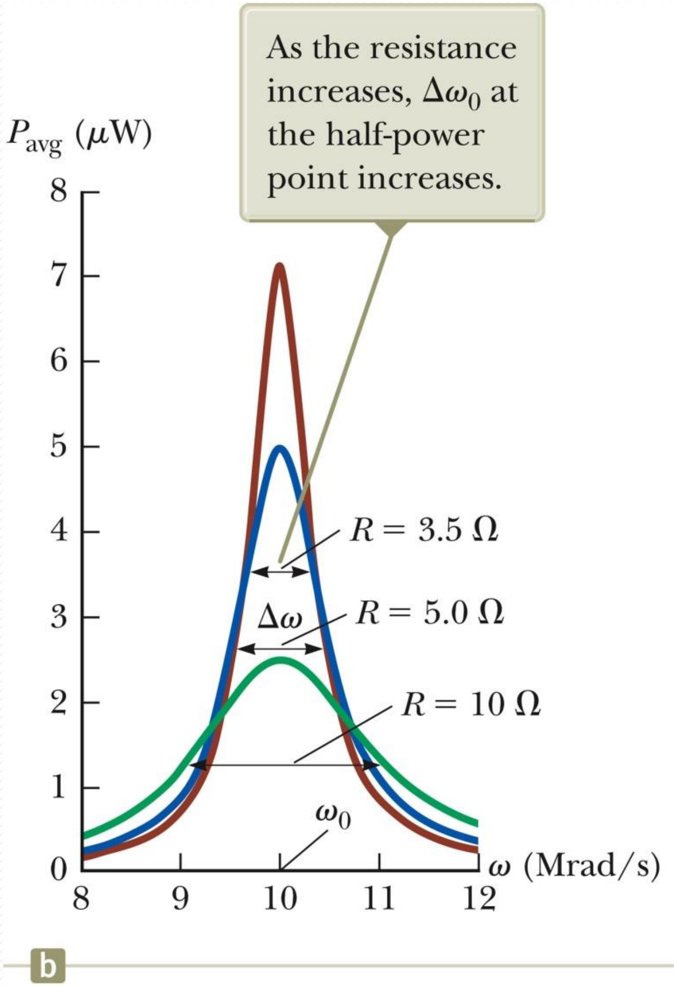
- The average power delivered by the source is converted to internal energy in the resistor.
- No power losses are associated with pure capacitors and pure inductors in an AC circuit.
  - In a capacitor, during one-half of a cycle, energy is stored and during the other half the energy is returned to the circuit and no power losses occur in the capacitor.
  - In an inductor, the source does work against the back emf of the inductor and energy is stored in the inductor, but when the current begins to decrease in the circuit, the energy is returned to the circuit.
- The power delivered by an AC circuit depends on the phase.
- Some applications include using capacitors to shift the phase to heavy motors or other inductive loads so that excessively high voltages are not needed.

➢ Power as a Function of Frequency

❖ Power can be expressed as a function of frequency in an *RLC* circuit.

$$P_{av} = \frac{(\Delta V_{rms})^2 R\omega^2}{R^2\omega^2 + L^2(\omega^2 - \omega_o^2)^2}$$

❖ This shows that at resonance, the average power is a maximum.



UNIT-IV
RESONANCE

Learning Objectives

- Become familiar with the frequency response of a series resonant circuit and how to calculate the resonant and cutoff frequencies.
- Be able to calculate a tuned network's quality factor, bandwidth, and power levels at important frequency levels.

- Become familiar with the frequency response of a parallel resonant circuit and how to calculate the resonant and cutoff frequencies.
- Understand the impact of the quality factor on the frequency response of a series or parallel resonant network.

Resonance In Electric Circuits

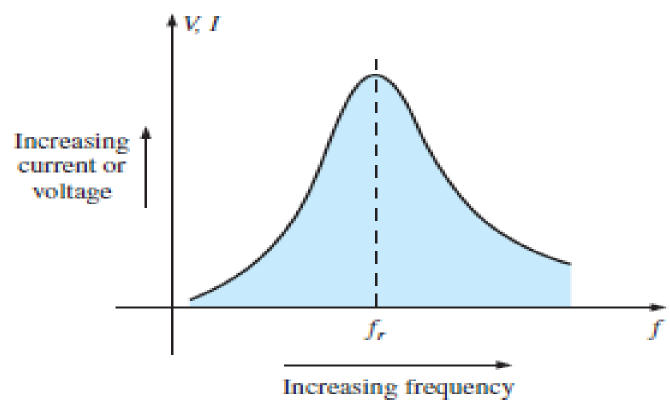
- ☑ Any passive electric circuit will resonate if it has an inductor and capacitor.

- ☑ Resonance is characterized by the input voltage and current being in phase. The driving point impedance (or admittance) is completely real when this condition exists.

- ☑ In this presentation we will consider (a) series resonance, and (b) parallel resonance.

Resonance Introduction

- ***Resonant*** (or ***tuned***) ***circuits,*** are fundamental to the operation of a wide variety of electrical and electronic systems in use today.
- The resonant circuit is a combination of ***R, L,*** and ***C*** elements having a frequency response characteristic similar to the one below:



Resonance Introduction

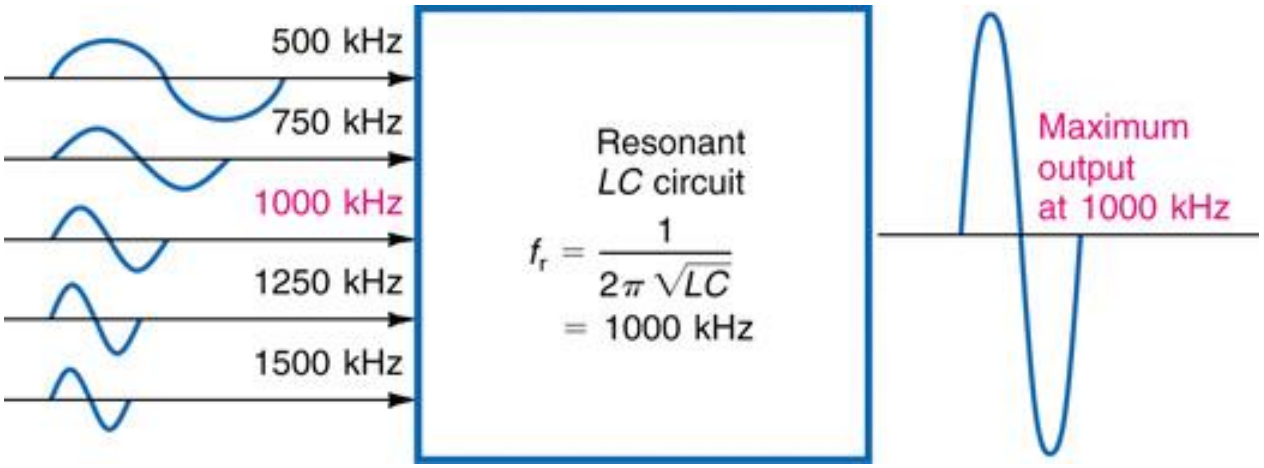
- The resonant electrical circuit *must* have ***both inductance and capacitance***.
- In addition, ***resistance*** will always be present due either to the lack of ideal elements or to the control offered on the shape of the resonance curve.
- When resonance occurs due to the application of the proper frequency (f_r), the energy absorbed by one reactive element is the same as that released by another reactive element within the system.
- Remember:

$$X_C = \frac{1}{wC} = \frac{1}{2\pi fC} \Rightarrow Z_C = -jX_C$$

$$X_L = wL = 2\pi fL \Rightarrow Z_L = jX_L$$

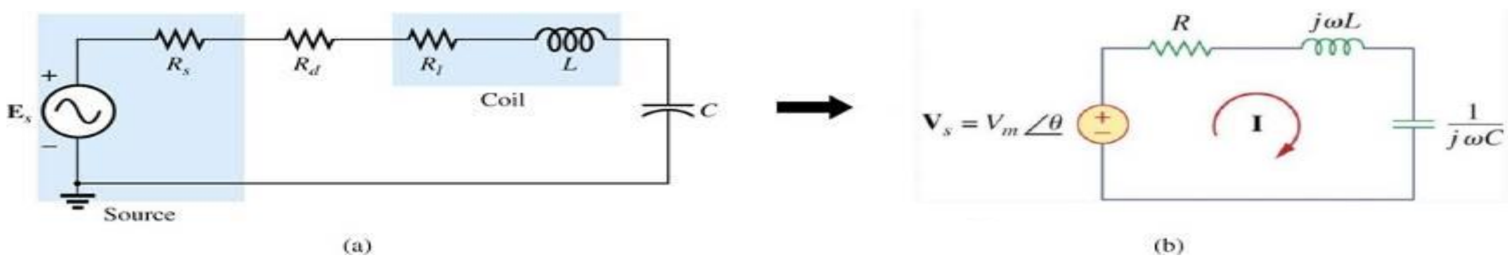
The Resonance Effect

- The most common application of resonance in rf circuits is called <u>tuning</u>.
- In Fig. below, the LC circuit is resonant at 1000 kHz.
- The result is **maximum output at 1000 kHz**, compared with lower or higher frequencies.

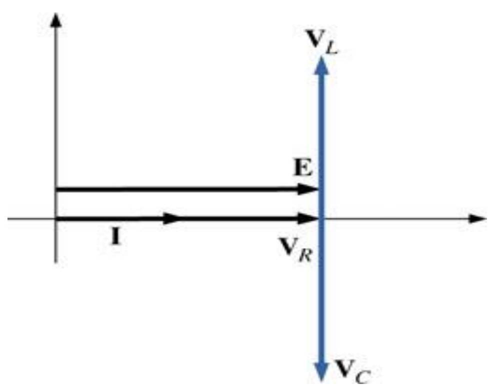


Series Resonant Circuit

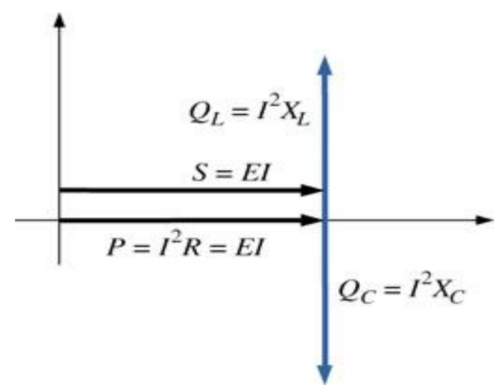
The basic format of the series resonant circuit is a series *R*-*L*-*C* combination in series with an applied voltage source.



Series resonant circuit.



Phasor diagram for the series resonant circuit at resonance.



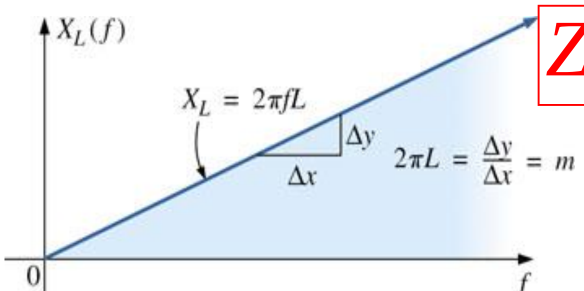
Power triangle for the series resonant circuit at resonance.

Z_T Versus Frequency

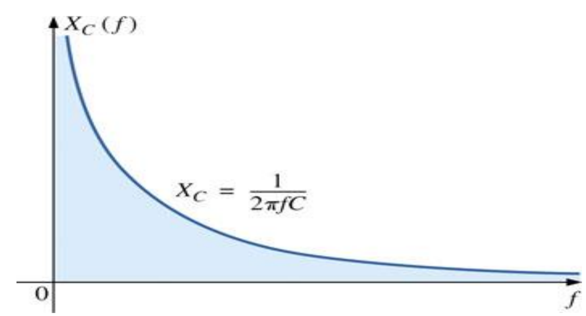
- Knowing that X_C and X_L are dependent upon frequency it can be stated:
  - Capacitor Impedance decreases as frequency increases.
  - Inductor Impedance increases as frequency increases.
- This implies that the total impedance of the series *R*-*L*-*C* circuit below, at any frequency, is determined by:

$$Z_T = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\boxed{Z_T = R + jX_L - jX_C}$$



X_L *versus f*



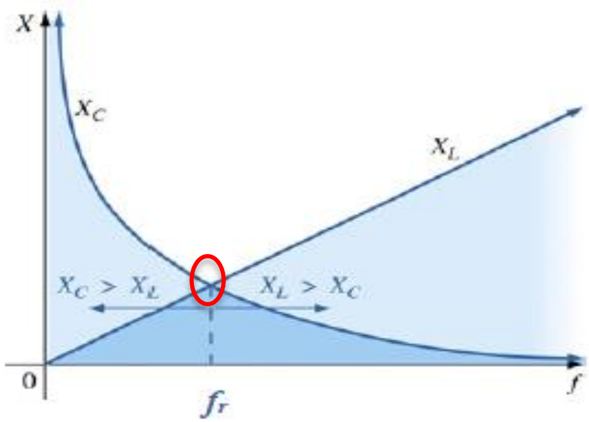
X_C *versus f*

Z_T Versus Frequency

- The total-impedance-versus-frequency curve for the series resonant circuit below can be found by applying the impedance-versus-frequency curve for each element of the equation previously shown, written in the following form:

- When X_L=X_C the resonant frequency (***f***$_r$) can be found.

$$Z_T(f) = \sqrt{[R(f)]^2 + [X_L(f) - X_C(f)]^2}$$



Frequency response of X_L and X_C of a series R-L-C circuit on the same set of axes

The Resonant Frequency (f_r)

- To find f_r, set the impedances equal and solve:

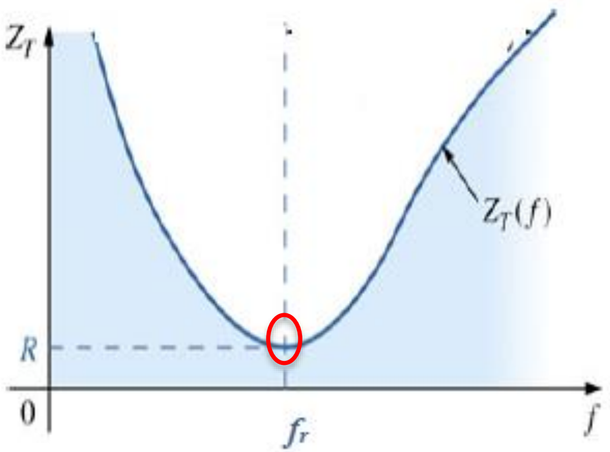
$$X_C = X_L$$

$$\frac{1}{\omega C} = \omega L \Rightarrow \omega C = \frac{1}{\omega L} \Rightarrow \omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}}, \text{since } \omega = 2\pi f$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

- This is the key equation for resonance. Total impedance at this point is shown to the right:



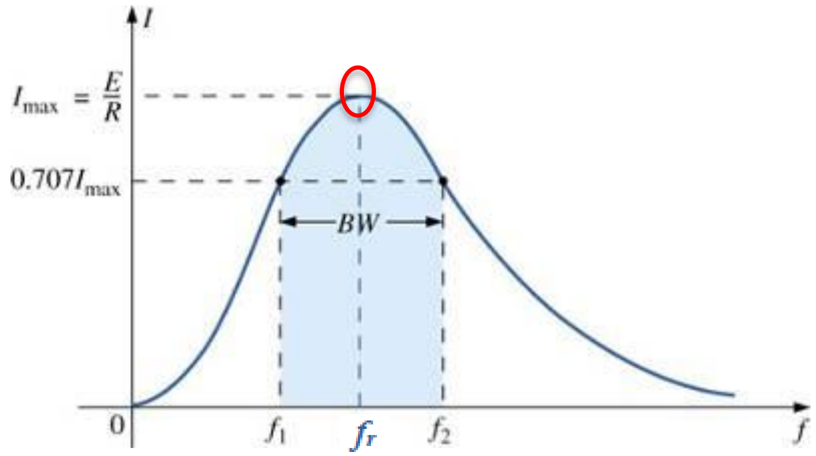
Z_T *versus f*

Current Versus Frequency

- If impedance is minimum at $\boldsymbol{f_r}$, current will be at a maximum:

$$I = \frac{E}{Z_T}$$

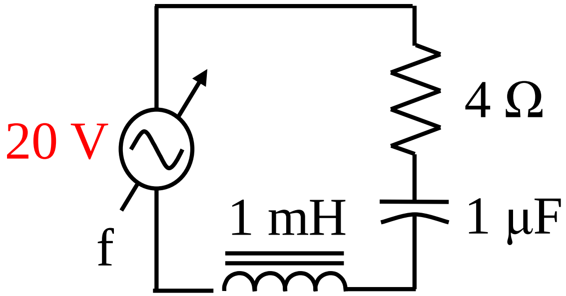
- If we now plot the magnitude of the current versus frequency for a *fixed* applied voltage *E*, we obtain the curve showing that current is maximum at $\boldsymbol{f_r}$:



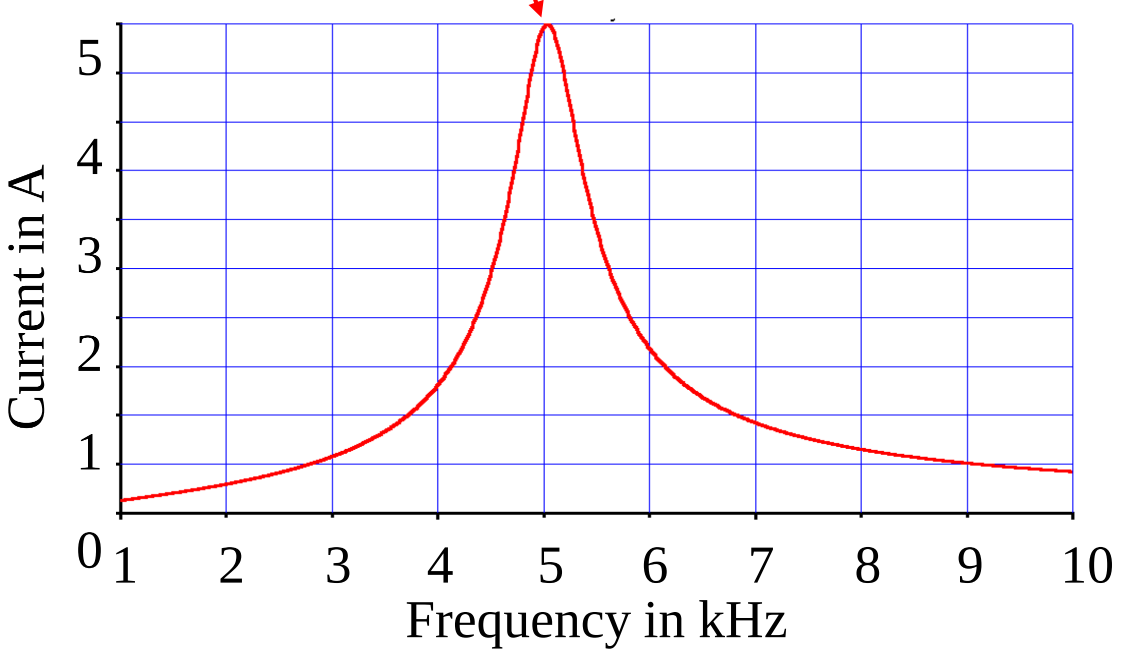
I versus f for the series resonant circuit

Cont…

Example:



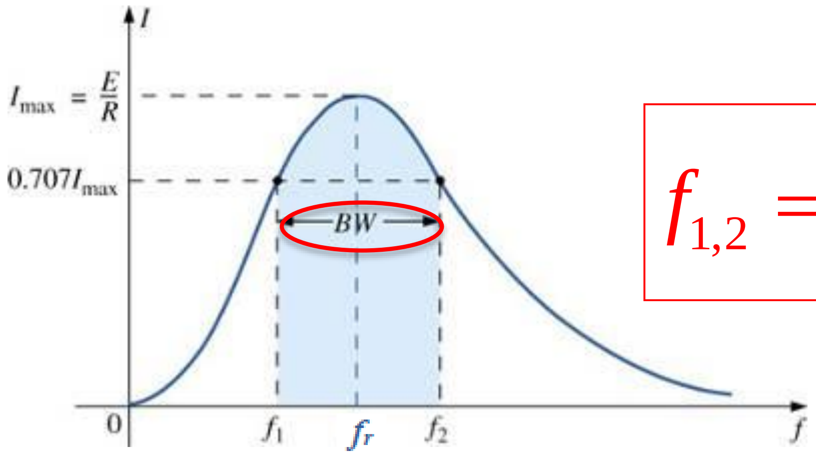
$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{1\times 10^{-3}\times 1\times 10^{-6}}} = 5.03 \text{ kHz}$$



Bandwidth (*BW*)

- Band frequencies are those that define the points on the resonance curve that are 0.707($\frac{1}{\sqrt{2}} = 0.707$ the peak current or voltage.
- Bandwidth (***BW***) is the range of frequencies between the band, or ½ power frequencies. Defined by:

$$BW = f_2 - f_1$$



$$\boxed{f_{1,2} = f_r \pm \frac{BW}{2}}$$

The Quality Factor (*Q*)

- The **quality factor (*Q*)** of a series resonant circuit is defined as the ratio of the reactive power of either the inductor or the capacitor to the average power of the resistor at resonance.
- *Q* can be found several ways:

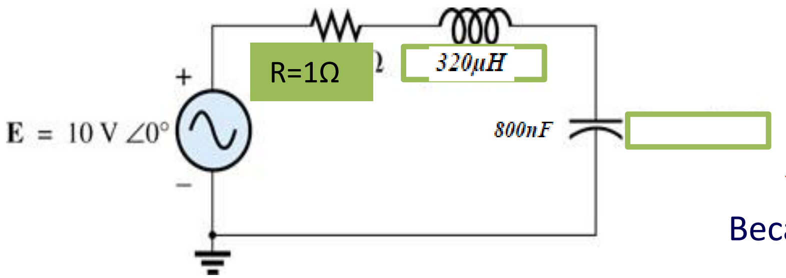
$$Q = \frac{f_r}{BW} = \frac{X_L}{R} = \frac{\omega_r L}{R} = \frac{2\pi f_r L}{R}$$

- This also gives an alternate way to find *BW*:

$$BW = \frac{f_r}{Q}$$

Example Problem 1

Determine ***f***$\boldsymbol{f_r}$***, Q, BW*** and the current (***I***) at resonance. Plot the current vs. frequency and label $\boldsymbol{f_r}$, $\boldsymbol{f_1}$, $\boldsymbol{f_2}$ and ***BW***.



$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(320\mu H)(800nF)}} = \boxed{10kHz}$$

$$X_L = \omega_r L = 2\pi f_r L = 2\pi(10kHz)(320\mu H) = 20\Omega$$

Because we are at f_r we know that $X_L = X_C$, but just to show it:

$$X_C = \frac{1}{\omega_r C} = \frac{1}{2\pi f_r C} = \frac{1}{2\pi(10kHz)(800nH)} = 20\Omega$$

We know X_L so we can find Q:

$$Q = \frac{X_L}{R} = \frac{20\Omega}{1\Omega} = \boxed{20}$$

We know Q so we can find BW:

$$BW = \frac{f_r}{Q} = \frac{10kHZ}{20} = \boxed{500Hz}$$

Now find I_{max}:

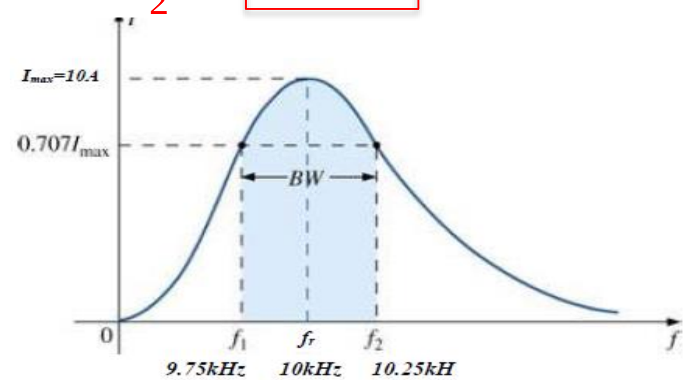
$$I = \frac{E}{Z_T} = \frac{10V}{1\Omega} = \boxed{10A}$$

Remember, since X_L=X_C at resonance, they cancel out for Z_T and only R is left.

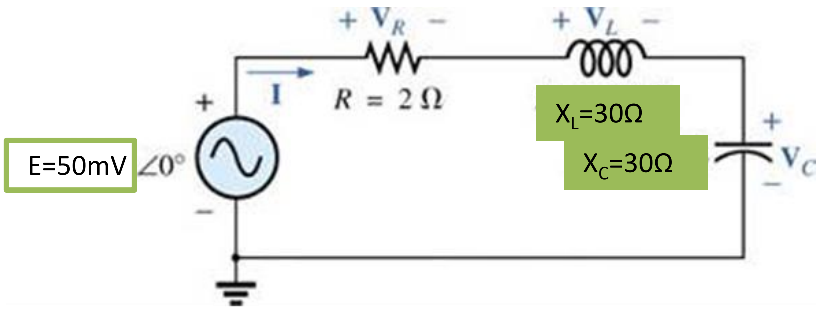
Now let's find f_1 and f_2 and then plot:

$$f_{1,2} = f_r \pm \frac{BW}{2} \Rightarrow f_2 = 10kHz + \frac{500Hz}{2} = \boxed{10.25kHz}$$

$$\Rightarrow f_1 = 10kHz - \frac{500Hz}{2} = \boxed{9.75kHz}$$



Example Problem 2



a) Find ***I, V_R, V_C, V_L*** at resonance.
b) Determine **Q** for the circuit.
c) If the $\boldsymbol{f_r}$ is 5 kHz, what is the ***BW***?
d) With $\boldsymbol{f_r}$ = 5kHz what are the values of **L** and **C**?
e) What is the power dissipated in the circuit at the ***half-power*** frequency?

a) $Z_T = R + jX_L - jX_C = 2\Omega + j30\Omega - j30\Omega = 2\Omega$

$$I = \frac{E}{Z_T} = \frac{50mV\angle 0°}{2\Omega\angle 0°} = \boxed{25mA\angle 0°}$$

$$V_R = I * R = (25mA\angle 0°)*(2\Omega\angle 0°) = \boxed{50mV\angle 0°}$$

$$V_L = I * X_L = (25mA\angle 0°)*(30\Omega\angle 90°) = \boxed{750mV\angle 90°}$$

$$V_C = I * X_C = (25mA\angle 0°)*(30\Omega\angle -90°) = \boxed{750mV\angle -90°}$$

This shows that at resonance $V_C = V_L$

b) $Q = \dfrac{X_L}{R} = \dfrac{30\Omega}{2\Omega} = \boxed{15}$

c) $BW = \dfrac{f_r}{Q} = \dfrac{5kHZ}{15} = \boxed{333.3Hz}$

d) $X_L = 2\pi f_r L =>$

$$L = \frac{X_L}{2\pi f_r} = \frac{30\Omega}{2\pi(5kHz)} = \boxed{955\mu H}$$

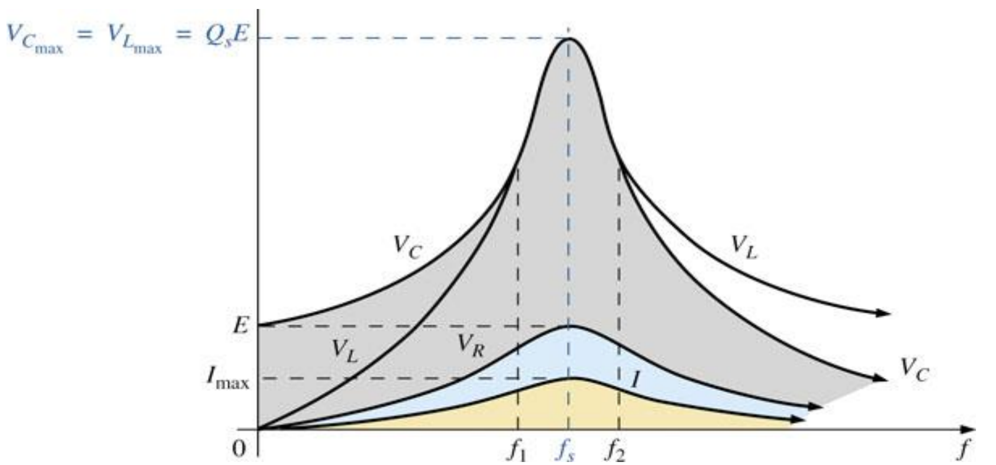
$$X_C = \frac{1}{2\pi f_r C} =>$$

$$C = \frac{1}{2\pi f_r X_C} = \frac{1}{2\pi(5kHz)(30\Omega)} = \boxed{1.06\mu F}$$

e) $P = I_{0.707}{}^2 * R = (25mA * 0.707)*(2\Omega) = \boxed{35.4mW}$

V_R, V_L, AND V_C

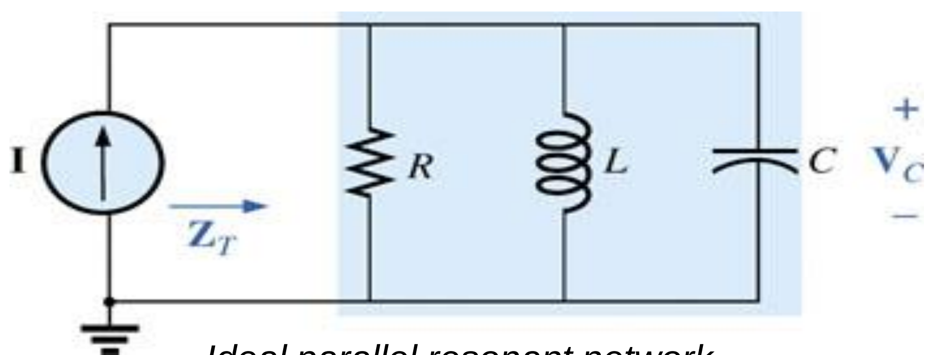
- In case you were wondering about KVL from the last problem, the below plot is what is happening with V_L and V_C at resonance.
  - V_R follows the *I* curve.
  - Until f_r is reached, V_C builds up from a value equal to the input voltage (E) because the reactance of the capacitor is infinite (open circuit) at zero frequency, but then decreases toward zero.
  - V_L increases from zero until f_r is reached, but then decreases to E.
- Notice, again, that V_L = V_C at the resonant frequency.



V_R, V_L, V_C, and I for a series resonant circuit where Qs ≥10

Parallel Resonant Circuit

- The basic format of the parallel resonant circuit is a parallel *R*-*L*-*C* combination with an applied current source.
- The parallel resonant circuit has the basic configuration shown below:

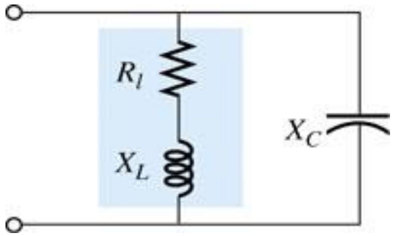


Ideal parallel resonant network

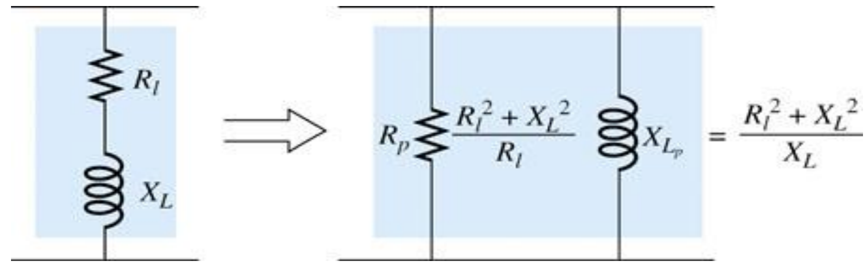
Parallel Resonance

- When L and C are in parallel and X_L equals X_C, the reactive branch currents are equal and opposite at resonance.
- Then they cancel each other to produce minimum current in the main line.
- Since the line current is minimum, the impedance is maximum.

Parallel Resonant Circuit



Practical parallel L-C network

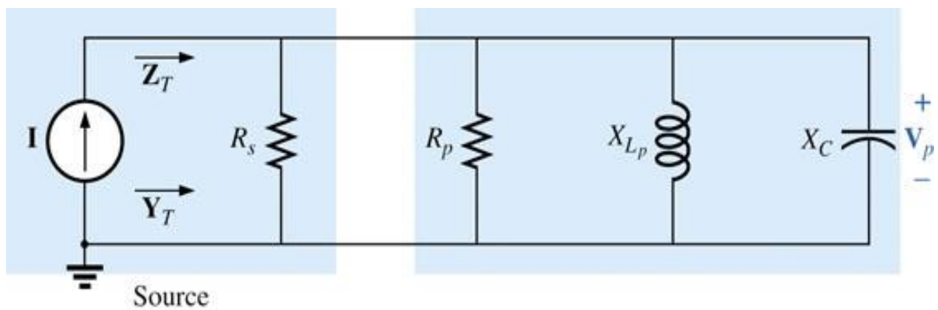


Equivalent parallel network for a series R-L combination

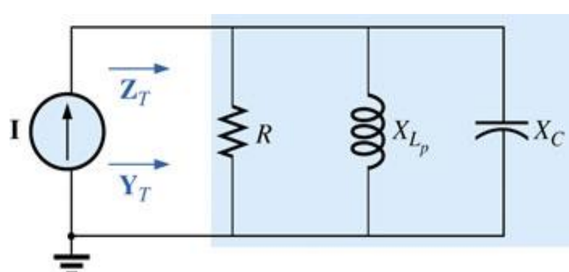
$$R_p = \frac{R_l^{\,2} + X_L^{\,2}}{R_l^{\,2}}$$

$$X_{L_p} = \frac{R_l^{\,2} + X_L^{\,2}}{X_L}$$

$$X_C = \frac{R_l^{\,2} + X_L^{\,2}}{X_L}$$



Substituting the equivalent parallel network for the series R-L combination



Substituting $R = R_s \parallel R_p$ *for the network*

Parallel Resonant Circuit

- **Unity Power Factor, f_p:**

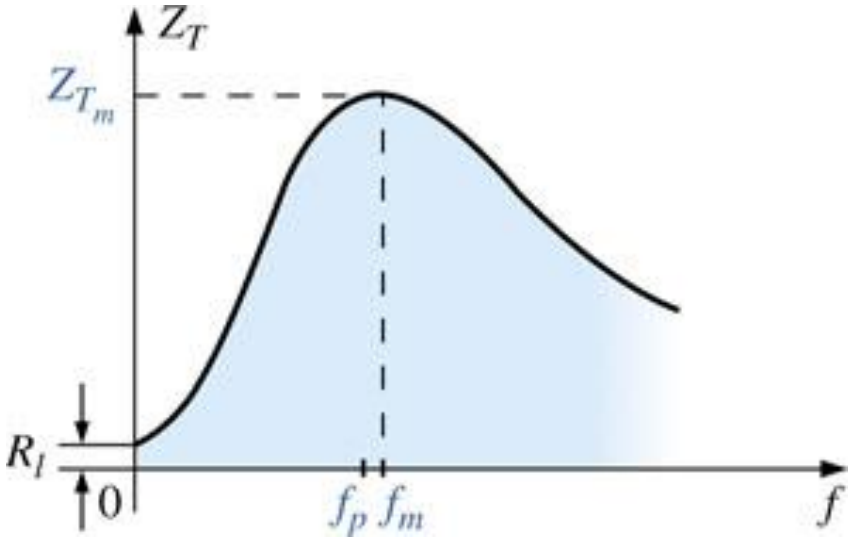
$$f_p = f_r\sqrt{1-\frac{R_l^2 C}{L}}$$

- **Maximum Impedance, f_m:**

$$f_m = f_r\sqrt{1-\frac{1}{4}\left(\frac{R_l^2 C}{L}\right)}$$

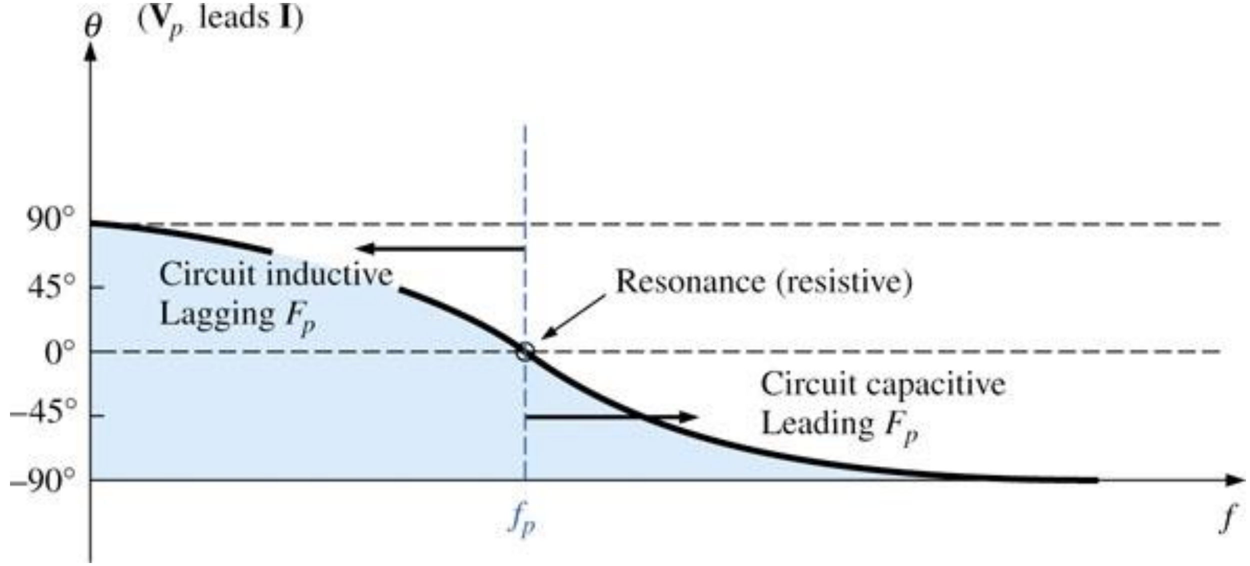
$$f_r > f_p > f_m$$

Parallel Resonant Circuit



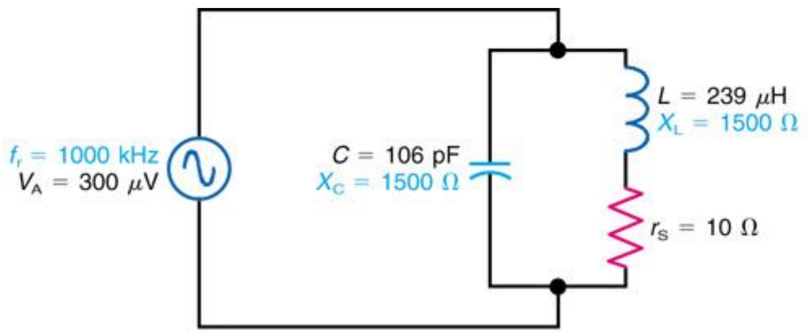
Z_T versus frequency for the parallel resonant circuit

Parallel Resonant Circuit

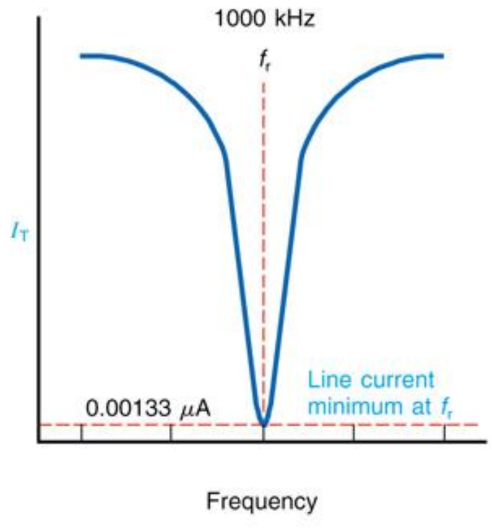


Phase plot for the parallel resonant circuit

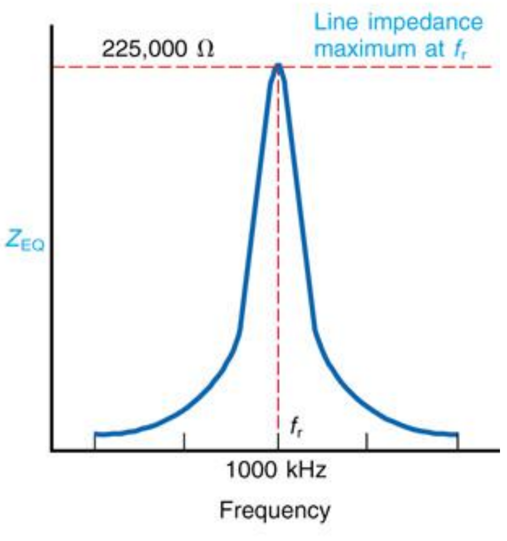
Parallel Resonance



(a)



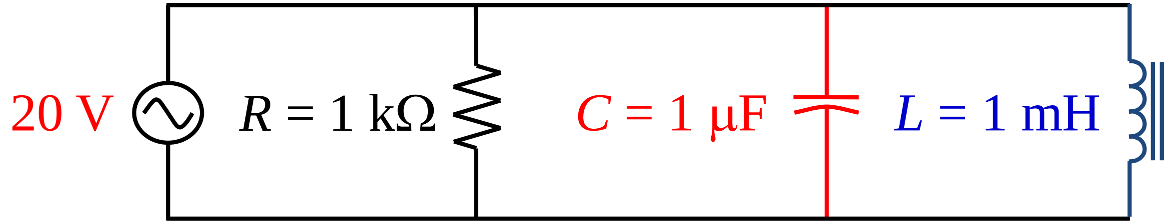
(b)

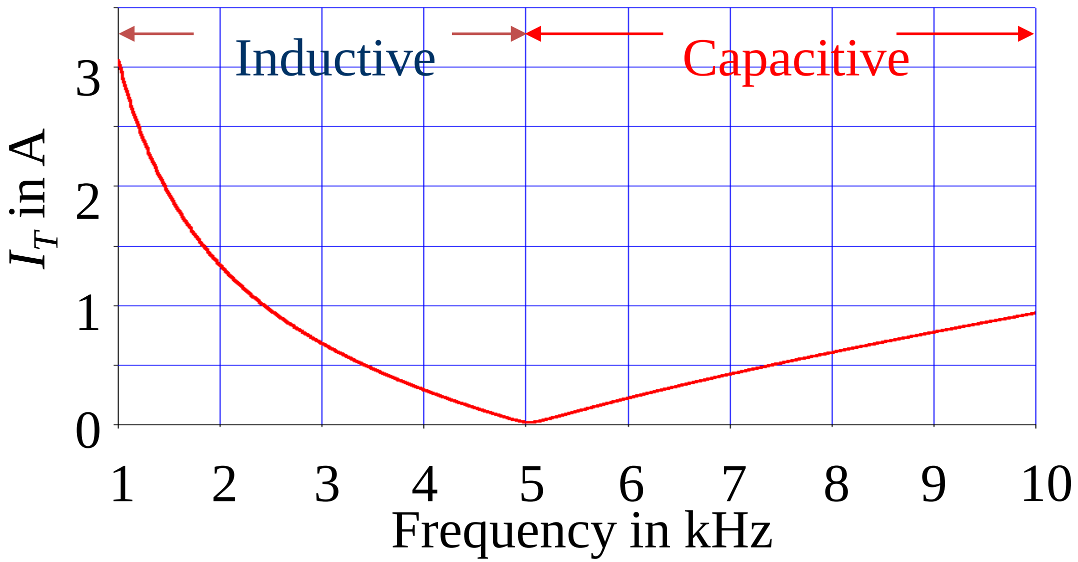


(c)

Parallel Resonance

Frequency Response





Resonant Frequency

- The formula for the resonant frequency is derived from $X_L = X_C$.

▪ For any series or parallel LC circuit, the f_r equal to

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

is the resonant frequency that makes the inductive and capacitive reactances equal.

Q Magnification Factor of Resonant Circuit

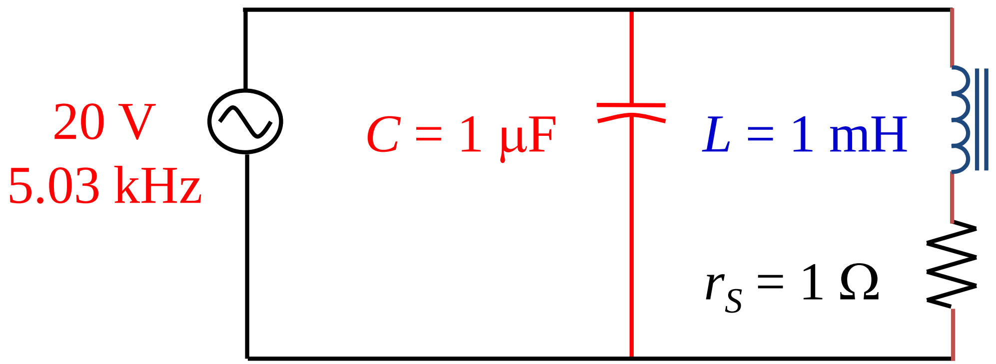
- The quality, or figure of merit, of the resonant circuit, the sharpness of resonance curve, is indicated by dimensionless parameter factor **Q**.
- The higher the ratio of the reactance at resonance to the series resistance, the higher the Q and the sharper the resonance effect.
- The Q of the resonant circuit can be considered a **magnification** factor that determines how much the voltage across L or C is increased by the resonant rise of current in a series circuit.

Cont..

- $Q = \omega_o / \Delta\omega = (\omega_o L) / R$
  - $\Delta\omega$ is the width of the curve, measured between the two values of ω for which P_{avg} has half its maximum value.
    - These points are called the *half-power points.*
- A high-*Q* circuit responds only to a narrow range of frequencies.
  - Narrow peak
- A low-*Q* circuit can detect a much broader range of frequencies.
- A radio's receiving circuit is an important application of a resonant circuit.

Q Magnification Factor of Resonant Circuit

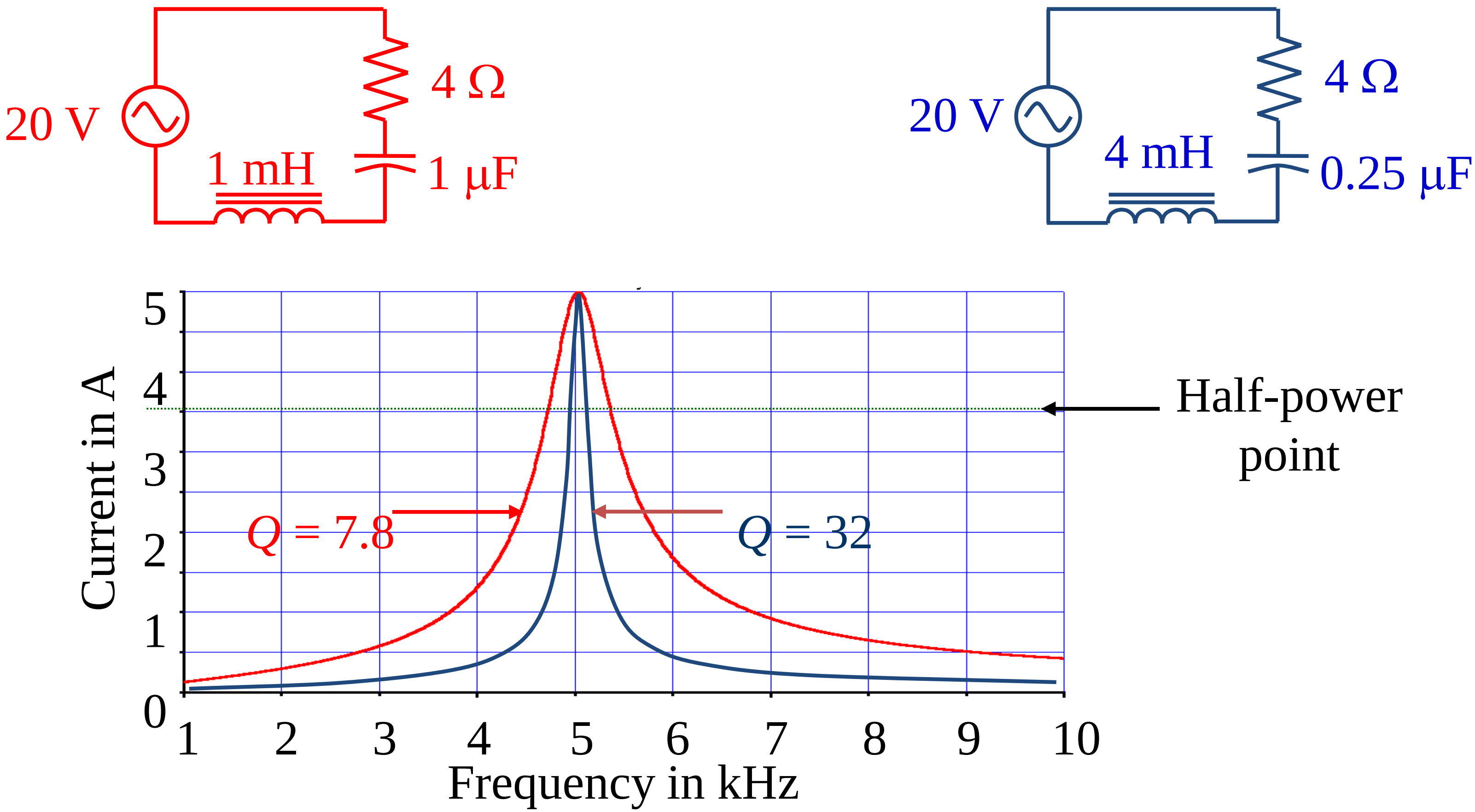
Q is often established by coil resistance.



$$Q = \frac{X_L}{r_S} = \frac{31.6}{1} = 31.6$$

Q Magnification Factor of Resonant Circuit

Increasing the *L*/*C* Ratio Raises the *Q*



Effect Of $Q_l \geq 10$

- The content of the previous section may suggest that the analysis of parallel resonant circuits is significantly more complex than that encountered for series resonant circuits.
- Fortunately, however, this is not the case since, for the majority of parallel resonant circuits, the quality factor of the coil $\boldsymbol{Q_l}$ is sufficiently large ($\boldsymbol{Q_l \geq 10}$) to permit a number of approximations that simplify the required analysis.

Effect Of $Q_l \geq 10$

- **Inductive Reactance:**

$$X_{Lp} \cong X_L \qquad X_L \cong X_C$$

- **Resonant Frequency, f_p (Unity Power Factor) and Resonant Frequency, f_m (Max V_C):**

$$f_m \cong f_p \cong f_r$$

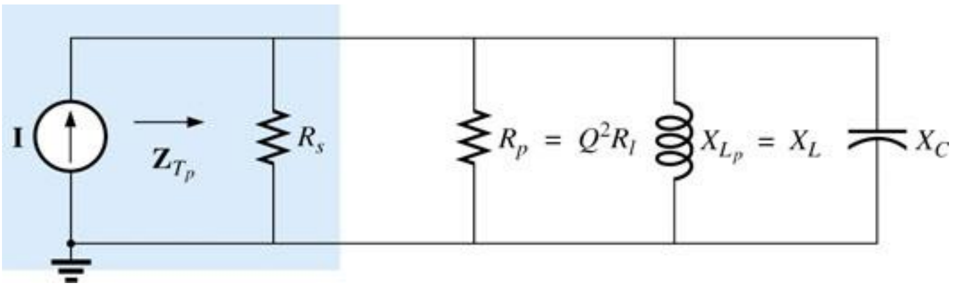
- ***R_p*** $\quad R_p \cong \dfrac{L}{R_l C}$

- ***Z_{Tp}*** $\quad Z_{T_p} \cong R_s \parallel Q_l^2 R_l$

- ***Q_p*** $\quad Q_p \cong \dfrac{R_s \parallel (Q_l^2 R_l)}{X_L}$

if $R_s >> R_p$

$Q_p \cong Q_l$



Approximate equivalent circuit for $Q_l \geq 10$

Effect Of $Q_l \geq 10$

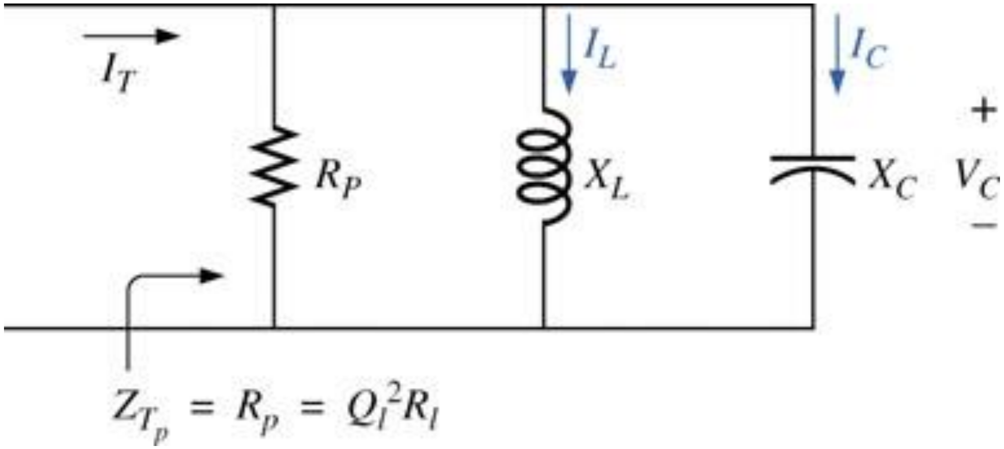
- **BW**

$$BW = f_2 - f_1 = \frac{R_l}{2\pi L}_{(R_s = \infty\Omega)} = \frac{f_p}{Q_p}$$

- **I_L and I_C**

$$I_L \cong Q_l I_T$$

$$I_C \cong Q_l I_T$$



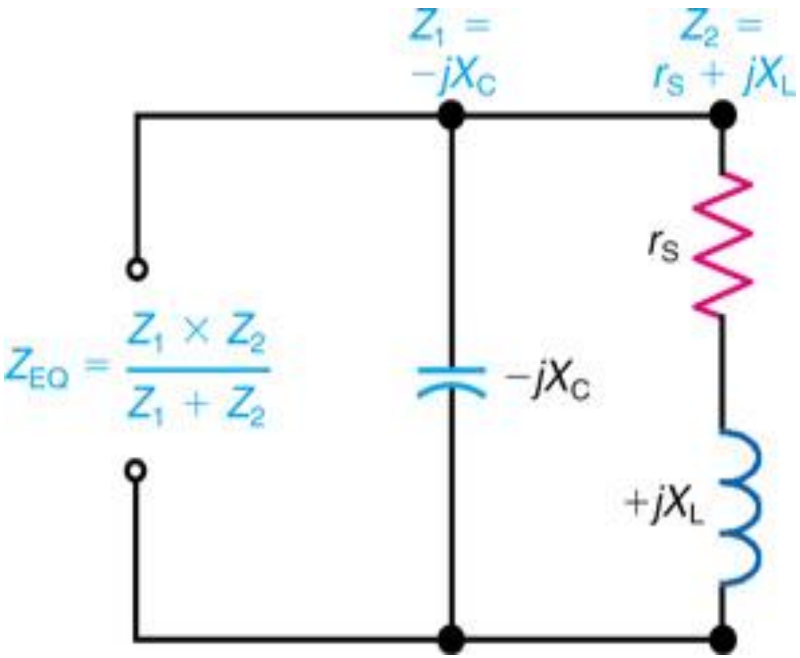
Establishing the relationship between I_C and I_L and the current I_T

Analysis of Parallel Resonant Circuits

-
    - Parallel resonance is more complex than series resonance because the reactive branch currents are <u>not</u> exactly equal when X_L equals X_C.

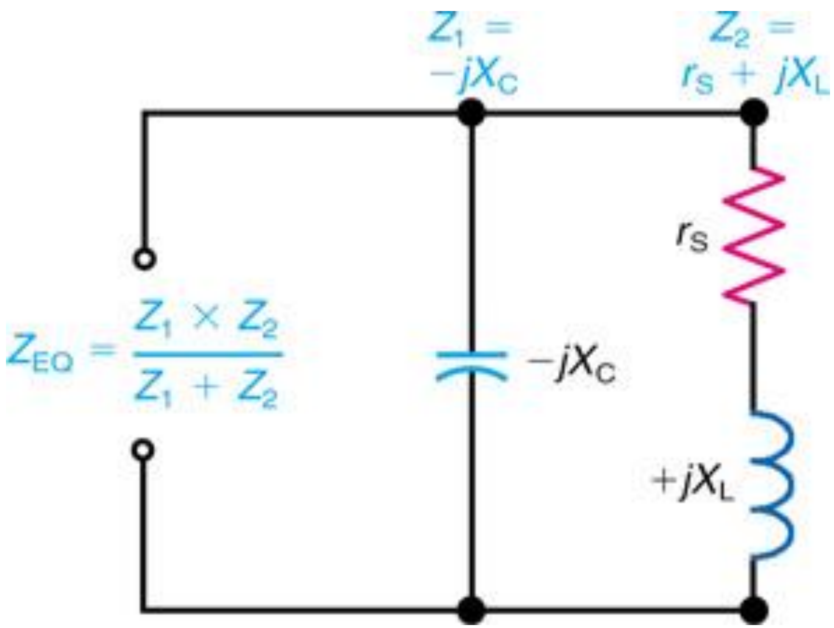
    - The coil has its series resistance r_s in the X_L branch, whereas the capacitor has only X_C in its branch.

    - For high-Q circuits, we consider r_s negligible.



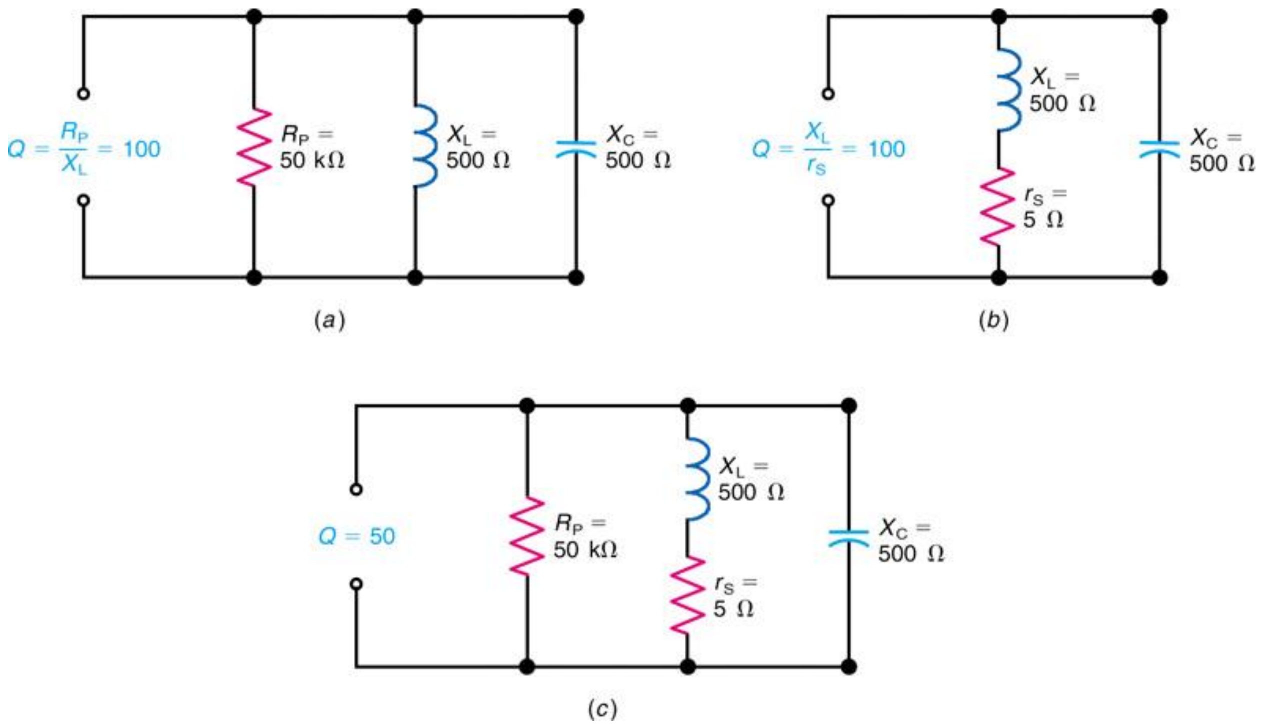
Analysis of Parallel Resonant Circuits

- In low-Q circuits, the inductive branch must be analyzed as a complex impedance with X_L and r_s in series.
- This impedance is in parallel with X_C, as shown in Fig. The total impedance Z_{EQ} can then be calculated by using complex numbers.



25-10: Damping of Parallel Resonant Circuits

- In Fig. (a), the shunt R_P across L and C is a **damping resistance** because it lowers the Q of the tuned circuit.
- The R_P may represent the resistance of the external source driving the parallel resonant circuit, or R_p can be an actual resistor.
- Using the parallel R_P to reduce Q is better than increasing r_s.



Summary Table

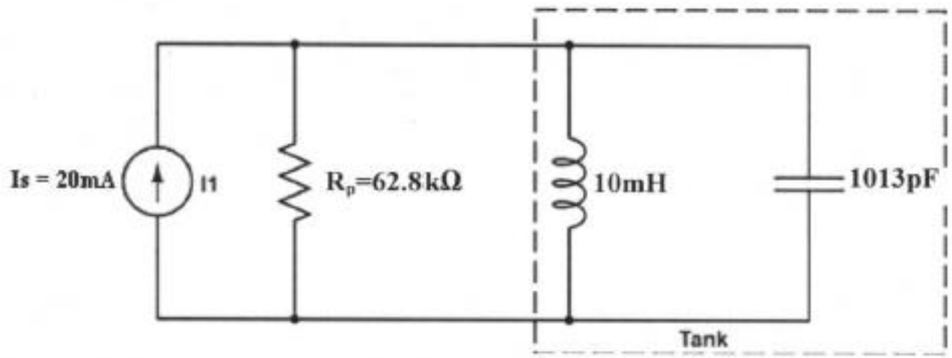
TABLE 20.1

Parallel resonant circuit $(f_s = 1/(2\pi\sqrt{LC}))$.

	Any Q_l	$Q_l \geq 10$	$Q_l \geq 10,\ R_s >> Q_l^2 R_l$
f_p	$f_s\sqrt{1 - \frac{R_l^2 C}{L}}$	f_s	f_s
f_m	$f_s\sqrt{1 - \frac{1}{4}\left[\frac{R_l^2 C}{L}\right]}$	f_s	f_s
Z_{T_p}	$R_s \parallel R_p = R_s \parallel \left(\frac{R_l^2 + X_L^2}{R_l}\right)$	$R_s \parallel Q_l^2 R_l$	$Q_l^2 R_l$
Z_{T_m}	$R_s \parallel \mathbf{Z}_{R\text{-}L} \parallel \mathbf{Z}_C$	$R_s \parallel Q_l^2 R_l$	$Q_l^2 R_l$
Q_p	$\frac{Z_{T_p}}{X_{L_p}} = \frac{Z_{T_p}}{X_C}$	$\frac{Z_{T_p}}{X_L} = \frac{Z_{T_p}}{X_C}$	Q_l
BW	$\frac{f_p}{Q_p}$ or $\frac{f_m}{Q_p}$	$\frac{f_p}{Q_p} = \frac{f_s}{Q_p}$	$\frac{f_p}{Q_l} = \frac{f_s}{Q_l}$
I_L, I_C	Network analysis	$I_L = I_C = Q_l I_T$	$I_L = I_C = Q_l I_T$

Example Problem 3

Find the resonant frequency (f_r), ***Q, BW, f_1, f_2*** and draw the frequency response for the circuit below:



$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10mH * 1013pF}} = \boxed{50kHz}$$

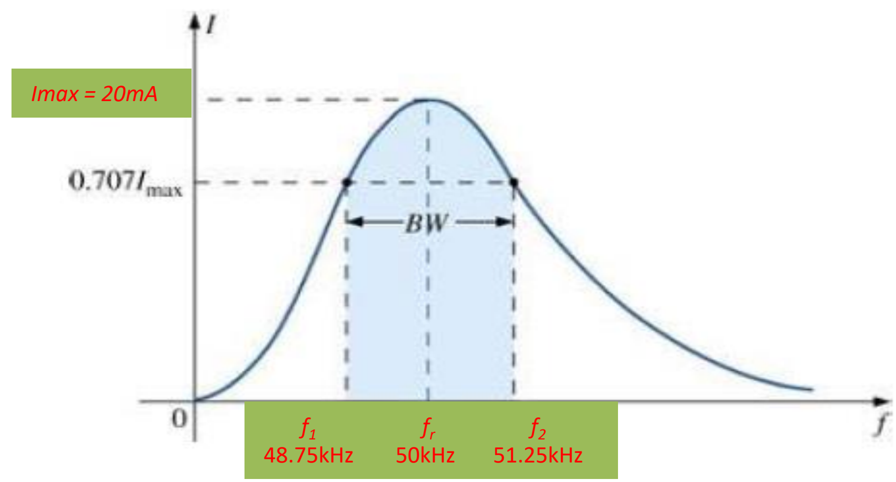
$$X_L = 2\pi f_r L = 2\pi(50kHz)(10mH) = 3141.5\Omega$$

$$Q_p \cong \frac{R_p}{X_L} = \frac{62.8k\Omega}{3145.5\Omega} = \boxed{20}$$

$$BW = \frac{f_p}{Q_p} = \frac{50kHz}{20} = \boxed{2.5kHz}$$

$$f_{1,2} = f_r \pm \frac{BW}{2} \Rightarrow f_2 = 50kHz + \frac{2.5kHz}{2} = \boxed{51.25kHz}$$

$$\Rightarrow f_1 = 50kHz - \frac{2.5kHz}{2} = \boxed{48.75kHz}$$



Choosing *L* and *C* for a Resonant Circuit

- A known value for either L or C is needed to calculate the other.
- In some cases, particularly at very high frequencies, C must be the minimum possible value.
- At medium frequencies, we can choose L for the general case when an X_L of 1000 Ω is desirable and can be obtained.
- For resonance at 159 kHz with a 1-mH L, the required C is 0.001 μF.
- This value of C can be calculated for an X_C of 1000 Ω, equal to X_L at the f_r of 159 kHz.

Tuning

- **Tuning** means obtaining resonance at different frequencies by varying either L or C.
- As illustrated in Fig. 25-12, the variable capacitance C can be adjusted to tune the series LC circuit to resonance at any one of five different frequencies.

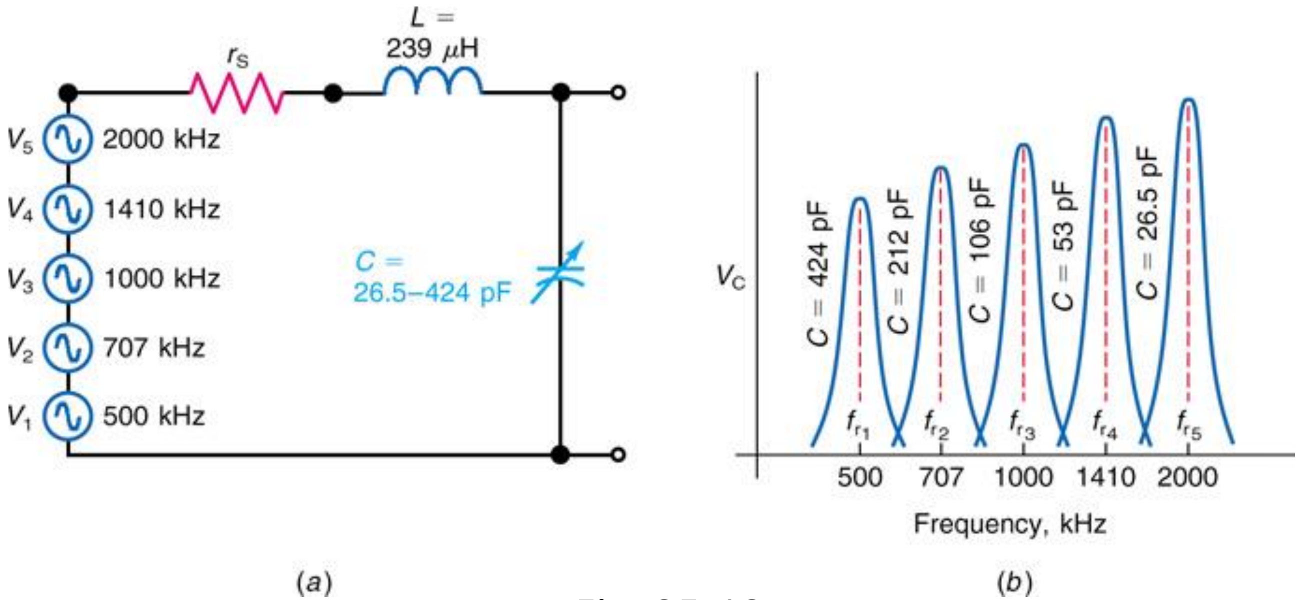
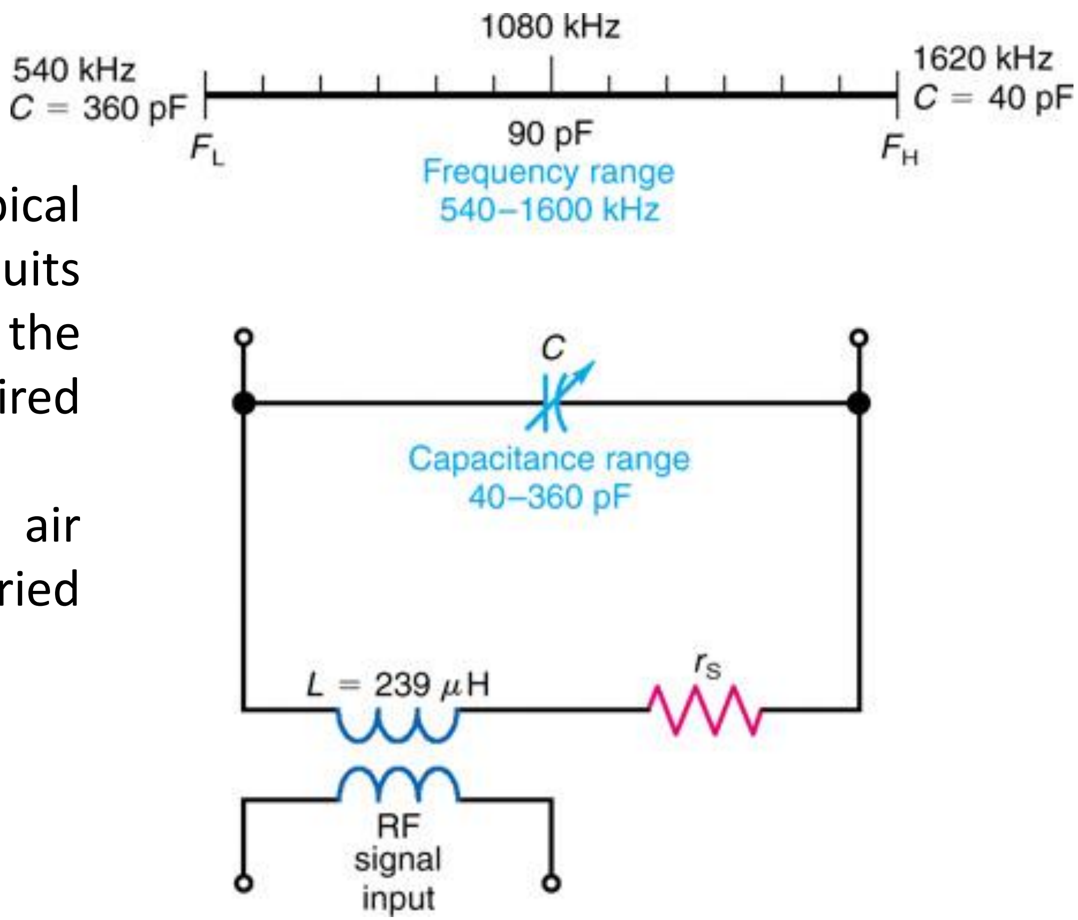


Fig. 25-12

Tuning



- Fig. illustrates a typical application of resonant circuits in tuning a receiver to the carrier frequency of a desired radio station.
- The tuning is done by the air capacitor C, which can be varied from 360 pF to 40 pF.

Mistuning

- When the frequency of the input voltage and the resonant frequency of a series LC circuit are <u>not</u> the same, the mistuned circuit has very little output compared with the Q rise in voltage at resonance.
- Similarly, when a parallel circuit is mistuned, it does <u>not</u> have a high value of impedance
- The net reactance of resonance makes the LC circuit either inductive or capacitive.

Points to remember:

- Characteristics of series resonance circuit:
  • Minimum impedance
  • Maximum circuit current
  • $\cos(\phi) = 1$, hence current and voltage becomes in phase.
  • Circuit current becomes proportional to circuit resistance i.e. $I \sim 1/R$

- Uses of series resonance circuit:
  • As frequency selection circuit in radio and TV tuner circuits.
  • As band pass filter circuit.

Points to remember:

- Characteristics of parallel resonance circuit:
  - Maximum impedance
  - Minimum circuit current
  - $\cos(\phi) = 1$, hence voltage and current becomes in phase
  - Circuit current depends on circuit impedance, $Z = L/C$ or $I \sim -(1/R)$

- Uses of parallel resonance circuit:
  - As a Band Stop Filter
  - As a tank circuit in Oscillators
  - As a plate load in IF and RF amplifiers
  - As I.F. trap in aerial circuit of radio as well as TV receivers.

Comparison between series resonance and parallel resonance circuits

Specifications	**Series resonance circuit(Acceptors)**	**Parallel resonance circuit(Rejectors)**
Impedance at resonance	Minimum	Maximum
Current at resonance	Maximum	Minimum
Effective impedance	R	L/CR
Resonant frequency	$1/(2*\pi*(LC)^{0.5})$	$(1/2*\pi)*\{(1/LC)- R^2/L^2\}^{0.5}$
It magnifies	Voltage	Current
It is known as	Acceptor circuit	Rejector circuit
Power Factor	Unity	Unity

Work done problems

1. A constant voltage of frequency, 1 MHz is applied to a lossy inductor (r in series with L), in series with variable capacitor, C The current drawn is maximum, when C = 400 pF; while current is reduced to (2/1) of the above value, when C = 450 pF. Find the values of r and L. Calculate also the quality factor of the coil, and the bandwidth.

- Solution:

$f = 1\text{ MHz} = 10^6\text{ Hz} \qquad \omega = 2\pi f \qquad C = 400\, pF = 400 \cdot 10^{-12}\, F$

$I_{max} = V/r$ as $X_L = X_C$ $\qquad X_c = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi \cdot 10^6 \times 400 \cdot 10^{-12}} = 398\,\Omega$

$X_L = X_c = 2\pi f L = 398\,\Omega \qquad L = \dfrac{398.0}{2\pi \cdot 10^6} = 63.34\,\mu H$

$C_1 = 450\,pF \qquad X_{C1} == \dfrac{1}{2\pi \cdot 10^6 \times 450 \cdot 10^{-12}} = 353.7\,\Omega$

$Z\angle\phi = r + j(X_L - X_{C1}) = r + j(398.0 - 353.7) = (r + j44.3)\,\Omega$

$I = \dfrac{I_{max}}{\sqrt{2}} = \dfrac{V}{\sqrt{2}\cdot r} = \dfrac{V}{Z} = \dfrac{V}{\sqrt{r^2 + (44.3)^2}}$

From above, $\sqrt{2}\cdot r = \sqrt{r^2 + (44.3)^2}$ $\quad$ or $2r^2 = r^2 + (44.3)^2$

or $r = 44.3\,\Omega$

The quality factor of the coil is $Q = \dfrac{X_L}{r} = \dfrac{398.0}{44.3} = 8.984$

The band with is

$\Delta f = f_2 - f_1 = \dfrac{r}{2\pi L} = \dfrac{44.3}{2\pi \times 63.34 \cdot 10^{-6}} = \dfrac{44.3}{398 \cdot 10^{-6}} = 0.1113 \cdot 10^6 = 0.1113\, MHz$

$= 111.3 \cdot 10^3 = 111.3\, kHz$

- 2. A coil, having a resistance of 15 Ω and an inductance of 0.75 H, is connected in series with a capacitor (Fig. a.) The circuit draws maximum current, when a voltage of 200 V at 50 Hz is applied. A second capacitor is then connected in parallel to the circuit (Fig.b). What should be its value, such that the combination acts like a non-inductive resistance, with the same voltage (200 V) at 100 Hz? Calculate also the current drawn by the two circuits.

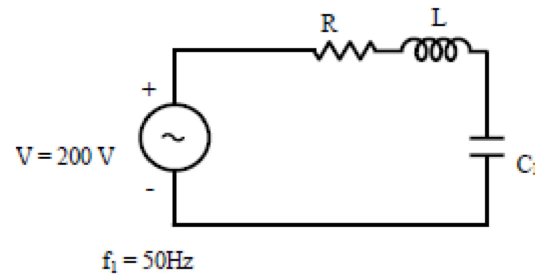


Fig.a

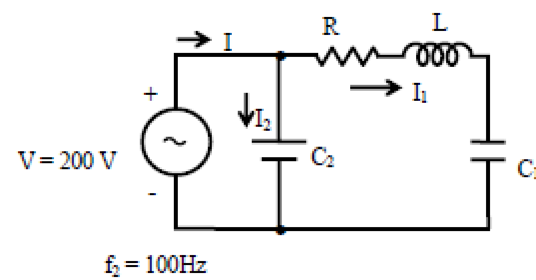


Fig.b

Solution

$f_1 = 50$ Hz V = 200 V R = 15 Ω L = 0.75 H

From the condition of resonance at 50 Hz in the series circuit,

$$X_{L1} = \omega_1 L = 2\pi f_1 L = X_{C1} = \frac{1}{\omega_1 C_1} = \frac{1}{2\pi f_1 C_1}$$

$$\text{So, } C_1 = \frac{1}{(2\pi f_1)^2 L} = \frac{1}{(2\pi \cdot 50)^2 \times 0.75} = 13.5 \cdot 10^{-6} = 13.5\ \mu F$$

The maximum current drawn from the supply is, $I_{max} = V / R = 200/15 = 13.33\ A$

$f_2 = 100$ Hz $\omega_2 = 2\pi f_2 = 2\pi \cdot 100 = 628.3\ rad/s$

$X_{L2} = 2\pi f_2 L = 2\pi \cdot 100 \cdot 0.75 = 471.24\ \Omega$

$$X_{C2} = \frac{1}{2\pi f_2 C_1} = \frac{1}{2\pi \cdot 100 \cdot 13.5 \cdot 10^{-6}} = 117.8.\ \Omega$$

$Z_1 \angle \phi_1 = R + j(X_{L2} - X_{C2}) = 15 + j(471.24 - 117.8) = 15 + j353.44$

$= 353.75 \angle 87.57°\ \Omega$

$$Y_1 \angle -\phi_1 = \frac{1}{Z_1 \angle \phi_1} = \frac{1}{15 + j353.44} = \frac{1}{353.75 \angle 87.57°} = 2.827 \cdot 10^{-3} \angle -87.57°$$

$= (0.12 - j2.824) \cdot 10^{-3}\ \Omega^{-1}$

$Y_2 = 1/Z_2 = j(\omega_2 C_2)$

As the combination is resistive in nature, the total admittance is

$Y \angle 0° = Y + j0 = Y_1 + Y_2 = (0.12 - j2.824) \cdot 10^{-3} + j\omega_2 C_2$

From the above expression, $\omega_2 C_2 = 628.3 \cdot C_2 = 2.824 \cdot 10^{-3}$

$$\text{or, } C_2 = \frac{2.824 \cdot 10^{-3}}{628.3} = 4.5 \cdot 10^{-6} = 4.5\ \mu F$$

The total admittance is $Y = 0.12 \cdot 10^{-3}\ \Omega^{-1}$

The total impedance is $Z = 1/Y = 1/(0.12 \cdot 10^{-3}) = 8.33 \cdot 10^3\ \Omega = 8.33\ k\Omega$

The total current drawn from the supply is

$$I = V \cdot Y = V / Z = 200 \times 0.12 \cdot 10^{-3} = 0.024\ A = 24 \cdot 10^{-3} = 24\ mA$$

Resonance in an AC Circuit

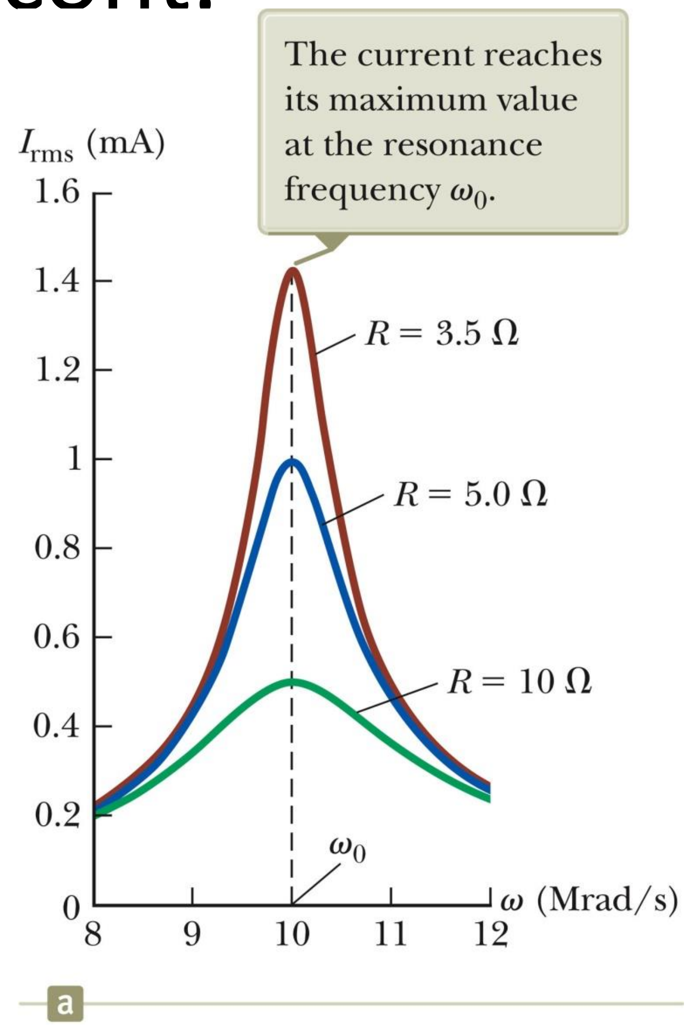
- *Resonance* occurs at the frequency ω_o where the current has its maximum value.
  - To achieve maximum current, the impedance must have a minimum value.
  - This occurs when $X_L = X_C$
  - Solving for the frequency gives

$$\omega_o = \frac{1}{\sqrt{LC}}$$

- The resonance frequency also corresponds to the natural frequency of oscillation of an *LC* circuit.
- The rms current has a maximum value when the frequency of the applied voltage matches the natural oscillator frequency.
- At the resonance frequency, the current is in phase with the applied voltage.

Resonance, cont.

- Resonance occurs at the same frequency regardless of the value of *R.*
- As *R* decreases, the curve becomes narrower and taller.
- Theoretically, if *R* = 0 the current would be infinite at resonance.
  - Real circuits always have some resistance.



Quality Factor

- The sharpness of the resonance curve is usually described by a dimensionless parameter known as the quality factor, Q.
- $Q = \omega_o / \Delta\omega = (\omega_o L) / R$
  - $\Delta\omega$ is the width of the curve, measured between the two values of ω for which P_{avg} has half its maximum value.
    - These points are called the *half-power points.*
- A high-*Q* circuit responds only to a narrow range of frequencies.
  - Narrow peak
- A low-*Q* circuit can detect a much broader range of frequencies.
- A radio’s receiving circuit is an important application of a resonant circuit.

UNIT-V
Network Theorems

Network Theorems

- Introduction
- Linear Circuits and Superposition
- Thevenin’s theorem
- Norton’s theorem
- Maximum power transfer theorem
- Millman's theorem
- Reciprocity theorem
- Substitution theorem
- Compensation theorem
- Tellegen’s theorem

Introduction

A large complex circuits

Simplify circuit analysis

Circuit Theorems

Thevenin's theorem
Linearity & Superposition
Millman's theorem
Compensation theorem
Tellegen's theorem

Norton's theorem
Max. Power transfer
Substitution theorem
Reciprocity theorem

Network Theorems

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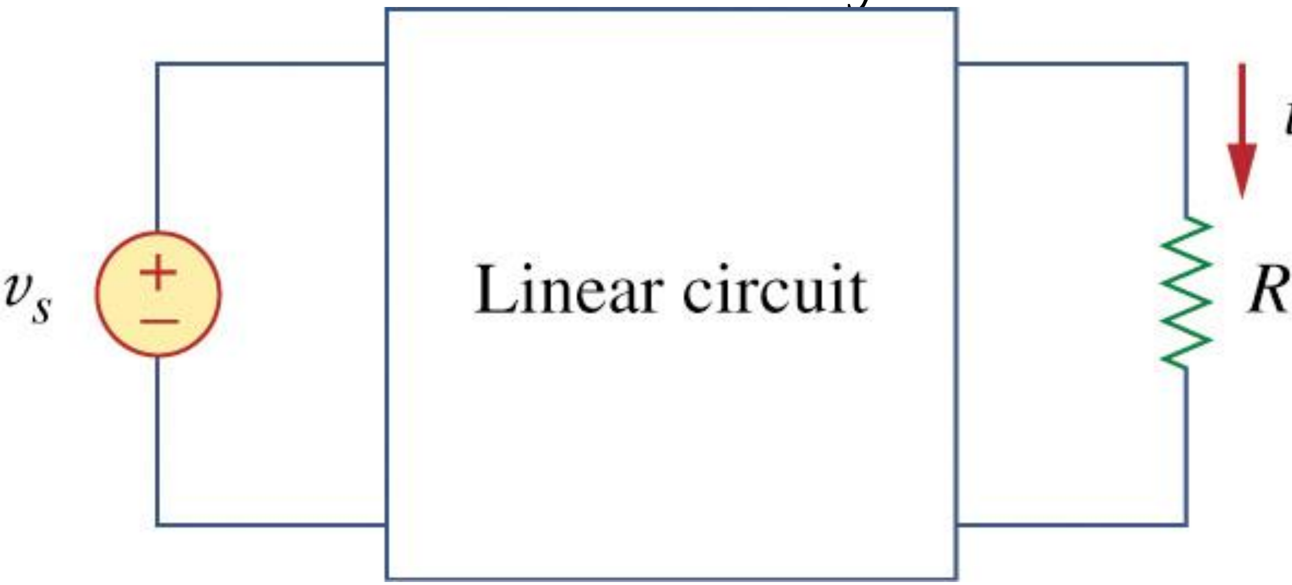
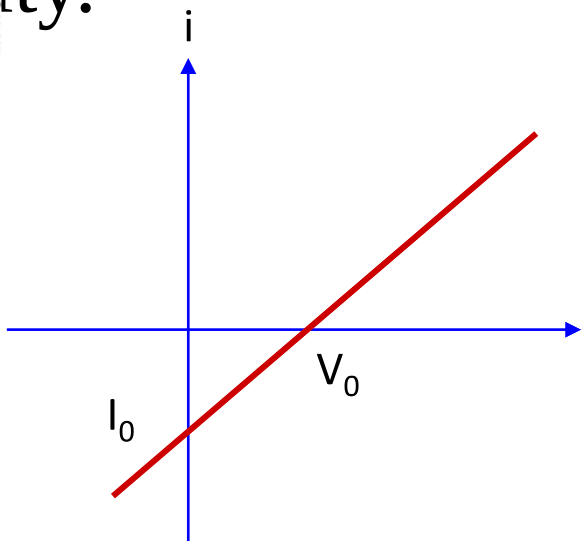
Linear Circuits

- A linear circuit is one whose output is linearly related (or directly proportional) to its input Fig. 1
- Linear circuits obey both the properties of



ity.



Linearity Property

Homogeneity property (Scaling)

$$i \rightarrow v = iR$$

$$ki \rightarrow kv = kiR$$

Additivity property

$$i_1 \rightarrow \quad v_1 = i_1 R$$

$$i_2 \rightarrow v_2 = i_2 R$$

$$i_1 + i_2 \rightarrow (i_1 + i_2)R = i_1 R + i_2 R = v_1 + v_2$$

Linear Circuits

- Linear circuit consist of
  - linear elements
  - linear dependent sources
  - independent sources

$$v_s = 10\text{V} \rightarrow i = 2\text{A}$$

$$v_s = 1\text{V} \rightarrow i = 0.2\text{A}$$

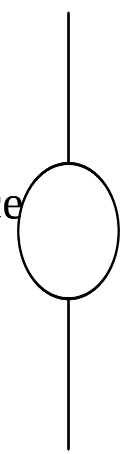
$$v_s = 5\text{mV} \leftarrow i = 1\text{mA}$$

$$p = i^2 R = \frac{v^2}{R} \ : nonlinear$$

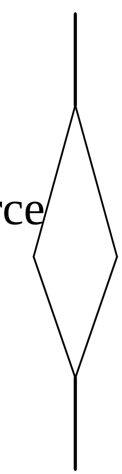
Independent and Dependent sources

- We also will classified sources as Independent and Dependent sources
- Independent source establishes a voltage or a current in a circuit without relying on a voltage or current elsewhere in the circuit
- Dependent sources establishes a voltage or a current in a circuit whose value depends on the value of a voltage or a current elsewhere in the circuit
- We will use circle to represent Independent source and diamond shape to represent Dependent sources

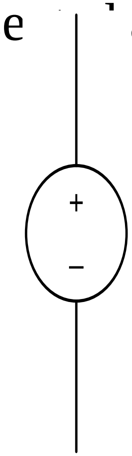
Independent source



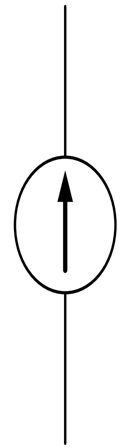
Dependent source



- Independent and dependent voltage and current sources can be represented as



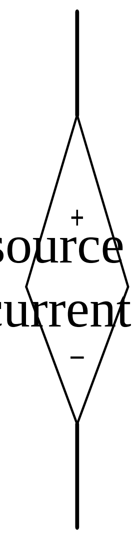
5
V



3
A

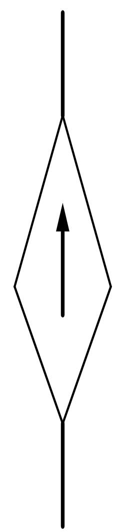
Independent voltage source Independent current source

Dependent voltage source
Voltage depend on current



4 i_x
V

where i_x is some
current
through an
element

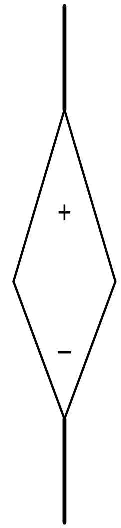


Dependent current sour
Current depend on volt

4 v_x
A

where v_x is some
voltage
across an element

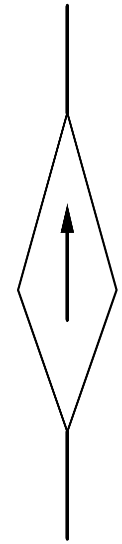
The dependent sources can be also as



$4\ v_x$
V

where v_x is some current through an element

Dependent voltage source
Voltage depend on voltage



$7\ i_x$
A

where i_x is some voltage across an element

Dependent current source
Current depend on current

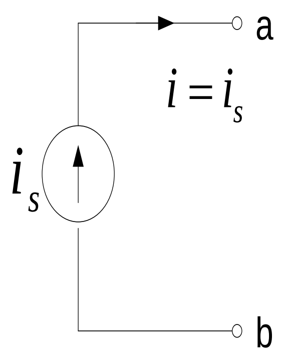
Superposition Principle

- Because the circuit is linear we can find the response of the circuit to each source acting alone, and then add them up to find the response of the circuit to all sources acting together. This is known as the **superposition principle**.

- *The **superposition principle** states that the voltage across (or the current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.*

Turning sources off

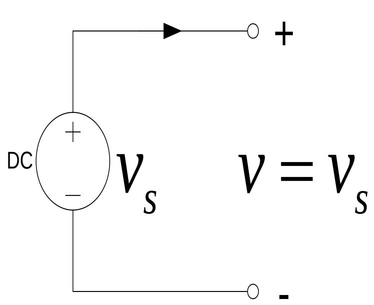
Current source:



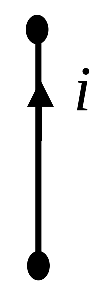
We replace it by a current source where $i_s = 0$

An open-circuit

Voltage source:



We replace it by a voltage source where $v_s = 0$



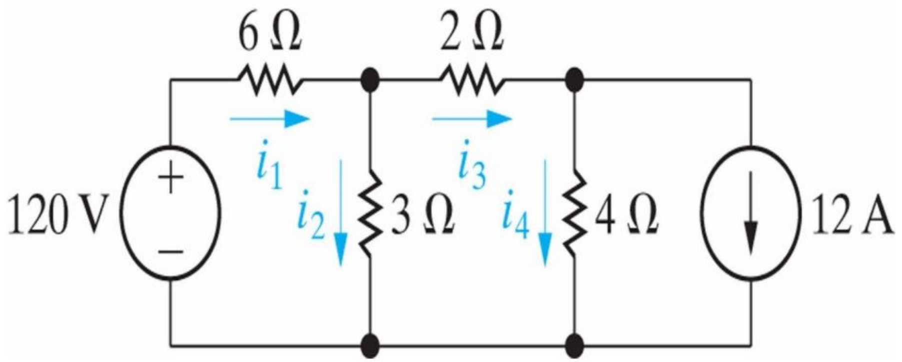
An short-circuit

Steps in Applying the Superposition Principle

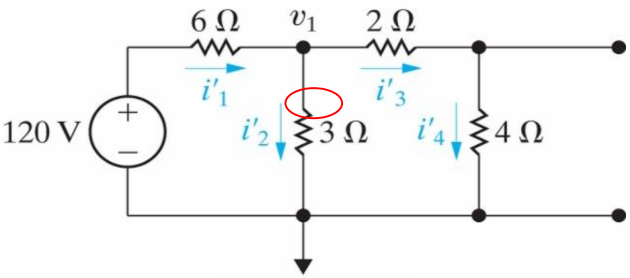
1. Turn off all independent sources except one. Find the output (voltage or current) due to the active source.
2. Repeat step 1 for each of the other independent sources.
3. Find the total output by adding algebraically all of the results found in steps 1 & 2 above.

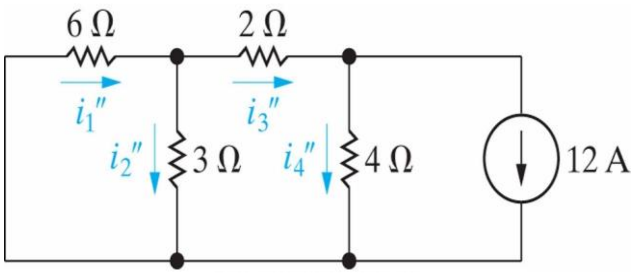
- In some cases, but certainly not all, superposition can simplify the analysis.

Superposition



The branch currents i_1, i_2, i_3 and i_4	=	The branch currents resulting from the 120 V voltage source i'_1, i'_2, i'_3 and i'_4	+	The branch currents resulting from the 12 A current source i''_1, i''_2, i''_3 and i''_4





$$\frac{v_1 - 120}{6} + \frac{v_1}{3} + \frac{v_1}{2+4} = 0, \qquad v_1 = 30 \text{ V}.$$

$$\frac{v_3}{3} + \frac{v_3}{6} + \frac{v_3 - v_4}{2} = 0,$$

$$\frac{v_4 - v_3}{2} + \frac{v_4}{4} + 12 = 0.$$

$$i_1'' = \frac{-v_3}{6} = \frac{12}{6} = 2 \text{ A},$$

$$i_1' = \frac{120 - 30}{6} = 15 \text{ A},$$

$$i_2'' = \frac{v_3}{3} = \frac{-12}{3} = -4 \text{ A},$$

$$v_3 = -12 \text{ V},$$

$$i_2' = \frac{30}{3} = 10 \text{ A},$$

$$i_3'' = \frac{v_3 - v_4}{2} = \frac{-12 + 24}{2} = 6 \text{ A},$$

$$v_4 = -24 \text{ V}.$$

$$i_3' = i_4' = \frac{30}{6} = 5 \text{ A}.$$

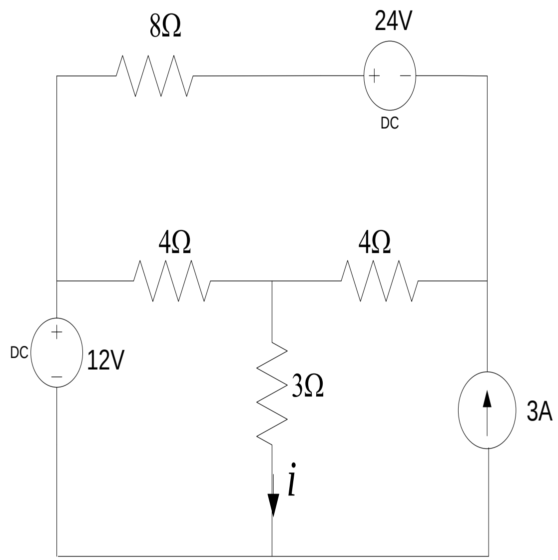
$$i_4'' = \frac{v_4}{4} = \frac{-24}{4} = -6 \text{ A}.$$

$$i_1 = i_1' + i_1'' = 15 + 2 = 17 \text{ A}, \qquad i_2 = i_2' + i_2'' = 10 - 4 = 6 \text{ A},$$

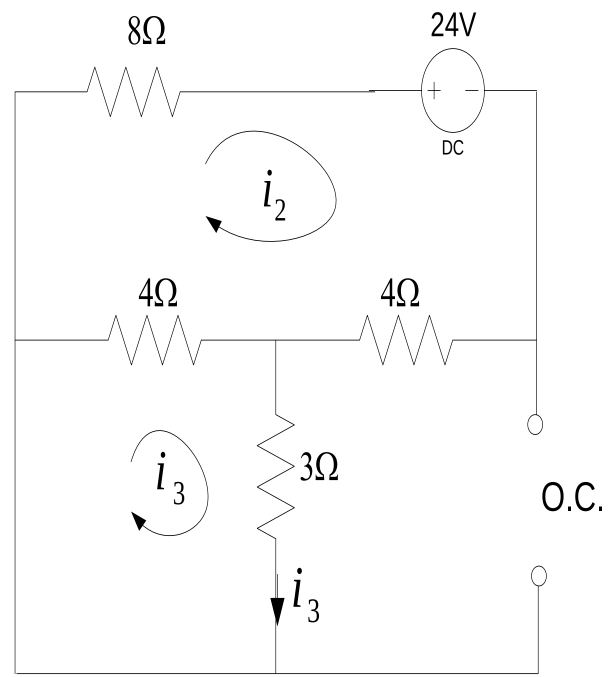
$$i_3 = i_3' + i_3'' = 5 + 6 = 11 \text{ A}, \qquad i_4 = i_4' + i_4'' = 5 - 6 = -1 \text{ A}.$$

Please, solve it using the node-voltage method

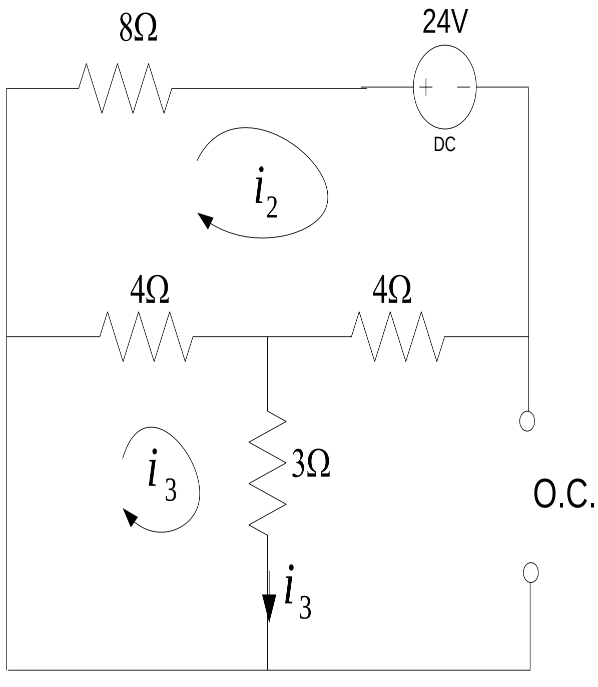
Example: In the circuit below, find the current *i* by superposition



Turn off the 3A & 12V sources:



$$\begin{pmatrix} 4+8+4 & -4 \\ -4 & 4+3 \end{pmatrix} \begin{pmatrix} i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} -24 \\ 0 \end{pmatrix}$$



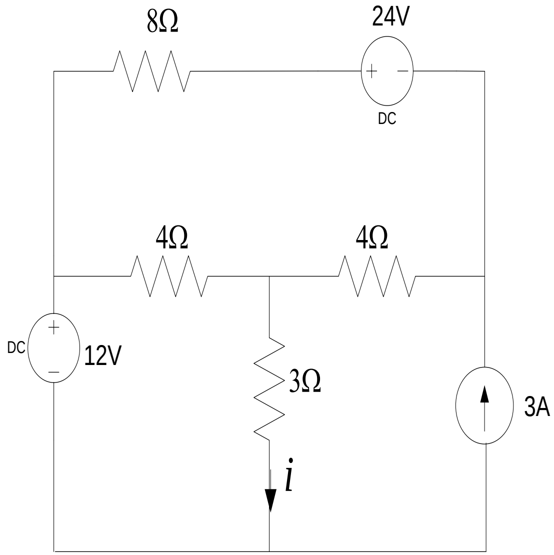
$$16i_2 - 4i_3 = -24$$

$$-4i_2 + 7i_3 = 0$$

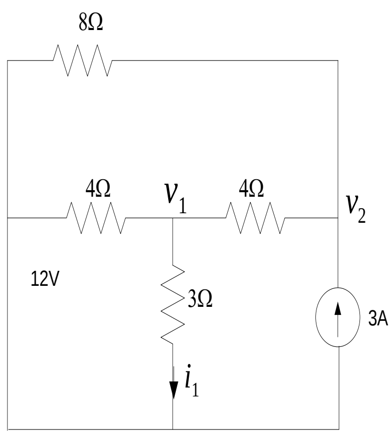
$$i_2 = \frac{7}{4} i_3$$

$$i_3(28-4) = -24$$

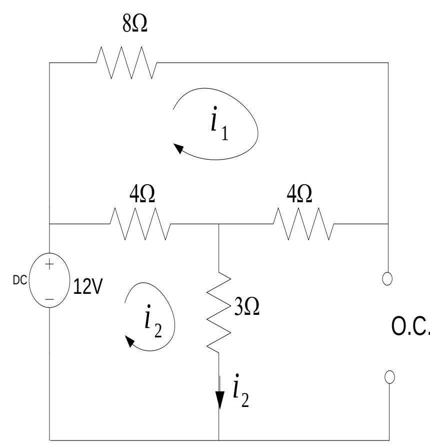
$$i_3 = -1$$



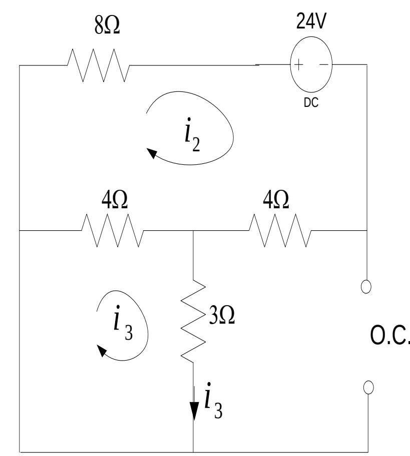
$$i = i_1 + i_2 + i_3 = 1\text{A} + 2\text{A} - 1\text{A} = 2\text{A}$$



$$i_1 = 1$$

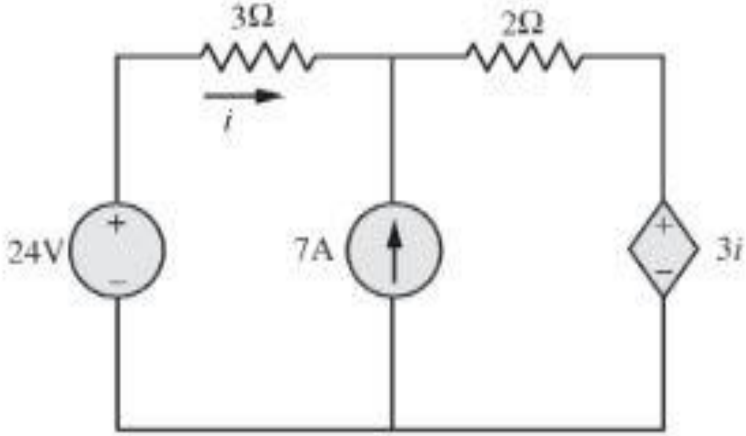


$$i_2 = 2$$



$$i_3 = -1$$

Find the current i using superposition theorem for the circuit shown in Fig



As a first step in the analysis, we will find the current resulting from the independent voltage source. The current source is deactivated and we have the circuit as shown in Fig. (a)

Applying KVL clockwise around loop , we find that

$5i_1+3i_1-24=0$

$i_1= 3\ A$

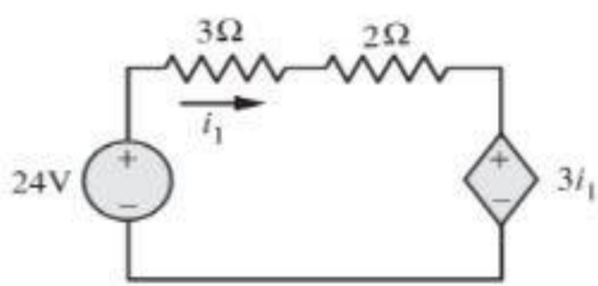


Fig. (a)

As a second step, we set the voltage source to zero and determine the current i_2 due to the current source as shown in Fig. (b).

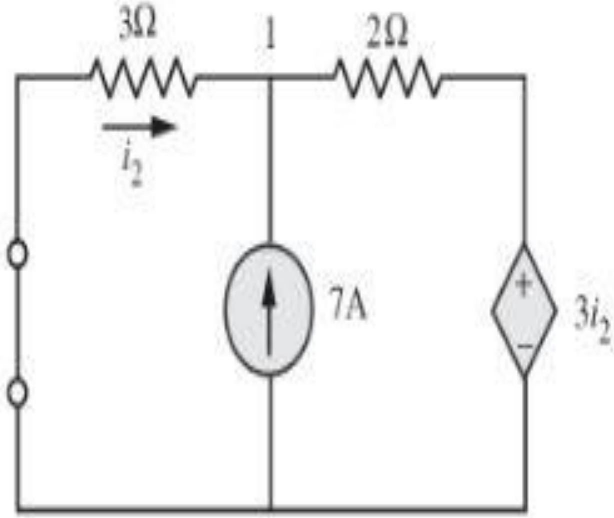


Fig. (b)

Applying KCL at node 1, we get

$i_2 + 7 = \frac{v_1 - 3i_2}{2}$

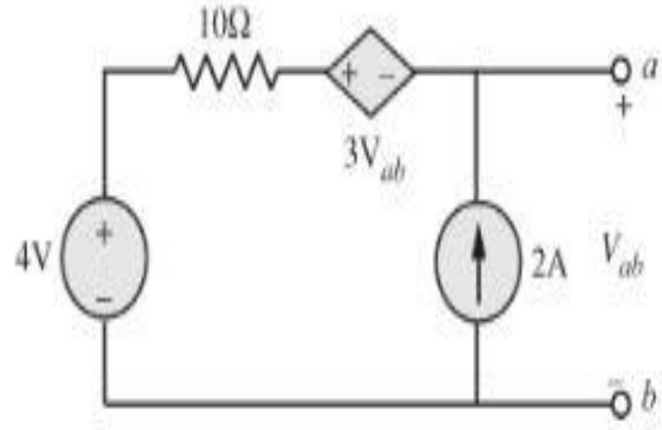
and $i_2 = \frac{0 - v_1}{3}$

we get , $v_1 = -3i_2$

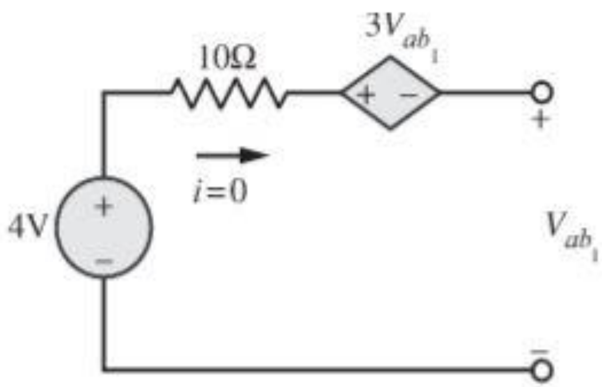
On substituting for v_1 , we get $i_2 = -\frac{7}{4}A$

Thus, the total current $i = i_1 + i_2 = \frac{5}{4}A$

For the circuit shown in Fig.. , find the terminal voltage V_{ab} using superposition principle.



Consider 4V source

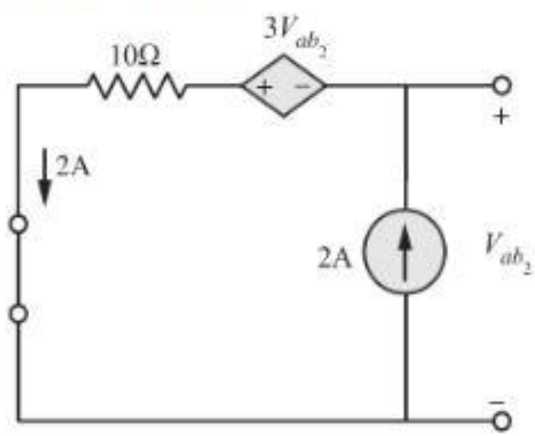


Apply KVL, we get

$4 - 10 \times 0 - 3V_{ab1} - V_{ab1} = 0$

$V_{ab1} = 1V$

Consider 2A source



Apply KVL, we get $-10 \times 2 + 3V_{ab2} + V_{ab2} = 0$

$V_{ab2} = 5V$

According to superposition principle, $V_{ab} = V_{ab1} + V_{ab2} = 6V$

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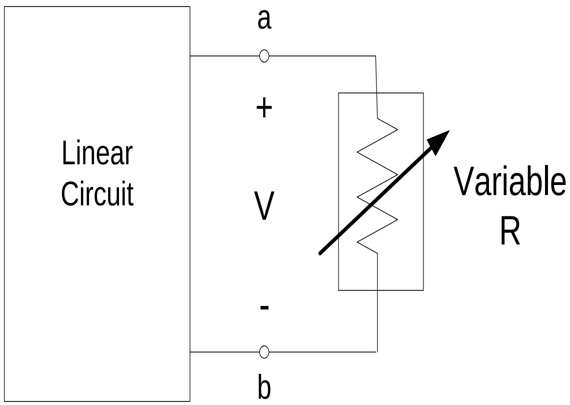
Thevenin's Theorem

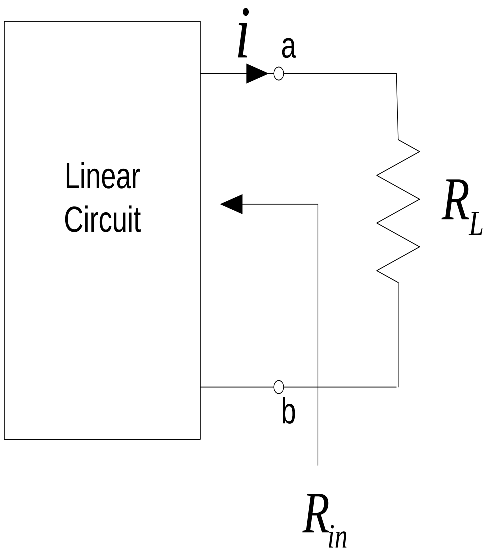
- In many applications we want to find the response to a particular element which may, at least at the design stage, be variable.

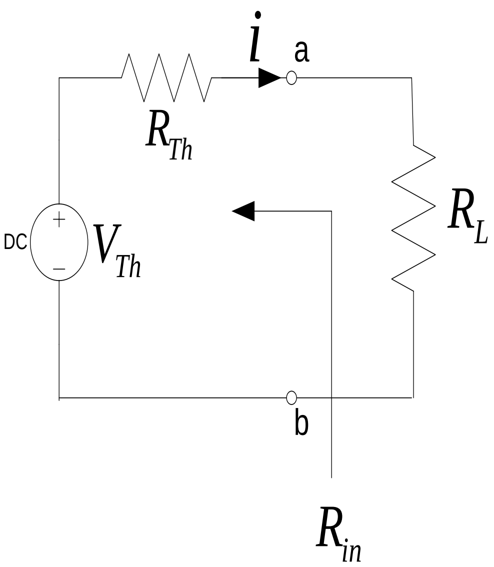


- Each time the variable element changes we have to re-analyze the entire circuit. To avoid this we would like to have a technique that replaces the linear circuit by something simple that facilitates the analysis.
- A good approach would be to have a simple **equivalent circuit** to replace everything in the circuit except for the variable part (the load).

Thevenin's Theorem

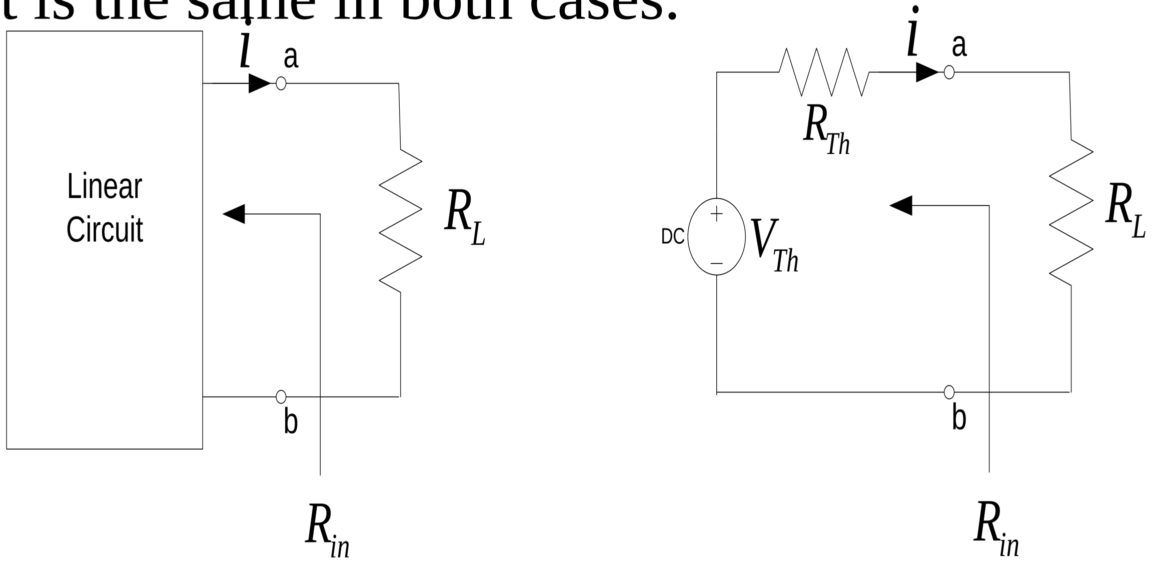
- **Thevenin's theorem** states that a linear two-terminal resistive circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals, and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are all **turned off**.





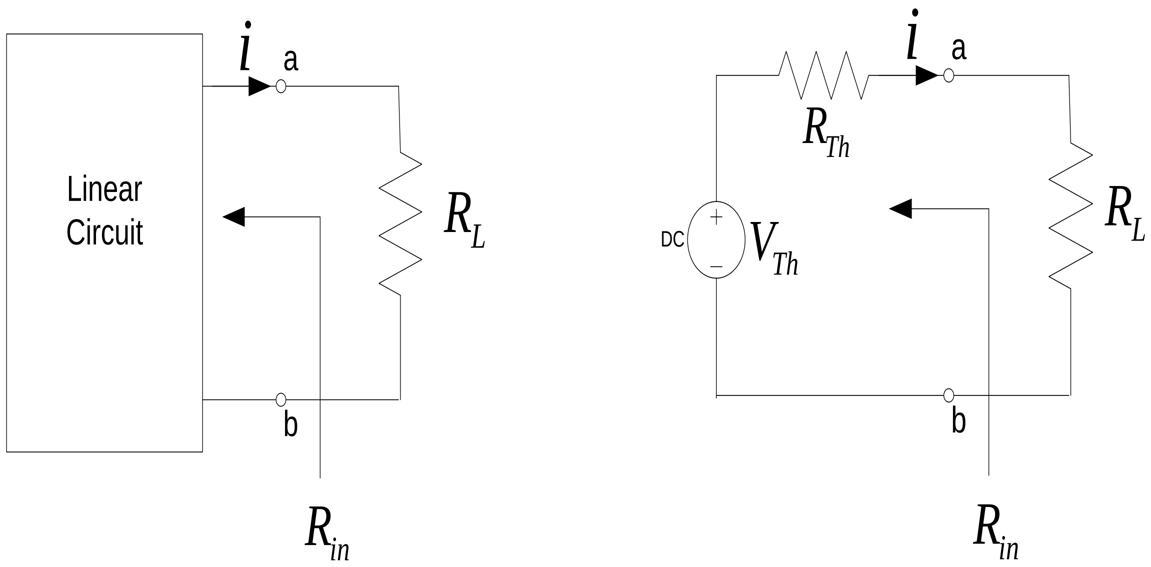
Thevenin's Theorem

- Thevenin's theorem states that the two circuits given below are equivalent as seen from the load R_L that is the same in both cases.



V_{Th} = Thevenin's voltage = V_{ab} with R_L disconnected (= ∞) = the open-circuit voltage = V_{OC}

Thevenin's Theorem



R_{Th} = Thevenin's resistance = the input resistance with all **independent** sources turned off (voltage sources replaced by short circuits and current sources replaced by open circuits). This is the resistance seen at the terminals *ab* when all independent sources are turned off.

Why are Independent energy sources kept off while calculating the Thevenin’s equivalent resistance of a two port circuit ?

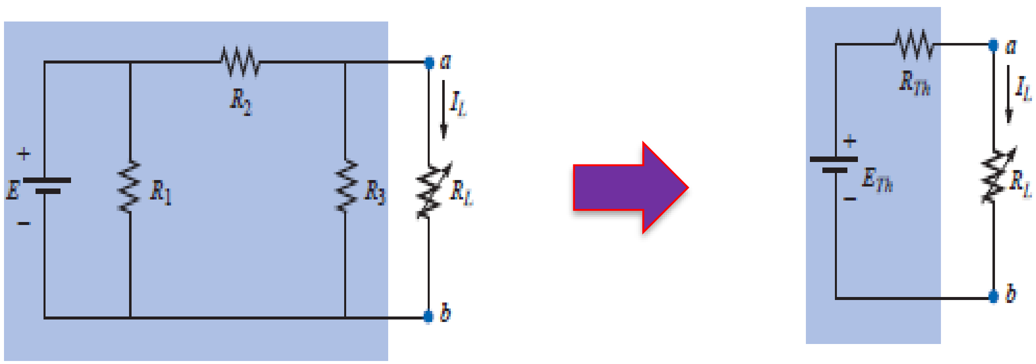
- We can do that only with linear circuits where the principle of superposition is applicable.
- The reason is that an independent source does not have a finite impedance to affect your calculations and will not impact the impedance of any other branch.
- At a mathematical level it boils down to linear superposition with voltages and currents being the variables and impedances being the coefficients.

Thevenin's Theorem

- Thevenin's theorem can be used to:
  - Analyze networks with sources that are not in series or parallel.
  - Reduce the number of components required to establish the same characteristics at the output terminals.
  - Investigate the effect of changing a particular component on the behavior of a network without having to analyze the entire network after each change.

Thevenin's Theorem

- Any complex two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source $\boldsymbol{V_{Th}}$ and a series resistor $\boldsymbol{R_{Th}}$.

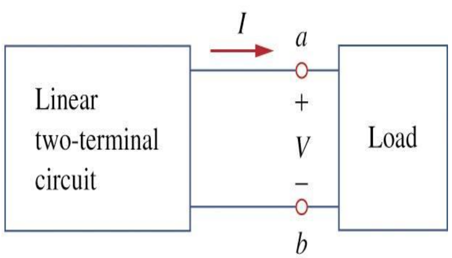


Original Circuit **Thévenin Equivalent Circuit**

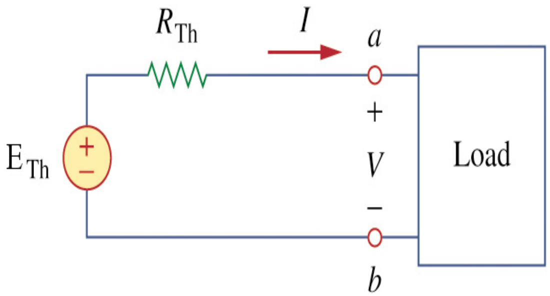
- The Thevenin equivalent circuit provides an equivalence at the **terminals only**.
  - The internal construction and characteristics of the original network and the Thévenin equivalent are usually quite different.

Thevenin's Theorem

- ***E_{Th}*** is the open circuit voltage at the terminals.
- ***R_{Th}*** is the input or equivalent resistance at the terminals when the sources are turned off.



Original Circuit



Thevenin's Equivalent Circuit

Thevenin's Theorem

Experimental Procedures

- Two popular experimental procedures for determining the parameters of the Thevenin's equivalent network:

1) Direct Measurement of E_{Th} and R_{Th}

- For any physical network, the value of E_{Th} can be determined experimentally by measuring the open-circuit voltage across the load terminals.
- The value of R_{Th} can then be determined by completing the network with a variable resistance R_L.

Thevenin's Theorem

2) Measuring V_{OC} and I_{SC}

- The Thevenin's voltage is again determined by measuring the open-circuit voltage across the terminals of interest; that is, $E_{Th} = V_{OC}$. To determine R_{Th}, a short-circuit condition is established across the terminals of interest and the current through the short circuit (I_{sc}) is measured with an ammeter.
- Using Ohm's law:

$$R_{Th} = V_{oc} / I_{sc}$$

Thevenin's Theorem

Steps to follow for finding V_{TH} and R_{TH}:

1. Remove the load &

2. Label the terminals **a** and **b**.

3. Solve for R_{TH} by setting all sources to zero.

4. Solve for V_{TH} by returning all sources to their original position and finding the open-circuit voltage between **a** and **b**.

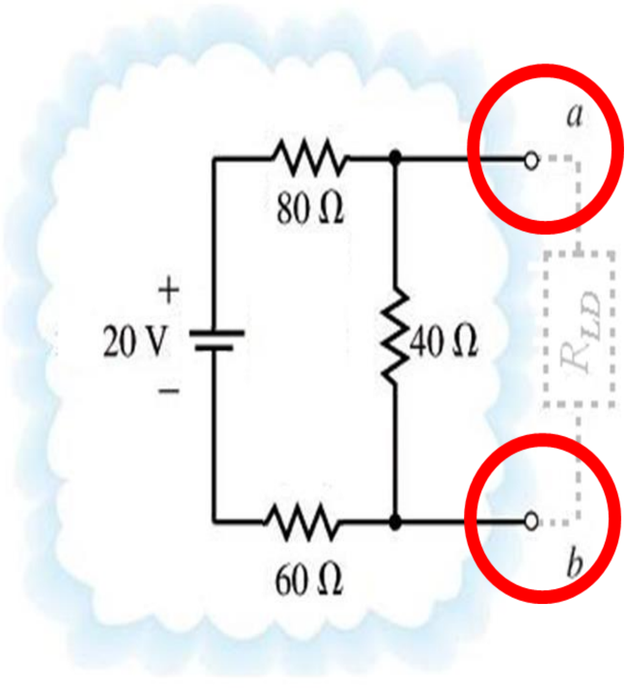
5. Draw the new equivalent circuit.

Thevenin's Theorem

Steps 1 & 2 :

Convert to a Thevenin's circuit:

1. Identify and remove the load from the circuit.
2. Label the resulting open terminals.



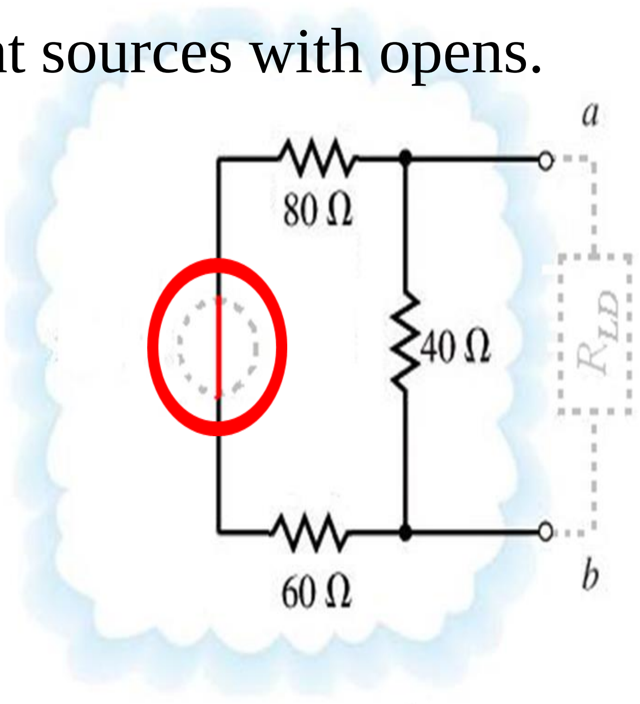
Thevenin's Theorem

Step 3:

Solve for $\boldsymbol{R_{TH}}$ and isolate the resistance from the source.

Set all sources to zero:

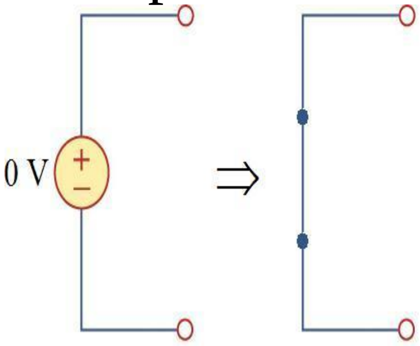
- Replace voltage sources with shorts.
- Replace current sources with opens.



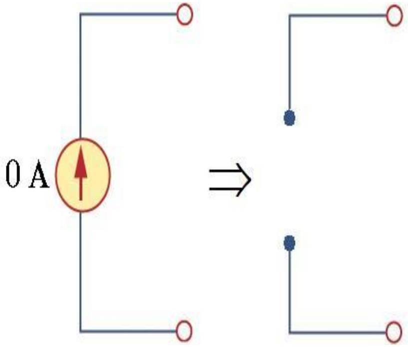
Zeroing Sources

Step-3: "Zeroing" a source means setting its value equal to zero.

- **Voltage sources** – 0 V is equivalent to a short-circuit.
- **Current sources** – 0 A is equivalent to a open-circuit.



Voltage Sources Become Short-Circuits

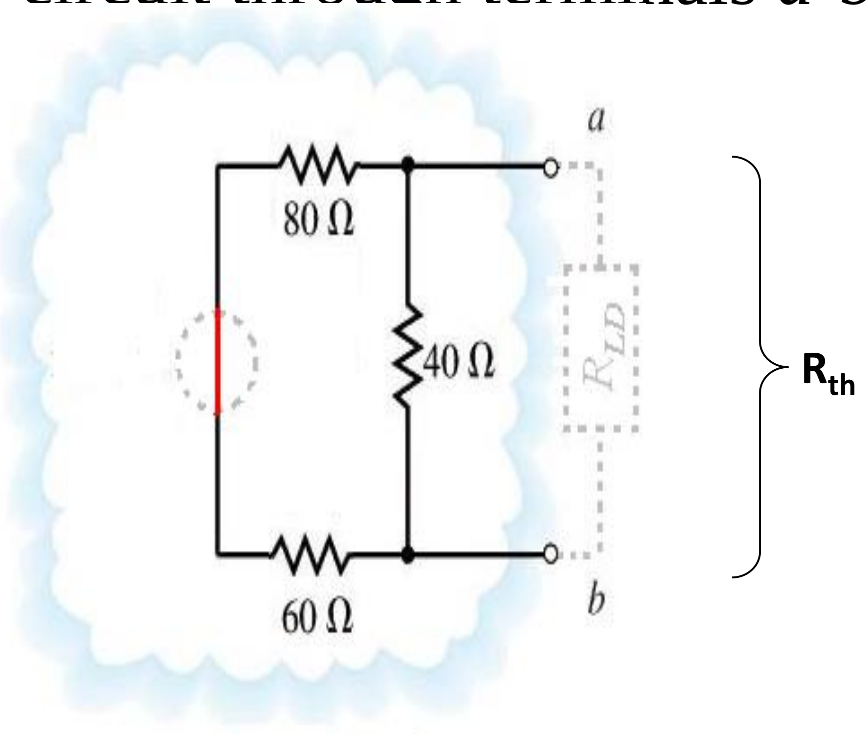


Current Sources Become Open-Circuits

Thevenin's Theorem

Step- 3:

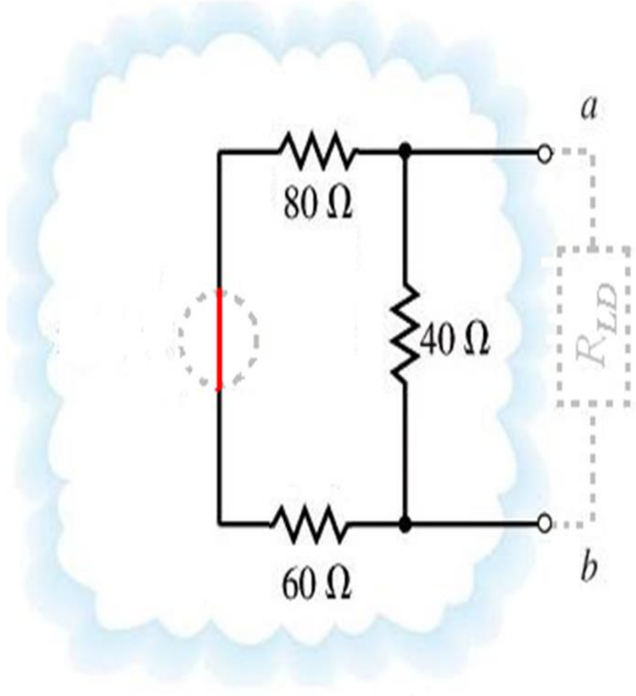
- With the load disconnected, turn off all source.
- $\boldsymbol{R}_{\mathbf{Th}}$ is the equivalent resistance **looking into** the "dead" circuit through terminals *a*-*b*.



Thevenin's Theorem

Step- 3: Set all sources to zero, and calculate $\boldsymbol{R}_{\mathbf{Th}}$.

$$R_{TH} = R_{ab} = \left(\frac{1}{80+60} + \frac{1}{40} \right)^{-1} = 31\Omega$$



Remember, calculate R_{TH} from the a and b perspective!

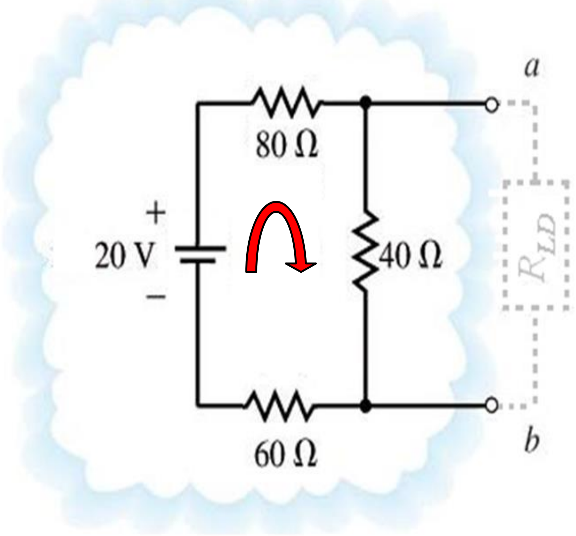
Thevenin's Theorem

<u>Step-4 :</u> Solve for $\boldsymbol{V_{TH}}$ and then, as needed:

- Calculate the voltage ($\boldsymbol{V_{LD}}$) across the $\boldsymbol{R_{LD}}$.
- Calculate the current ($\boldsymbol{I_{LD}}$) through $\boldsymbol{R_{LD}}$.

$$(VDR) \rightarrow E_{TH} = V_{TH} = V_{ab} = V_{40\Omega} =$$

$$V_{TH} = E * \frac{R_{40\Omega}}{R_T} = 20V * \frac{40\Omega}{40\Omega + 80\Omega + 60\Omega} = 4.44V$$



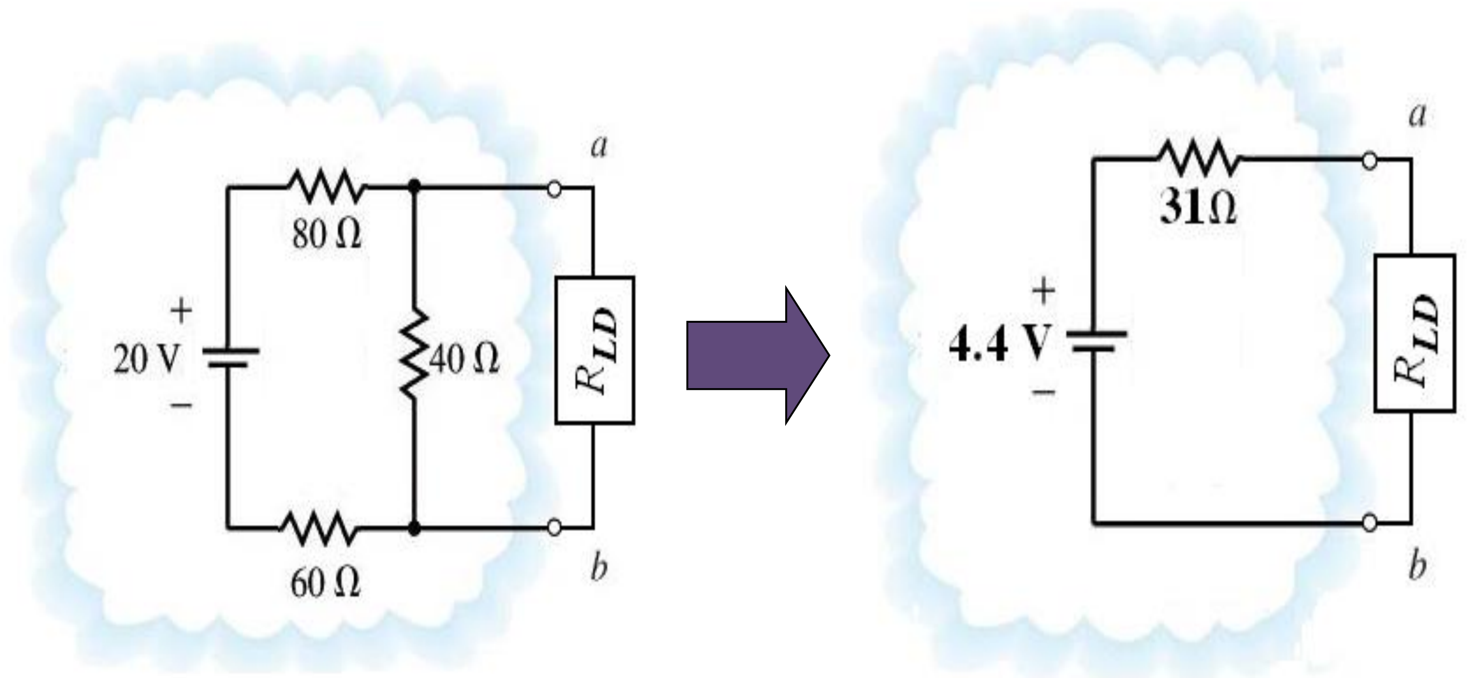
$$V_{LD} = \frac{R_{LD}}{R_{Th} + R_{LD}} E_{Th}$$

$$I_{LD} = \frac{E_{Th}}{R_{Th} + R_{LD}}$$

Thevenin's Theorem

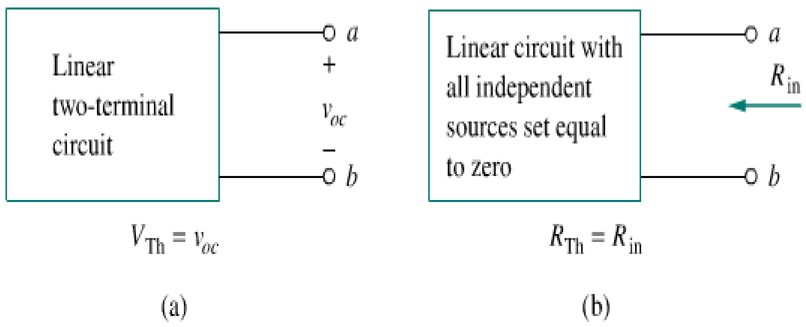
Step-5:

REDRAW the circuit showing the Thèvenin equivalents ($\boldsymbol{V_{TH}}$ and $\boldsymbol{R_{TH}}$) with the load installed.



Thevenin's Theorem

- In finding the Thevenin's resistance RTh, we need to co



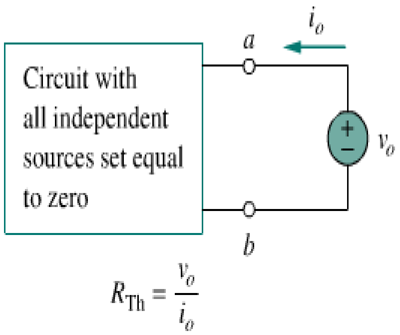
<u>Finding V_{Th} and R_{Th}</u>

- If the network has no dependent sources, we turn off all independent sources. R_{Th} is the input resistance of the network looking between terminals a and b.

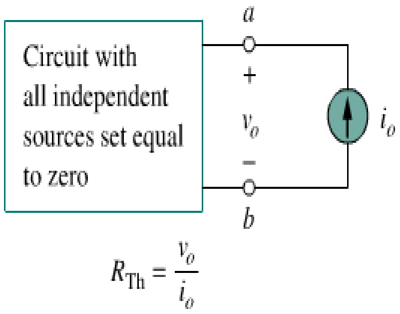
- If the network has dependent sources, we turn off all independent sources. We apply a voltage source *vo* at the terminals a and b determine the resulting current *io*.

Thevenin's Theorem

•Then R_{Th}= *vo/io* as shown below. Alternatively, we may insert a current source *io* at terminals a-b and find the terminal voltage *vo*. Again *RTh*= *vo/io* (either of the two approaches will give the same result.)



(a)

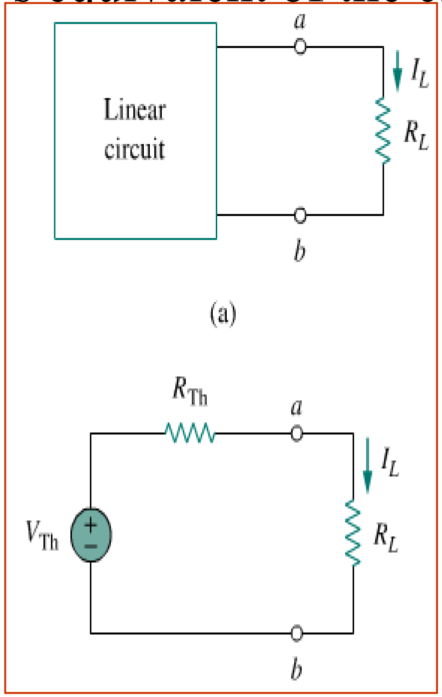


Thevenin’s Theorem

- Important of Thevenin’s theorem- helps simplify a circuit (a large circuit may be replaced by a single independent voltage source and a single resistor)
- The current I_L through the load and the voltage *VL* across the load are easily determined once the Thevenin’s equivalent of the circuit at the load’s terminals is obtained. :

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

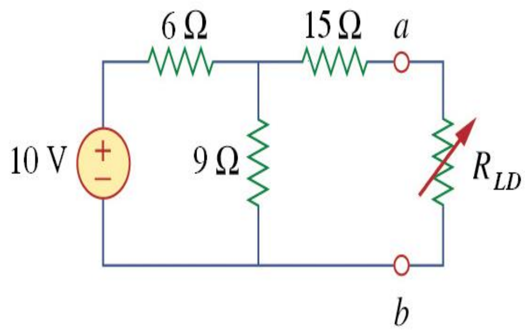
$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$

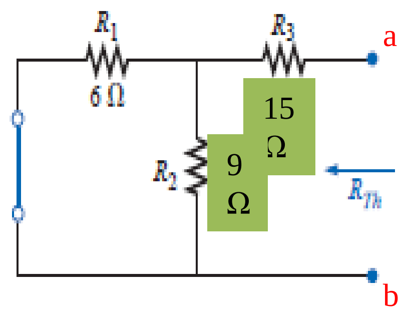


Note: A negative *RTh* value shows that the circuit is supplying power (circuit with dependent source)

Problem 1

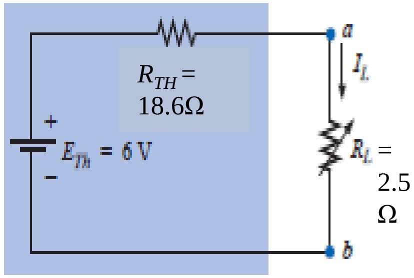
- Find the Thevenin equivalent circuit external to $\boldsymbol{R_{LD}}$. Determine $\boldsymbol{I_{LD}}$ and $\boldsymbol{V_{LD}}$ when $\boldsymbol{R_{LD}} = \mathbf{2.5\ \Omega}$.





$$(VDR) \rightarrow E_{TH} = V_{ab} = V_{9\Omega} = E * \frac{R_{9\Omega}}{R_T} = 10V * \frac{9\Omega}{6\Omega + 9\Omega} = \boxed{6V}$$

$$R_{TH} = R_{ab} = \left(\frac{1}{\frac{1}{6} + \frac{1}{9}} \right) + 15 = \boxed{18.6\Omega}$$



$$I_{LD} = \frac{E_{Th}}{R_{Th} + R_{LD}}$$

$$V_{LD} = E_{Th} \frac{R_{LD}}{R_{Th} + R_{LD}}$$

$$I_{LD} = \frac{6V}{18.16 + 2.5} = \boxed{284mA}$$

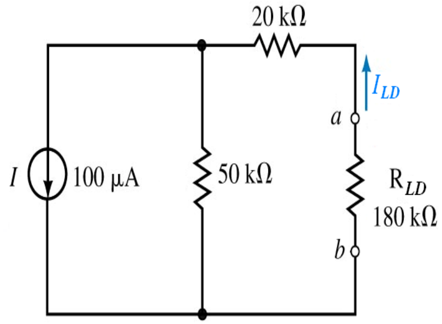
$$V_{LD} = 6V \frac{2.5}{18.6 + 2.5} = 0.71V$$

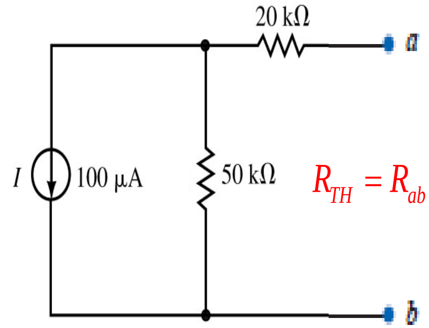
$$\text{OR} \quad V_{LD} = I_{LD} * R_{LD} = 284mA * 2.5\Omega = 0.71V$$

1/2. Remove the load label the terminals a and b.
3. Solve for RTH.
4. Solve for VTH.
5. Draw the new equivalent circuit.

Problem 2

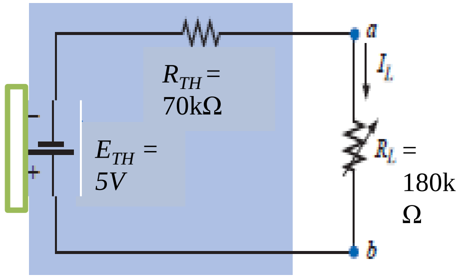
- Find the Thevenin equivalent circuit external to $\boldsymbol{R_{LD}}$ and determine $\boldsymbol{I_{LD}}$.





$$R_{TH} = R_{ab} = 20k + 50k = \boxed{70k\Omega}$$

$$E_{TH} = V_{ab} = V_{50k\Omega} = I_S * R_{50k} = 100\mu A * 50k\Omega = \boxed{5V}$$



$$I_{LD} = \frac{E_{Th}}{R_{Th} + R_{LD}}$$

$$I_{LD} = \frac{5V}{70k + 180k} = \boxed{20\mu A}$$

Thevenin’s Theorem

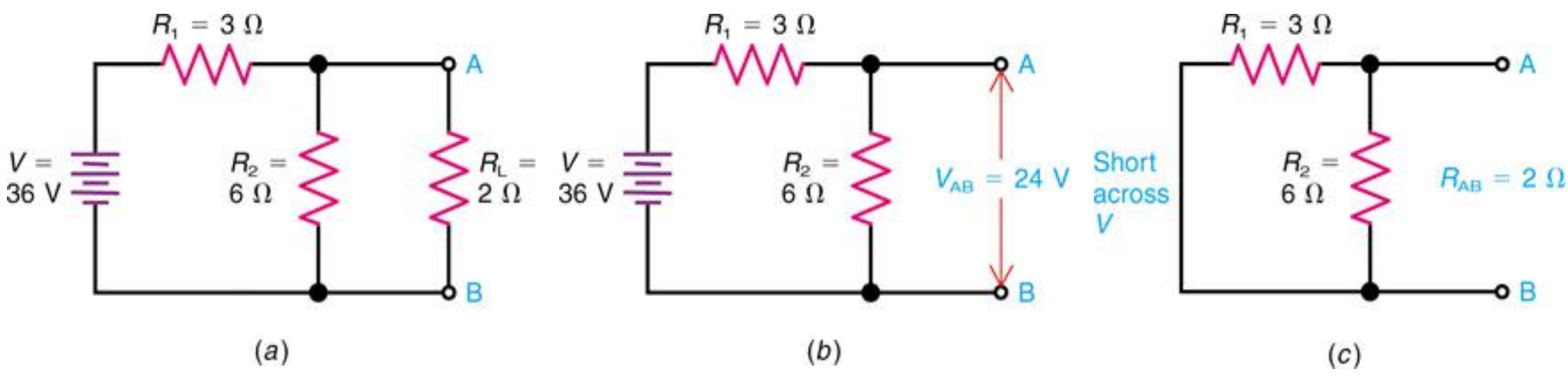


Fig. 1: Application of Thevenin’s theorem. (*a*) Actual circuit with terminals A and B across R_L. (*b*) Disconnect R_L to find that V_{AB} is 24V. (*c*) Short-circuit *V* to find that R_{AB} is 2Ω.

Thevenin's Theorem

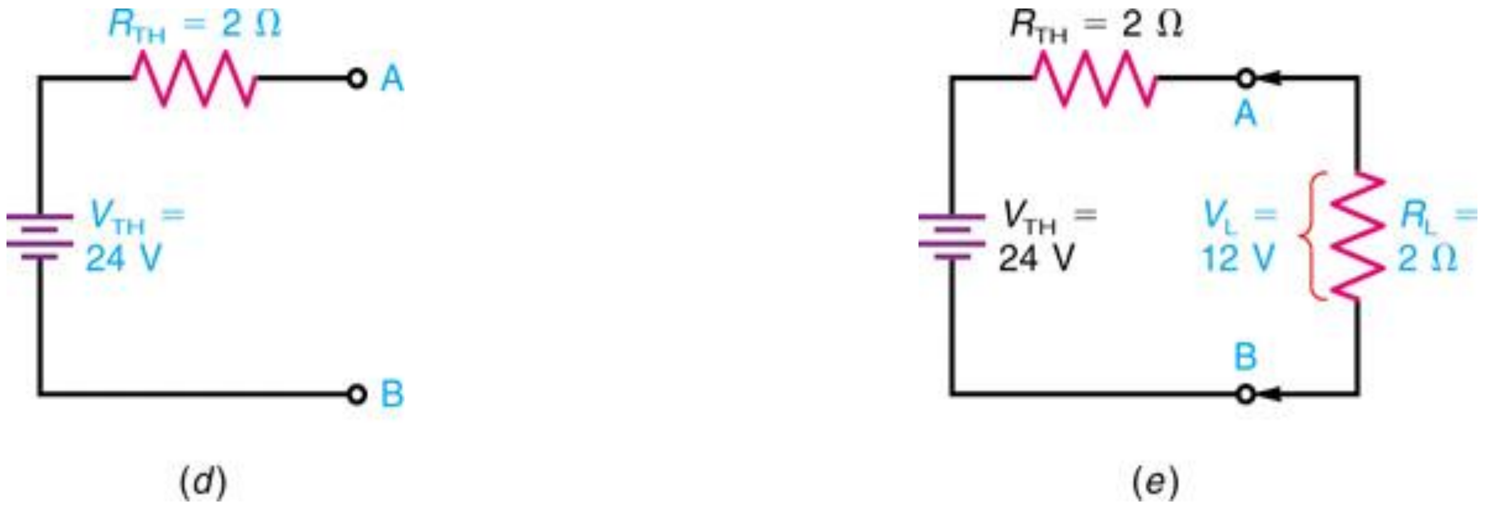
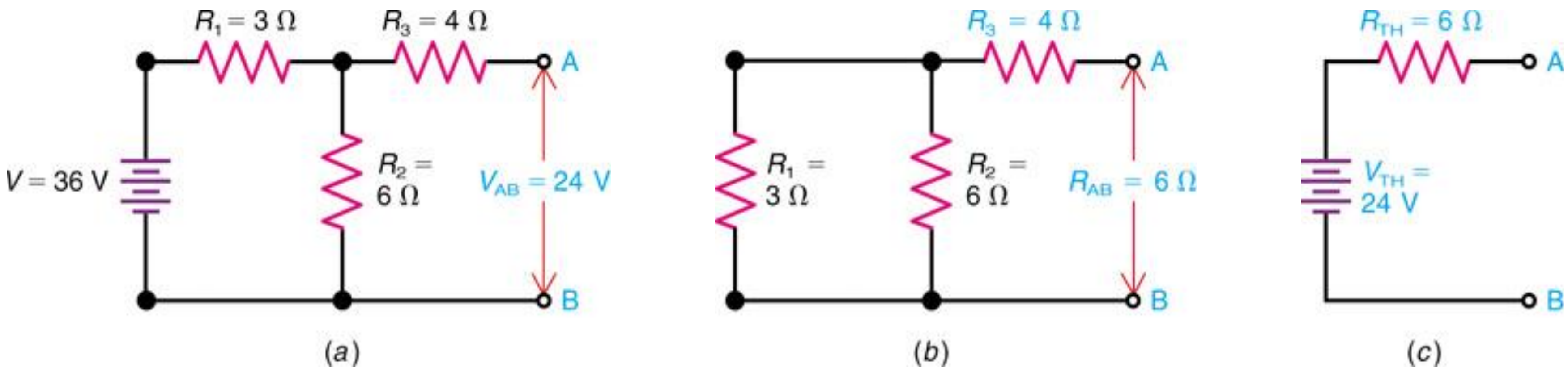


Fig. 1 (*d*) Thevenin equivalent circuit. (*e*) Reconnect R_L at terminals A and B to find that V_L is 12V.

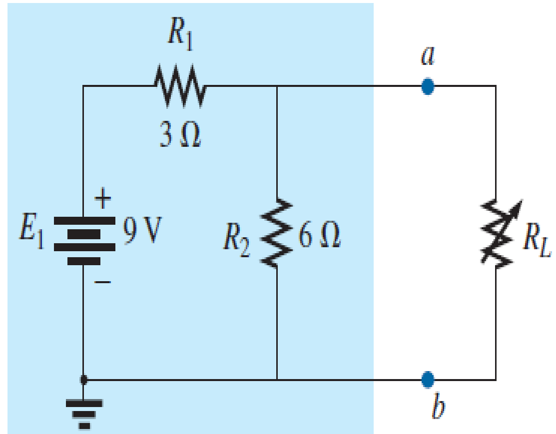
Thevenin's Theorem

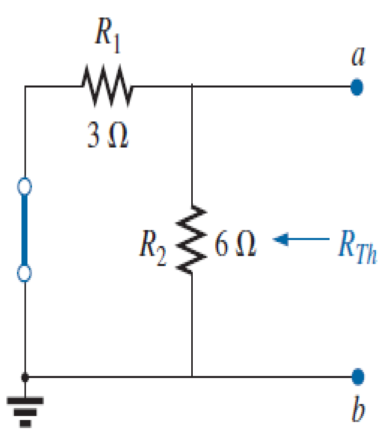


Note that $\mathbf{R_3}$ does not change the value of $\mathbf{V_{AB}}$ produced by the source V, but R_3 <u>does</u> increase the value of $\mathbf{R_{TH}}$.

Fig. 2: Thevenizing the circuit of Fig.1 but with a 4-Ω R_3 in series with the A terminal. (*a*) V_{AB} is still 24V. (*b*) Now the R_{AB} is 2 + 4 = 6 Ω. (*c*) Thevenin equivalent circuit.

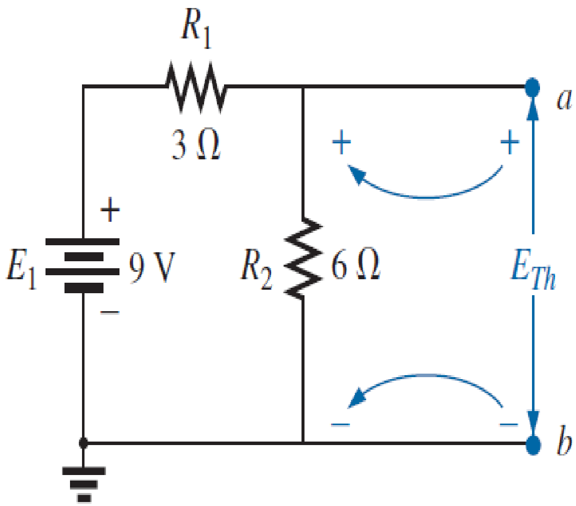
- **EXAMPLE (1)** Find the Thévenin equivalent circuit for the network in the shaded area of the network in Figure. Then find the current through R_L for values of 2 , 10 , and 100 .





$$R_{Th} = R_1 \| R_2 = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \mathbf{2\ \Omega}$$

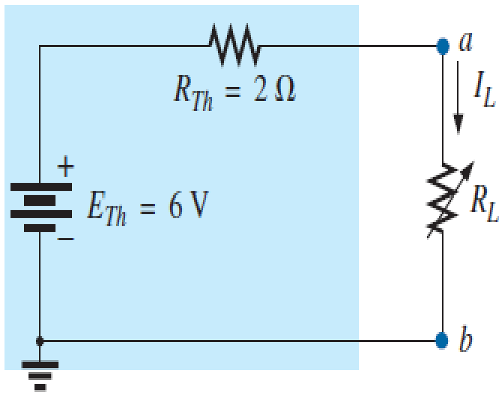
Determining R_{Th} for the network



$$E_{Th} = \frac{R_2 E_1}{R_2 + R_1} = \frac{(6\ \Omega)(9\ \text{V})}{6\ \Omega + 3\ \Omega} = \frac{54\ \text{V}}{9} = \mathbf{6\ V}$$

Determining E_{Th} for the network.

$$I_L = \frac{E_{Th}}{R_{Th} + R_L}$$



$$R_L = 2\ \Omega: \qquad I_L = \frac{6\ \text{V}}{2\ \Omega + 2\ \Omega} = \mathbf{1.5\ A}$$

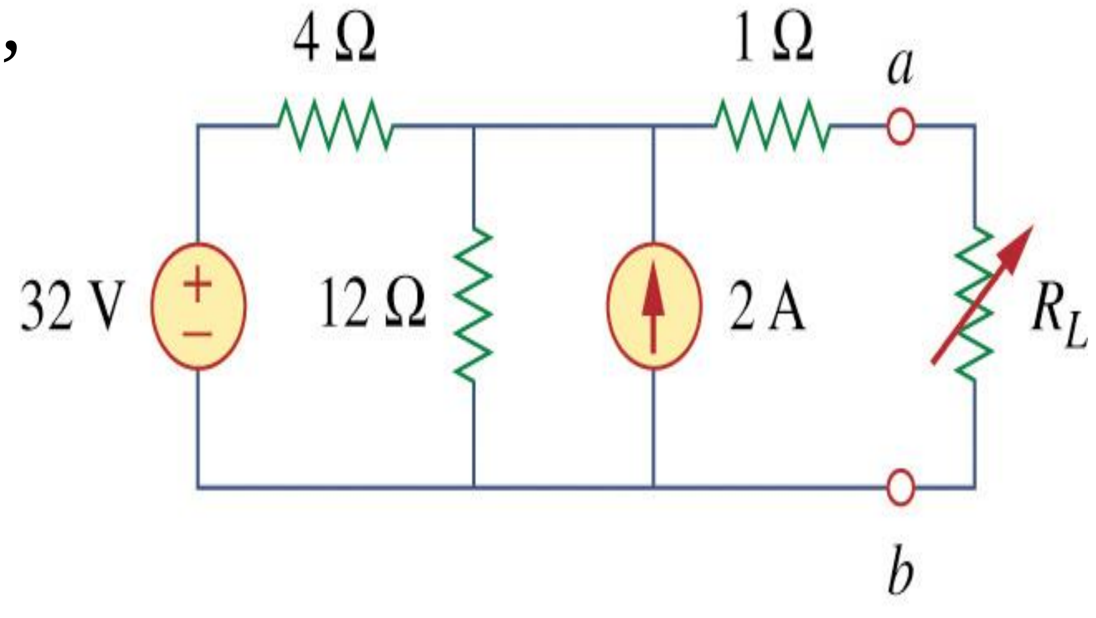
$$R_L = 10\ \Omega: \qquad I_L = \frac{6\ \text{V}}{2\ \Omega + 10\ \Omega} = \mathbf{0.5\ A}$$

$$R_L = 100\ \Omega: \qquad I_L = \frac{6\ \text{V}}{2\ \Omega + 100\ \Omega} = \mathbf{0.06\ A}$$

Substituting the Thevenin equivalent circuit for the network external to R_L

Example (2)

- Find the Thevenin's equivalent circuit of the circuit shown in Fig , to the left of the terminals *a*-*b*. Then find the current through R_L = 6,16,

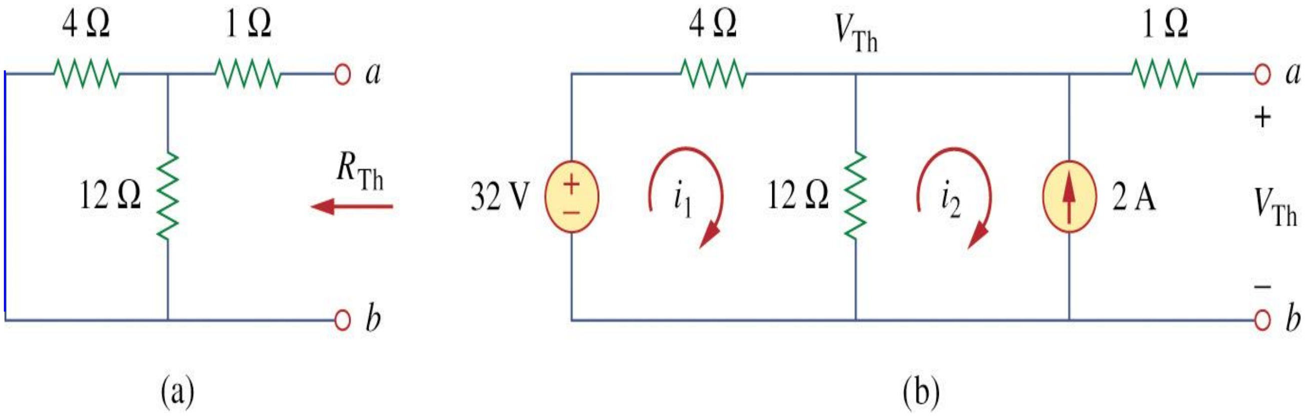


Find R_{th}

R_{Th} : 32V voltage source $\rightarrow$ short

2A current source $\rightarrow$ open

$$R_{Th} = 4 \| 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\Omega$$



Find V_{th}

V_{Th} :

(1) Mesh analysis

$$-32+4i_1+12(i_1-i_2)=0\ ,\ \ i_2=-2\mathrm{A}$$

$$\therefore i_1=0.5\mathrm{A}$$

$$V_{\mathrm{Th}}=12(i_1-i_2)=12(0.5+2.0)=30\mathrm{V}$$

(2) Alternatively, Nodal Analysis

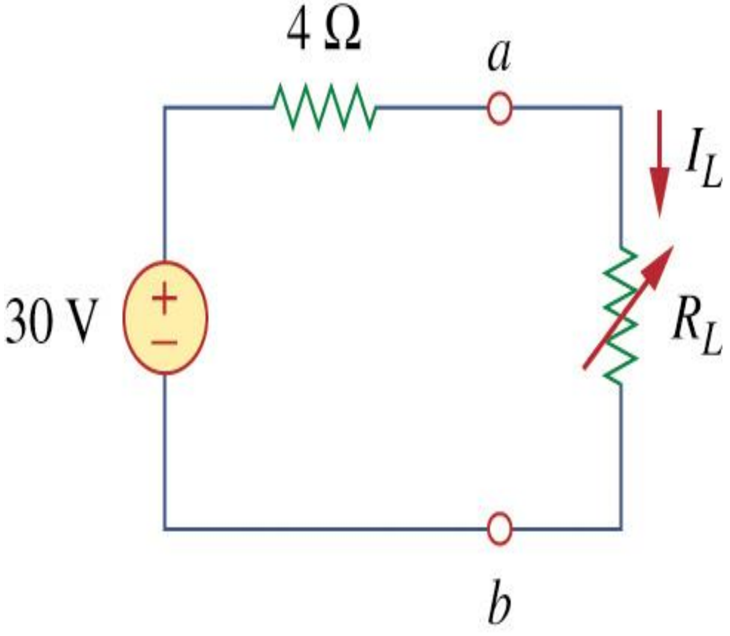
$$(32-V_{\mathrm{Th}})/4+2=V_{\mathrm{Th}}/12$$

$$\therefore V_{\mathrm{Th}}=30\mathrm{V}$$

(3) Alternatively, source transform

$$\frac{32 - V_{TH}}{4} + 2 = \frac{V_{TH}}{12}$$

$$96 - 3V_{TH} + 24 = V_{TH} \Rightarrow V_{TH} = 30\text{V}$$



To get i_L :

$$i_L = \frac{V_{\mathrm{Th}}}{R_{\mathrm{Th}} + R_L} = \frac{30}{4 + R_L}$$

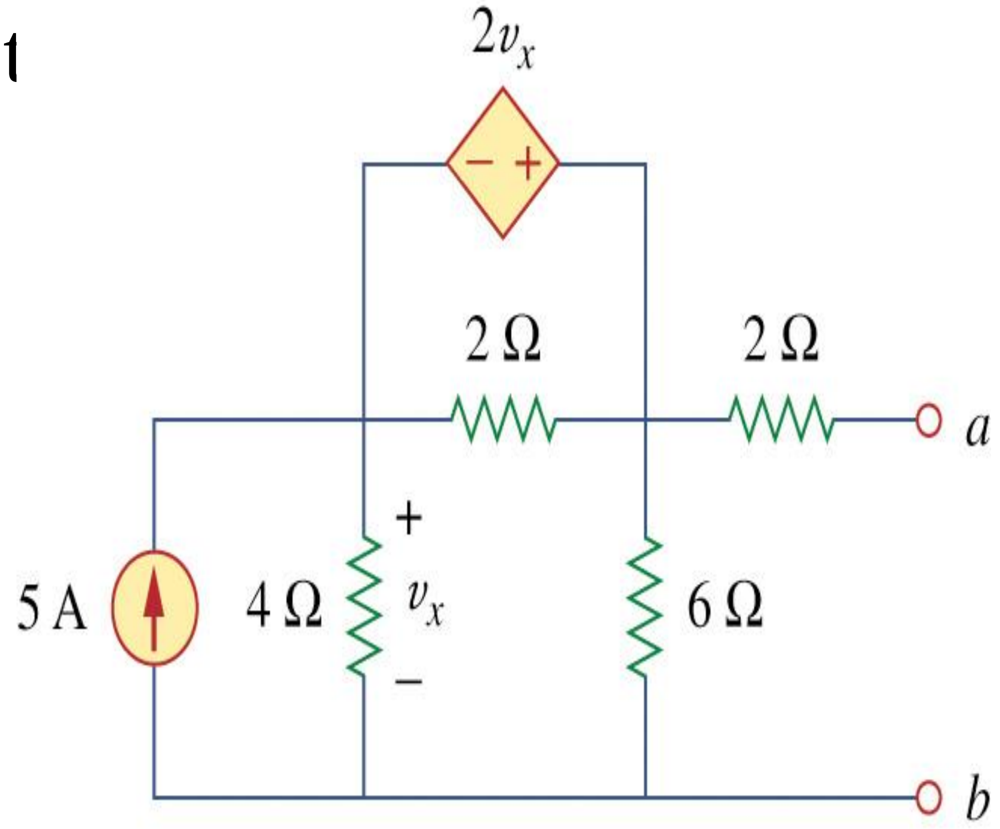
$$R_L = 6 \rightarrow I_L = 30/10 = 3\mathrm{A}$$

$$R_L = 16 \rightarrow I_L = 30/20 = 1.5\mathrm{A}$$

$$R_L = 36 \rightarrow I_L = 30/40 = 0.75\mathrm{A}$$

Example (3)

- Find the Thevenin's equivalent of the circuit in Fig. at t



- (independent + dependent source case)

$$\boxed{\text{To find } R_{\mathrm{Th}} \text{ : Fig(a)}}$$

$$\text{independent source} \to 0$$

$$\text{dependent source} \to \text{intact}$$

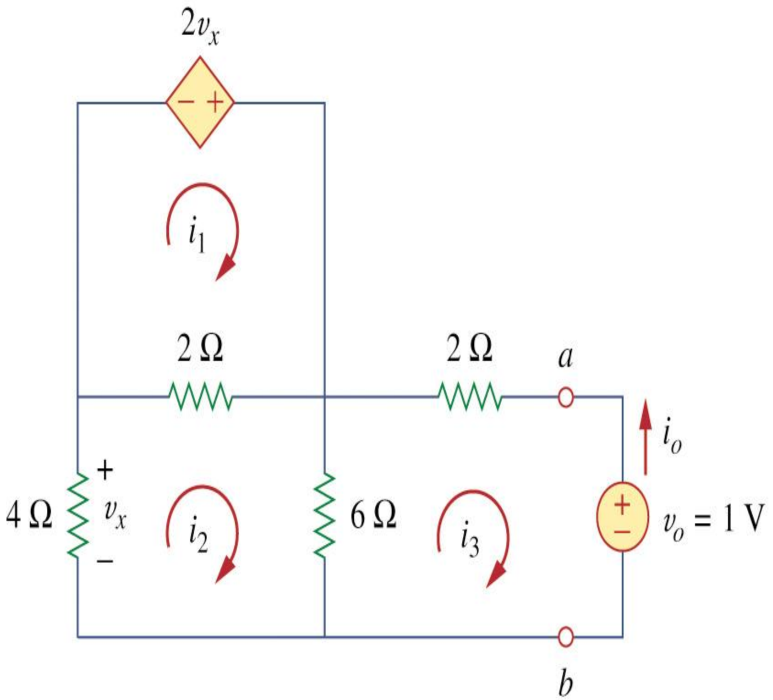
$$v_o = 1\mathrm{V}, \quad R_{\mathrm{Th}} = \frac{v_o}{i_o} = \frac{1}{i_o}$$

- For loop 1,

$$-2v_x + 2(i_1 - i_2) = 0 \text{ or } v_x = i_1 - i_2$$

$$\text{But } -4i = v_x = i_1 - i_2$$

$$\therefore i_1 = -3i_2$$



(a)

Loop 2 and 3:

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$$

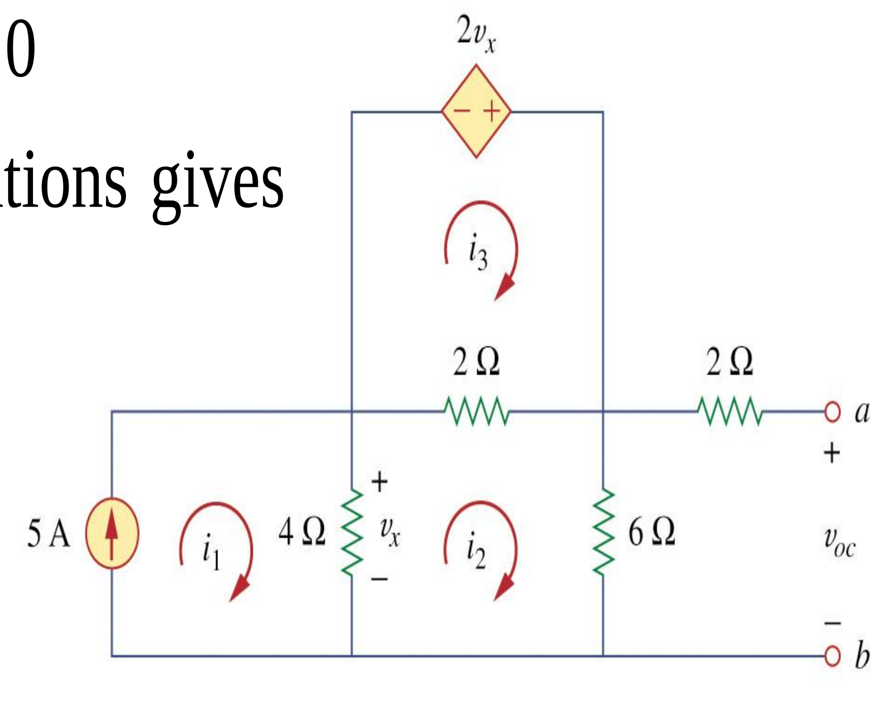
$$6(i_3 - i_2) + 2i_3 + 1 = 0$$

Solving these equations gives

$i_3 = -1/6\text{A}.$

But $i_o = -i_3 = \frac{1}{6}\text{A}$

$$\therefore R_{\text{Th}} = \frac{1V}{i_o} = 6\Omega$$



(b)

To get V_{Th} :Fig(b)   Mesh analysis

$$i_1 = 5$$

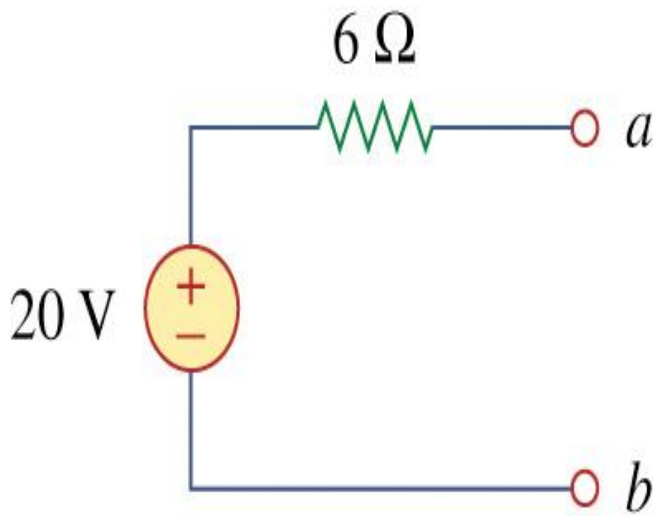
$$-2v_x + 2(i_3 - i_2) = 0 \Rightarrow v_x = i_3 - i_2$$

$$4(i_2 - i_1) + 2(i_2 - i_1) + 6i_2 = 0 \Rightarrow 12i_2 - 4i_1 - 2i_3 = 0$$

But $4(i_1 - i_2) = v_x$

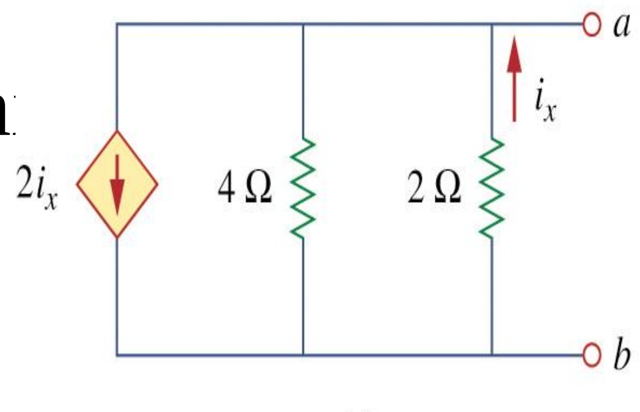
$$\therefore i_2 = 10/3.$$

$$V_{Th} = v_{oc} = 6i_2 = 20\text{V}$$



Example (4)

- Determine the Thevenin equivalent circuit in Fig.4.35(a).



(a)

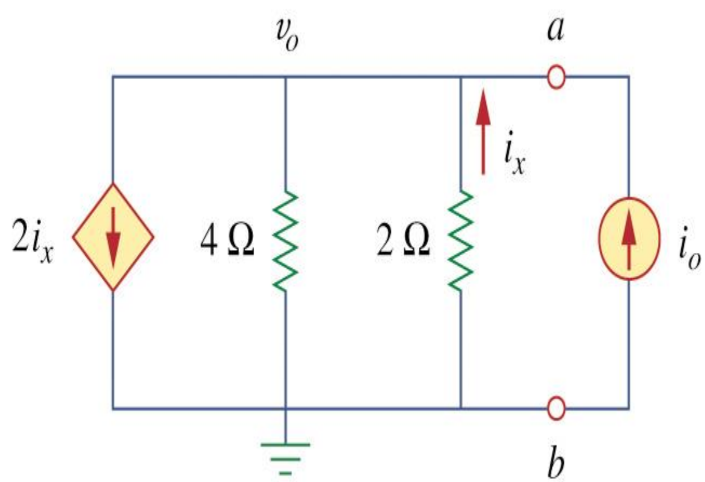
- Solution

(dependent source only case)

$$V_{\text{Th}} = 0 \qquad R_{\text{Th}} = \frac{v_o}{i_o}$$

Nodal anaysis :

$$i_o + i_x = 2i_x + v_o / 4$$

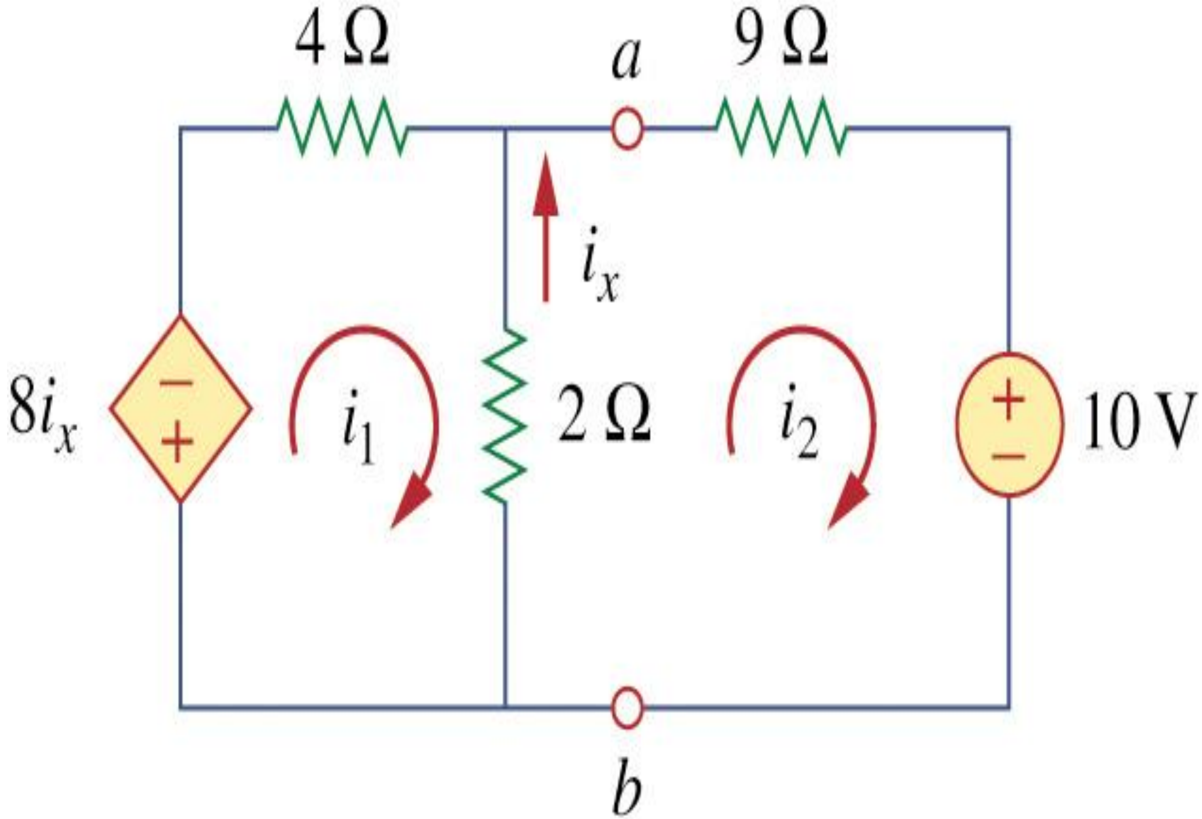


(b)

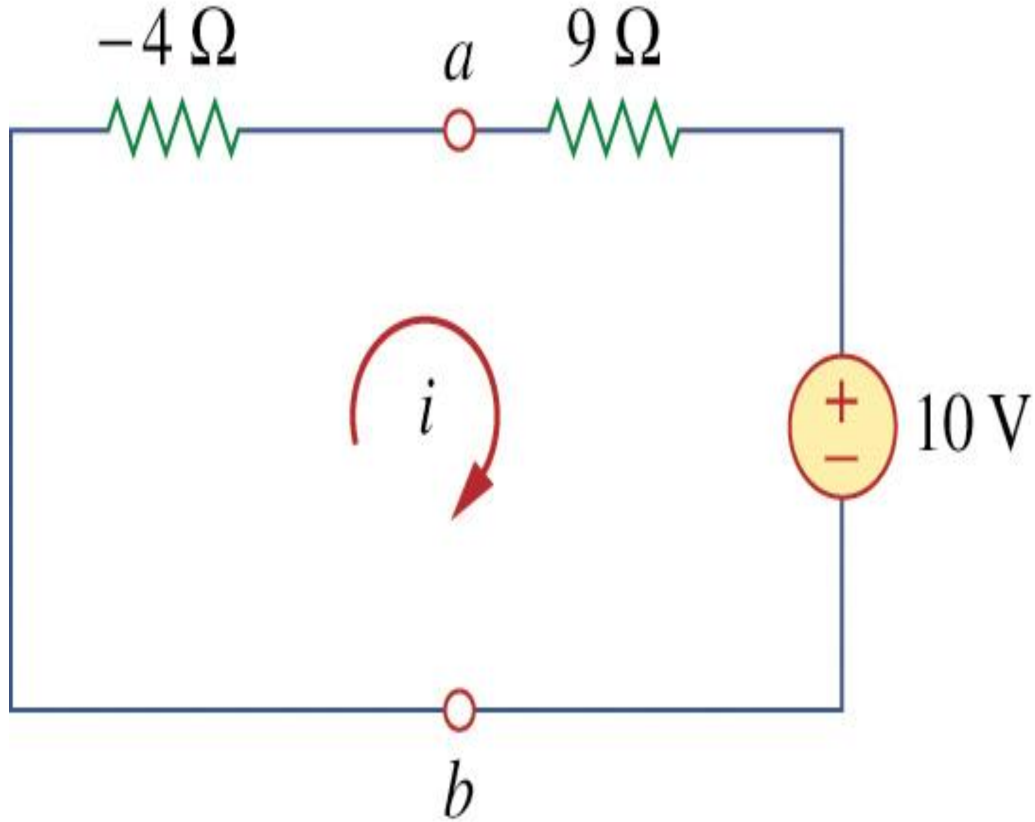
But $$i_x = \frac{0 - v_o}{2} = -\frac{v_o}{2}$$

$$i_o = i_x + \frac{v_o}{4} = -\frac{v_o}{2} + \frac{v_o}{4} = -\frac{v_o}{4} \quad \text{or } v_o = -4i_o$$

Thus $R_{\mathrm{Th}} = \frac{v_o}{i_o} = -4\Omega$: Supplying power



(c)



(d)

Thevenizing a Circuit with Two Voltage Sources

- The circuit in Figure 3 can be solved by Kirchhoff's laws, but **Thevenin's theorem** can be used to find the current I_3 through the middle resistance R_3.
  – Mark the terminals A and B across R_3.
  – Disconnect R_3.
  – To calculate V_{TH}, find V_{AB} across the open terminals

Thevenizing a Circuit with Two Voltage Sources

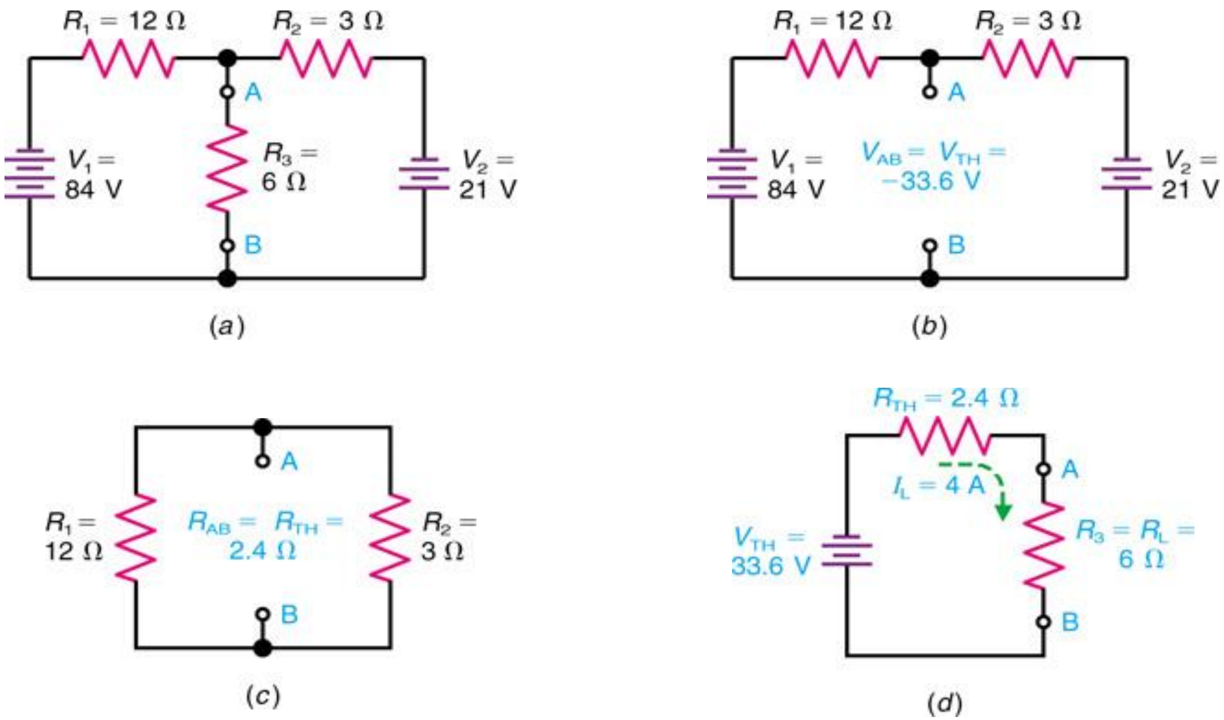
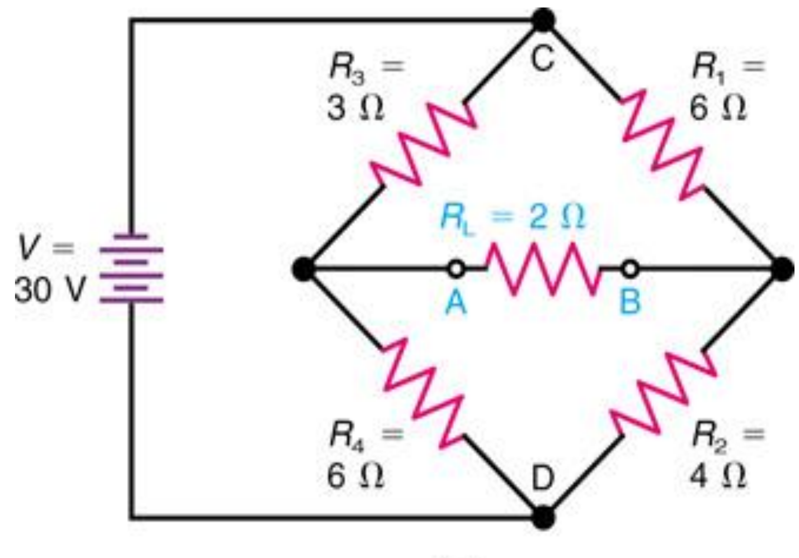


Fig. 3: Thevenizing a circuit with two voltage sources V_1 and V_2. (*a*) Original circuit with terminals A and B across the middle resistor R_3. (*b*) Disconnect R_3 to find that V_{AB} is −33.6V. (*c*) Short-circuit V_1 and V_2 to find that R_{AB} is 2.4 Ω. (*d*) Thevenin equivalent with R_L reconnected to terminals A and B.

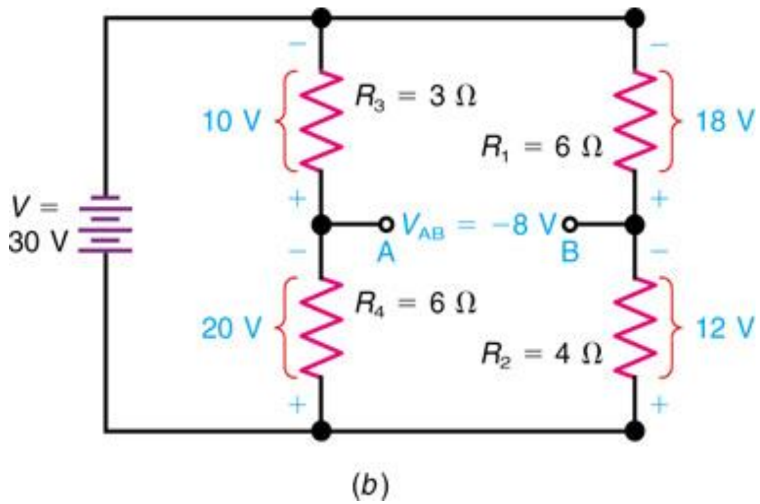
Thevenizing a Bridge Circuit

- A Wheatstone Bridge Can Be Thevenized.
  - Problem: Find the voltage drop across R_L.
  - The bridge is unbalanced and Thevenin's theorem is a good choice.
  - R_L will be removed in this procedure making **A** and **B** the Thevenin terminals.

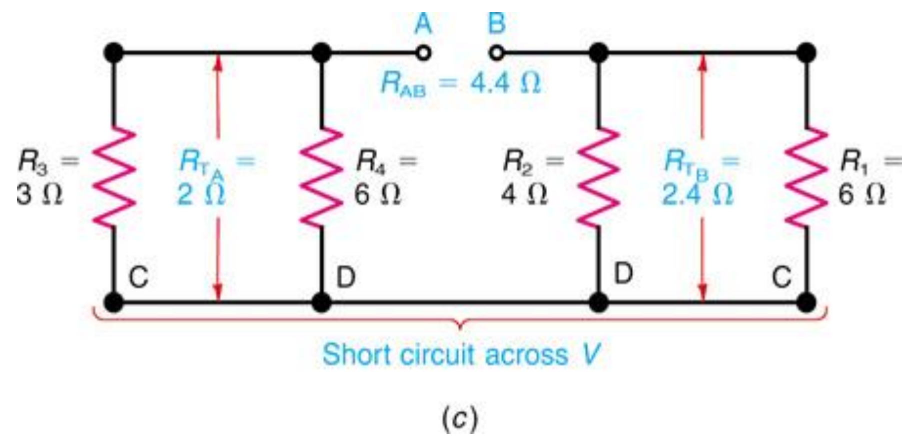


(a)

Thevenizing a Bridge Circuit



$V_{AB} = -20 - (-12) = -8V$



$R_{AB} = R_{TA} + R_{TB} = 2 + 2.4 = 4.4\ \Omega$

Fig. 4(*b*) Disconnect R_L to find V_{AB} of −8 V. (*c*) With source V short-circuited, R_{AB} is 2 + 2.4 = 4.4 Ω.

Thevenizing a Bridge Circuit

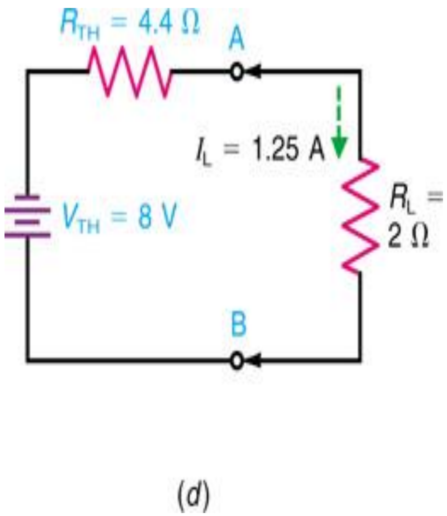


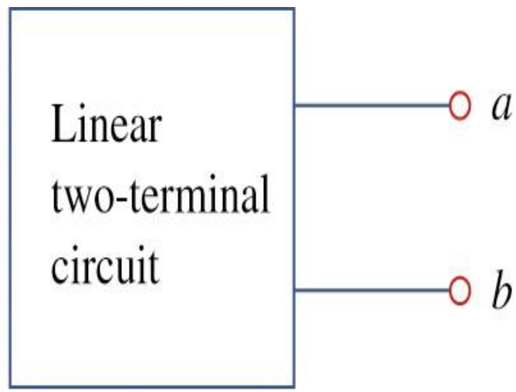
Fig. 4(*d*) Thevenin equivalent with R_L reconnected to terminals A and B.

Network Theorems

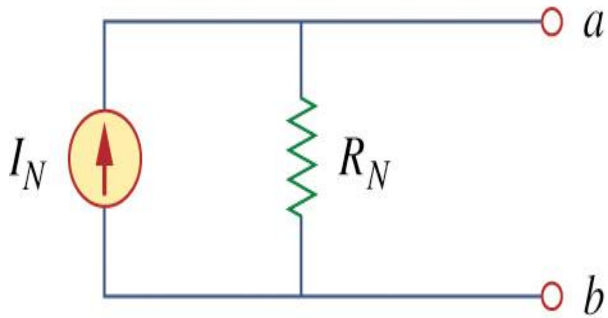
- Introduction
- Linear Circuits and Superposition
- Thevenin’s theorem
- Norton’s theorem
- Maximum power transfer theorem
- Millman's theorem
- Reciprocity theorem
- Substitution theorem
- Compensation theorem
- Tellegen’s theorem

Norton's Theorem

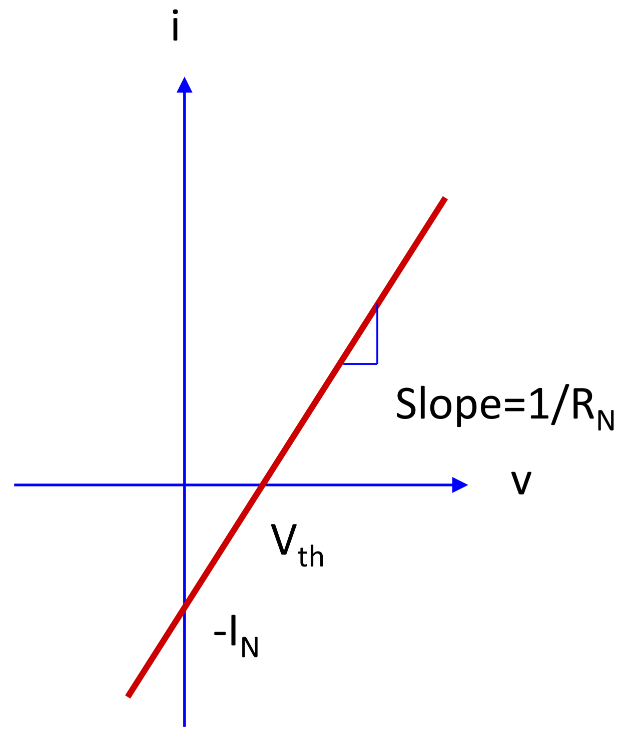
- Formally, Norton’s Theorem states that a linear two terminal resistive circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals, and R_N is the input or equivalent resistance at the terminals when all independent sources are all turned off.



(a)



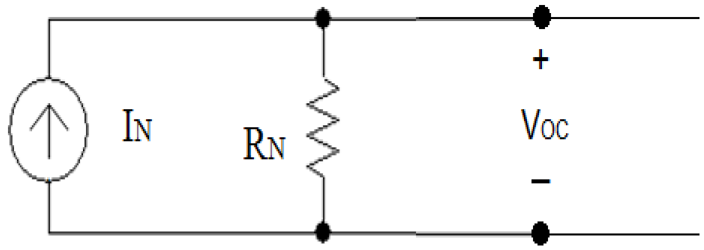
(b)



Norton's Theorem

- A linear two-terminal circuit can be replaced with an equivalent circuit of an ideal current source, I_N, in series with a resistor, R_N.
  - I_N is equal to the short-circuit current at the terminals.
  - R_N is the equivalent or input resistance when the independent sources are turned off.

Definitions for Norton's Theorem

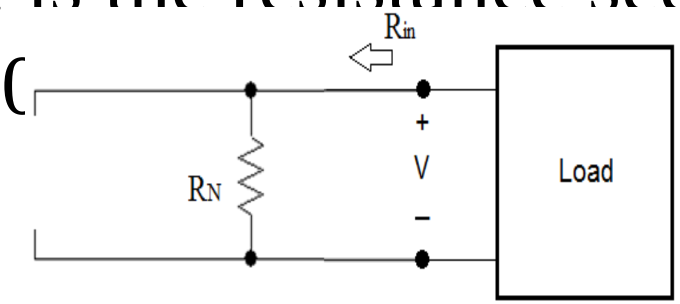


Open-circuit voltage I_{sc} is the current, *i*, when the load is a short circuit (i.e., R_L = 0W).

$$I_{SC} = I_N$$

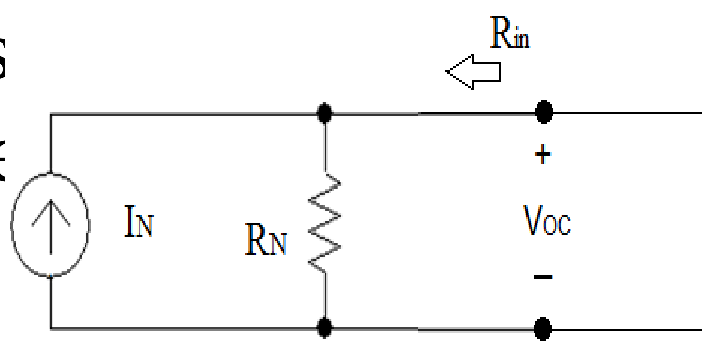
Definitions for Norton's Theorem

- Input resistance is the resistance seen by the load when I_N = (



- It is … f the linear circuit whe… n circuit ($R_L = \infty\ \Omega$).

$$R_{in} = R_N = V_{OC} / I_N$$



Steps to Determine I_N and R_N

1. Identify the load, which may be a resistor or a part of the circuit.
2. Replace the load with a short circuit .
3. Calculate I_{SC}. This is I_N.
4. Turn off all independent voltage and currents sources.
5. Calculate the equivalent resistance of the circuit. This is R_{TH}.
   - The current through and voltage across the load in parallel with I_N and R_N is the load's actual current and voltage in the originial circuit.

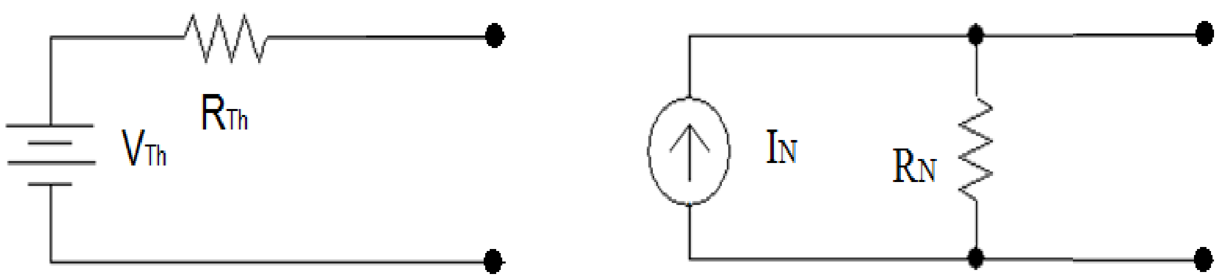
Norton's Theorem

- **Experimental Procedure**

The Norton current is measured in the same way as described for the short-circuit current (I_{sc}) for the Thevenin network. Since the Norton and Thevenin resistances are the same, the same procedures can be followed as described for the Thevenin network.

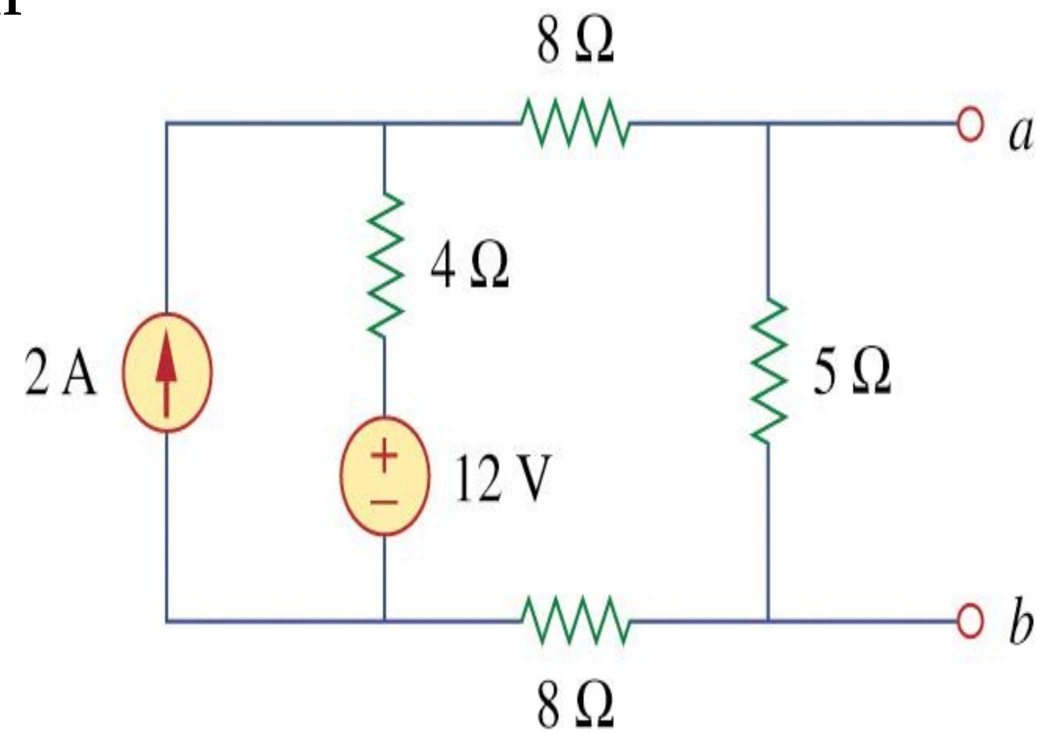
Source Conversion (Transformation)

- A Thevenin's equivalent circuit can easily be transformed to a Norton equivalent circuit (or visa versa).
  - If $R_{Th} = R_N$ then $V_{Th} = R_N I_N$ and $I_N = V_{Th}/R_{Th}$



Example (5)

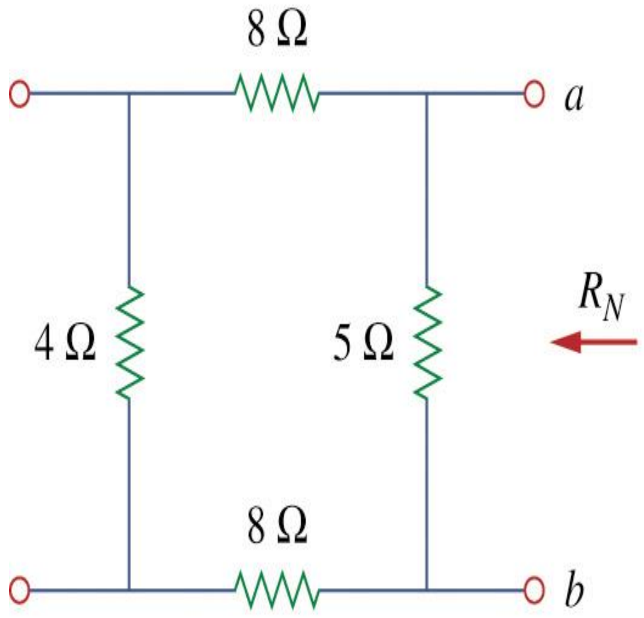
- Find the Norton equivalent circuit of the circuit in Fig



To find R_N Fig(a):

$$R_N = 5 \| (8+4+8)$$

$$= 5 \| 20 = \frac{20 \times 5}{25} = 4\Omega$$



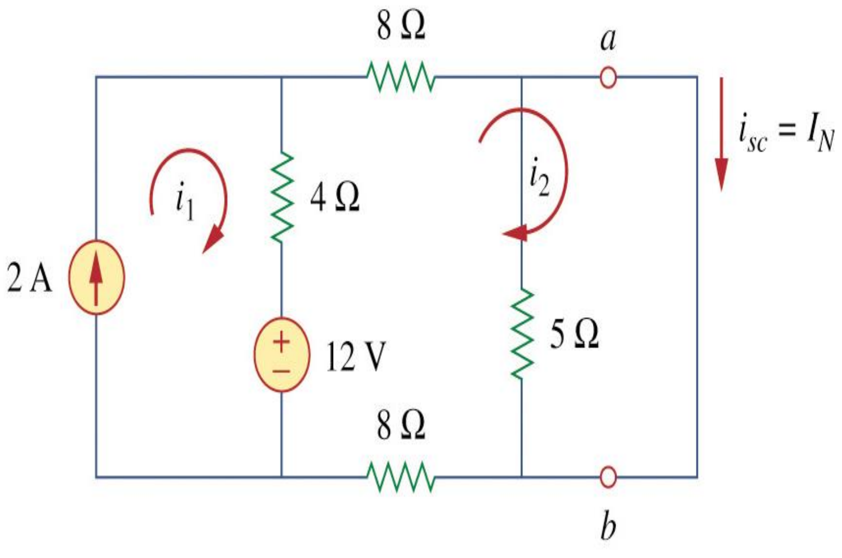
(a)

To find i_N (Fig.(b))

short – circuit terminals a and b.

Mesh: $i_1 = 2\text{A}, \quad 20i_2 - 4i_1 - i_2 = 0$

$i_2 = 1\text{A} = i_{sc} = I_N$



(b)

$\boxed{\text{Alternative method for } I_N}$ $$I_N = \frac{V_{Th}}{R_{Th}}$$

V_{Th}: open – circuit voltage across terminals a and b

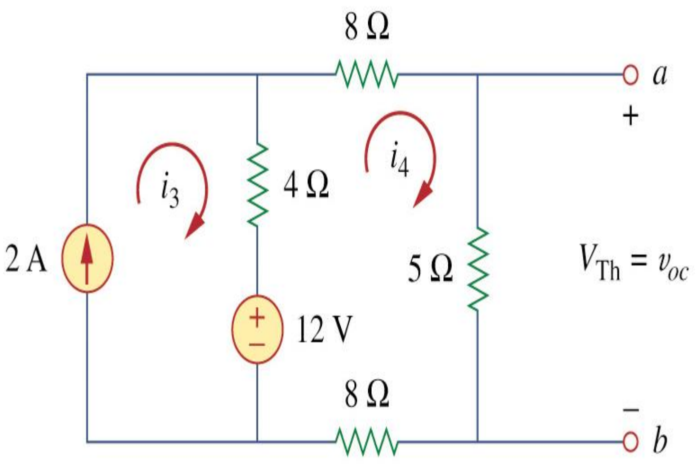
$(Fig(c))$:

Mesh analysis:

$$i_3 = 2A, \ \ 25i_4 - 4i_3 - 12 = 0$$

$$\therefore \ i_4 = 0.8\text{A}$$

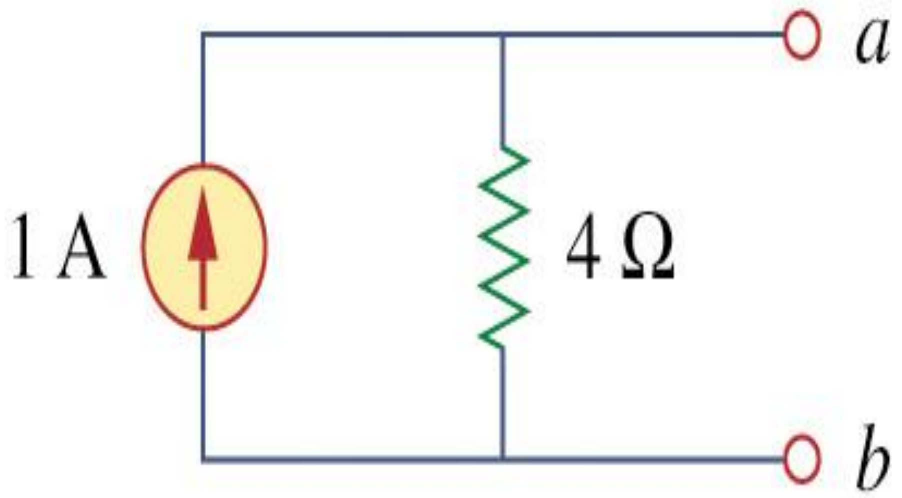
$$\therefore v_{oc} = V_{Th} = 5i_4 = 4\text{V}$$



(c)

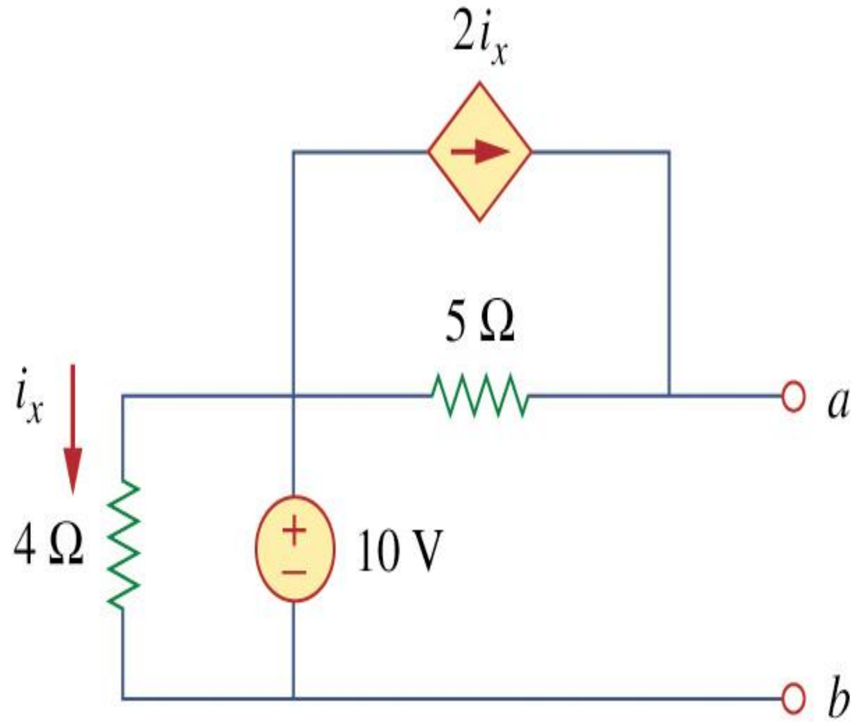
Hence,

$$I_N = \frac{V_{Th}}{R_{Th}} = 4/4 = 1\text{A}$$



Example (6)

Using Norton's theorem, find R_N and I_N of the circuit in Fig. at terminals *a*-*b*.

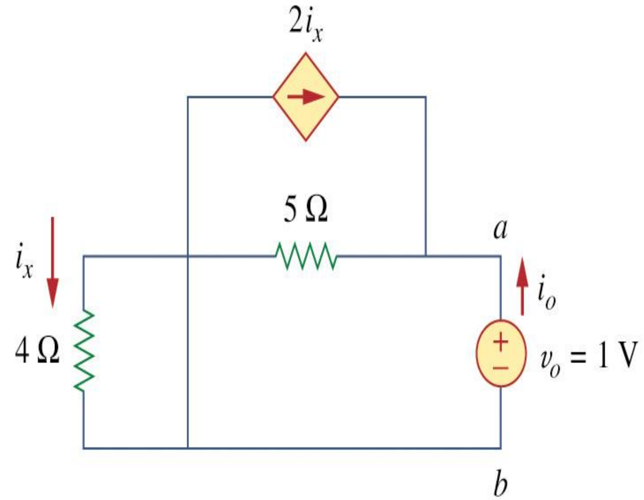


$\boxed{\text{To find } R_N}$ $Fig.(a)$

- 4Ω resistor shorted
- $5\Omega \| v_o \| 2i_x$: Parallel

Hence, $i_x = v_o/5 = 1/5 = 0.2$

$$\therefore R_N = \frac{v_o}{i_o} = \frac{1}{0.2} = 5\Omega$$



(a)

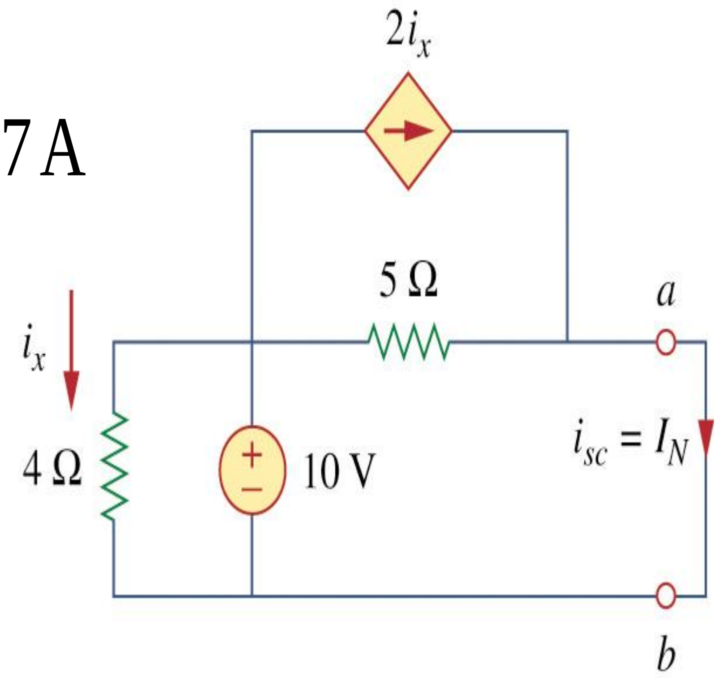
$\boxed{\text{To find } I_N}$ *Fig.(b)*

- $4\Omega \,\|\, 10v \,\|\, 5\Omega \,\|\, 2i_x$:Parallel

$$i_x = \frac{10-0}{4} = 2.5\text{A},$$

$$i_{sc} = i_x + 2i_x = \frac{10}{5} + 2(2.5) = 7\,\text{A}$$

$$\therefore I_N = 7\text{A}$$



(b)

Norton's Theorem

- A Wheatstone Bridge Can Be Nortonized.

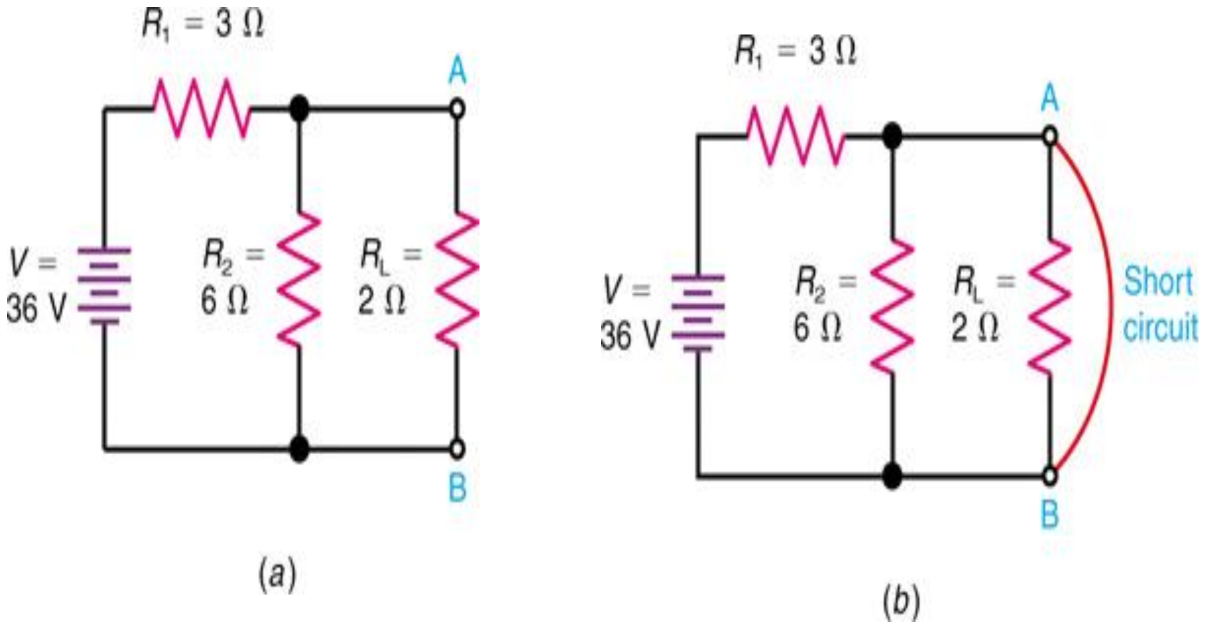


Fig. (*a*) Original circuit. (*b*) Short circuit across terminals A and B.

Norton's Theorem

- The Norton Equivalent Circuit
  - Replace R_2 with a short and determine I_N.
  - Apply the current divider.
  - Apply KCL.
  - $R_N = R_{TH}$.
  - The current source provides 12 A total flow, regardless of what is connected across it. With no load, all of the current will flow in R_N. When shorted, all of the current will flow in the short.
    - Connect R_2.
    - Apply the current divider.
    - Use Ohm's Law.

Norton's Theorem

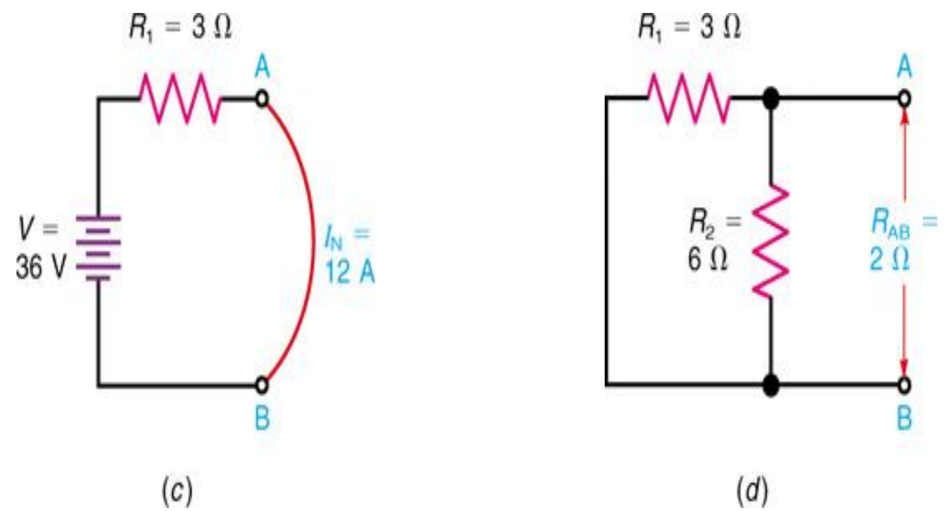
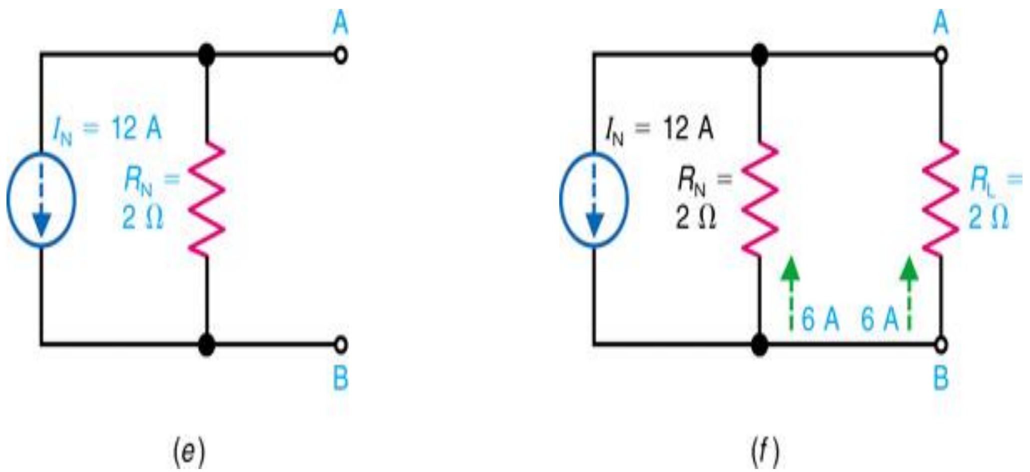


Fig. (*c*) The short-circuit current I_N is 36/3 = 12 A. (*d*) Open terminals A and B but short-circuit V to find R_{AB} is 2 Ω, the same as R_{TH}.

Norton's Theorem



$I_L = I_N \times R_N/R_N + R_L = 12 \times$
$2/4 =$ 6 A

Fig. (*e*) Norton equivalent circuit. (*f*) R_L reconnected to terminals A and B to find that I_L is 6A.

Example (7)

- Find the Norton equivalent circuit to the left of terminals A-B for the network shown below. Connect the Norton equivalent circuit to the load and find the current in the 50 Ω resistor.

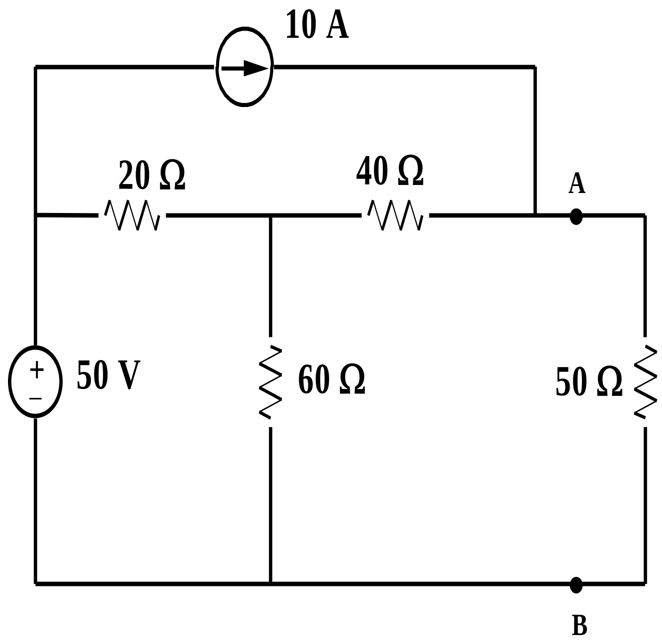


Figure 1 : Circuit for Example (7)

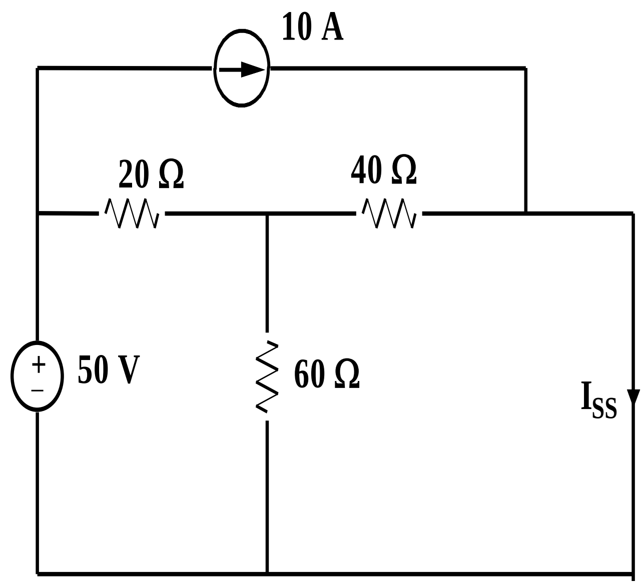


Figure 2: Circuit for find I_{NORTON}.

It can be shown by standard circuit analysis that

$$I_{SS} = 10.7\,A$$

- It can also be shown that by deactivating the sources, We find the resistance looking into terminals A-B is

$$R_N = 55\ \Omega$$

- R_N and R_{TH} will always be the same value for a given circuit. The Norton equivalent circuit tied to the load is shown below.

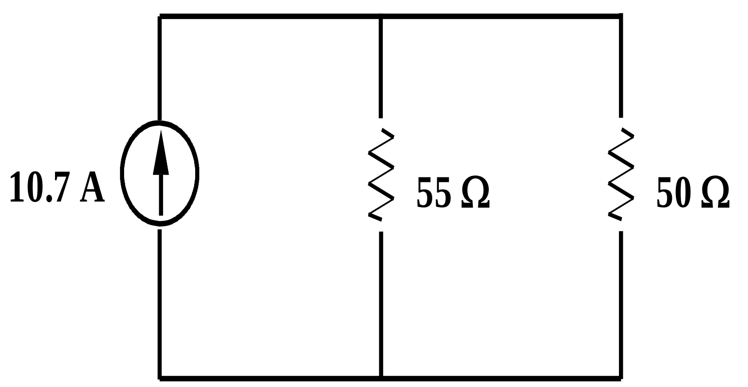


Figure 3: Final circuit for Example (7)

Example (8)

- For the circuit shown below, find the Norton equivalent circuit to the left of terminals A-B.

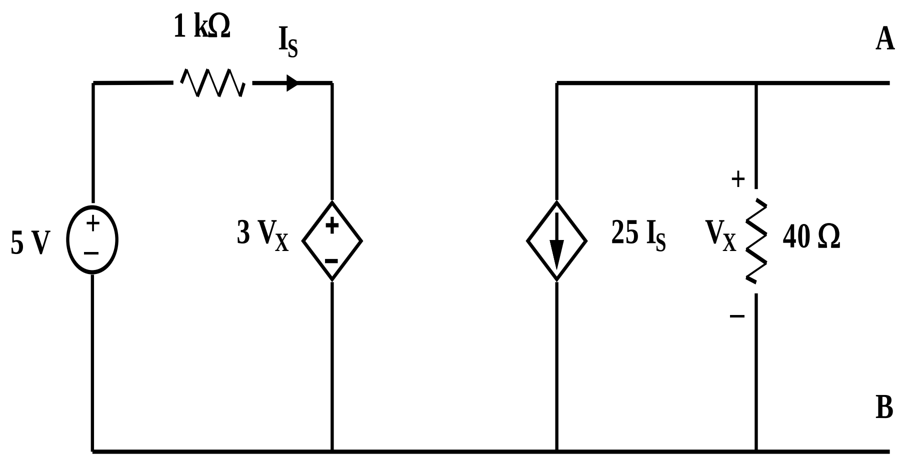
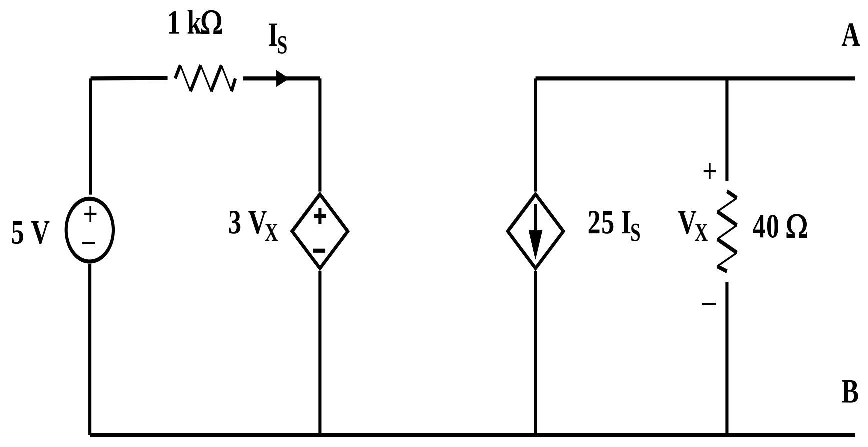


Figure 4: Circuit for Example (8).



We first find;

$$R_N = \frac{V_{OS}}{I_{SS}}$$

We first find V_{OS}:

$$V_{OS} = V_X = (-25I_S)(40) = -1000I_S$$

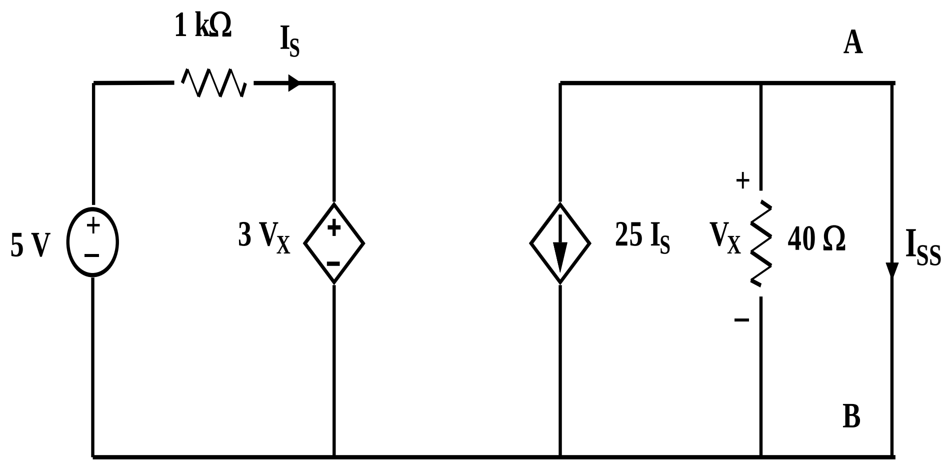


Figure 5: Circuit for find I_{SS}, Example (8).

We note that I_{SS} = - 25I_S. Thus,

$$R_N = \frac{V_{OS}}{I_{SS}} = \frac{-1000 I_S}{-25 I_S} = 40\ \Omega$$

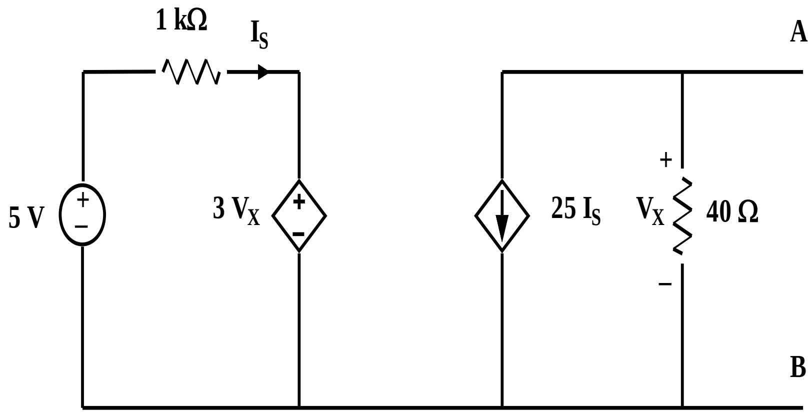


Figure 6: Circuit for find V_{OS}, Example (8).

From the mesh on the left we have;

$$-5+1000I_S+3(-1000I_S)=0$$

From which,

$$I_S=-2.5\ mA$$

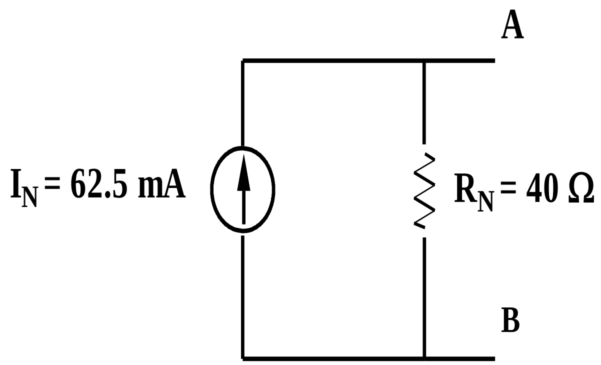
We saw earlier that,

$$I_{SS} = -25\, I_S$$

Therefore;

$$I_{SS} = 62.5\ mA$$

The Norton equivalent circuit is shown below.



Norton Circuit for Example (8)

Network Theorems

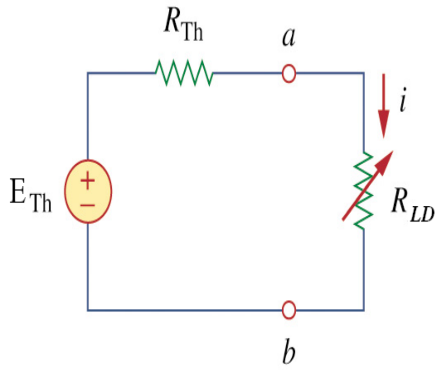
- Introduction
- Linear Circuits and Superposition
- Thevenin’s theorem
- Norton’s theorem
- Maximum power transfer theorem
- Millman's theorem
- Reciprocity theorem
- Substitution theorem
- Compensation theorem
- Tellegen’s theorem

Maximum Power Transfer

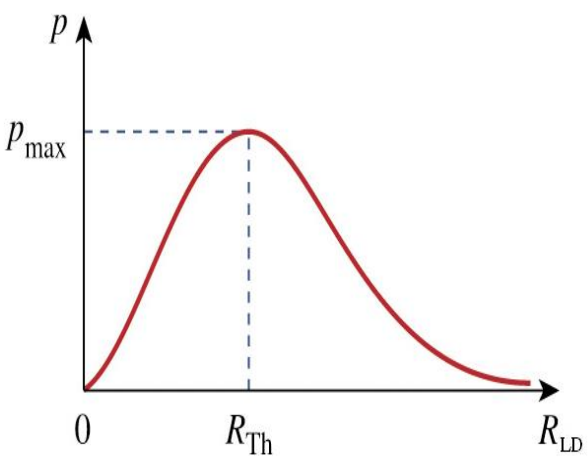
- In some applications, the purpose of a circuit is to provide maximum power to a load.
- Some examples:
  - Stereo amplifiers
  - Radio transmitters
  - Communications equipment
- **The question is:** If you have a system, what load should you connect to the system in order for the load to receive the maximum power that the system can deliver?

Maximizing P_{LD}

- How might we determine $\boldsymbol{R_{LD}}$ such that $\boldsymbol{P_{LD}}$ is maximized?

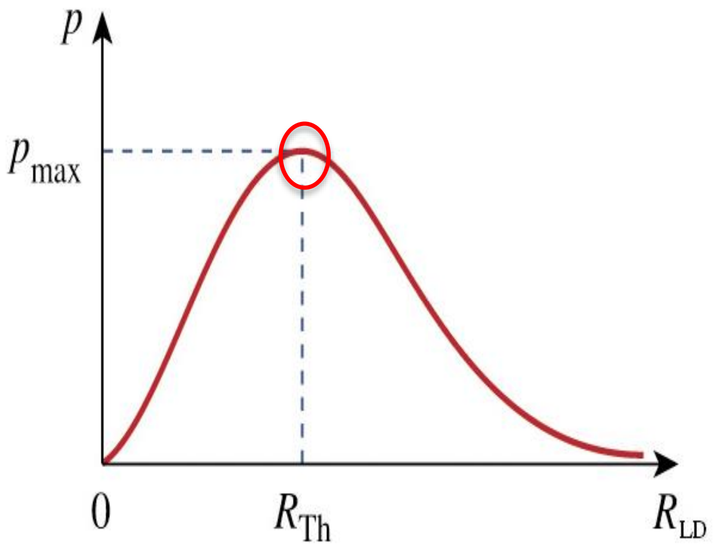


$$P_{LD} = I_{LD}^2 R_{LD} = \left(\frac{V_{Th}}{R_{Th} + R_{LD}} \right)^2 R_{LD}$$



Maximum Power Transfer

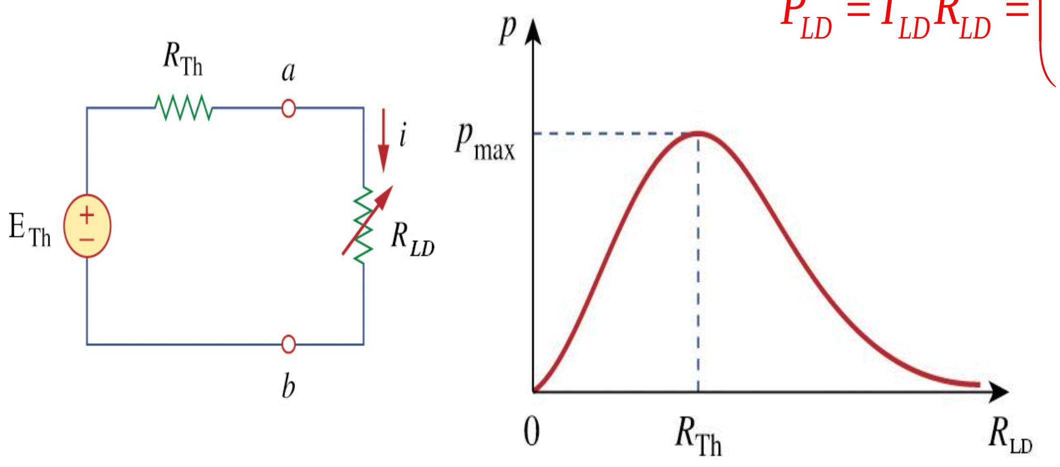
- **Maximum power** is transferred to the load when the load resistance equals the Thevenin's resistance as seen from the load ($\boldsymbol{R_{LD}} = \boldsymbol{R_{\mathrm{Th}}}$).
  - When $\boldsymbol{R_{LD}} = \boldsymbol{R_{\mathrm{Th}}}$, the source and load are said to be **matched**.

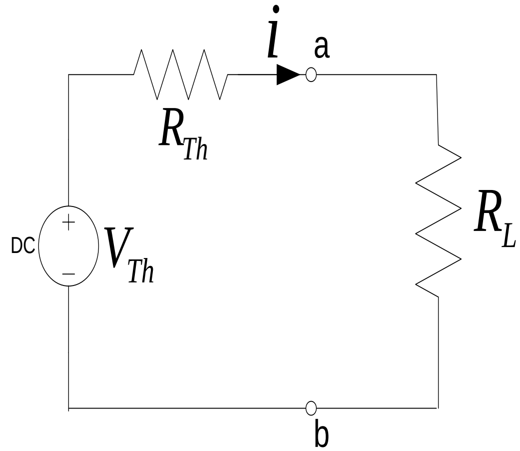


Maximizing P_{LD}

- As $\boldsymbol{R_{LD}}$ increases, a higher percentage of the total power is dissipated in the load resistor.
- But since the total resistance is increasing, the total current is dropping, and a point is reached where the total power dissipated by the entire circuit starts dropping.

$$P_{LD} = I_{LD}^2 R_{LD} = \left(\frac{V_{\mathrm{Th}}}{R_{\mathrm{Th}} + R_{LD}}\right)^2 R_{LD}$$





R_{LD} and R_L both are same

The power delivered to the load (absorbed by R_L) is

$$p = i^2 R_L = \left[V_{Th} / \left(R_{Th} + R_L \right) \right]^2 R_L$$

This power is maximum when $\partial p / \partial R_L = 0$

$$\frac{\partial p}{\partial R_L} = V_{Th}^{\ 2} \left[\left(R_{Th} + R_L \right)^{-2} - 2R_L \left(R_{Th} + R_L \right)^{-3} \right] = 0$$

$$\frac{dp}{dR_L} = V_{Th}^{\ 2}\left[\left(R_{Th}+R_L\right)^{-2} - 2R_L\left(R_{Th}+R_L\right)^{-3}\right] = 0$$

$$R_{Th} + R_L = 2R_L$$

$$R_L = R_{Th}$$

Thus, maximum power transfer takes place when the resistance of the load equals the Thevenin resistance R_{Th}. Note also that

$$p_{\max} = \left[V_{Th} \big/ \left(R_{Th}+R_L\right)\right]^2 R_L \Big|_{R_L = R_{Th}}$$

$$p_{\max} = \left[V_{Th} \big/ \left(2R_{Th}\right)\right]^2 R_{Th} = V_{Th}^{\ 2} \big/ 4R_{Th}$$

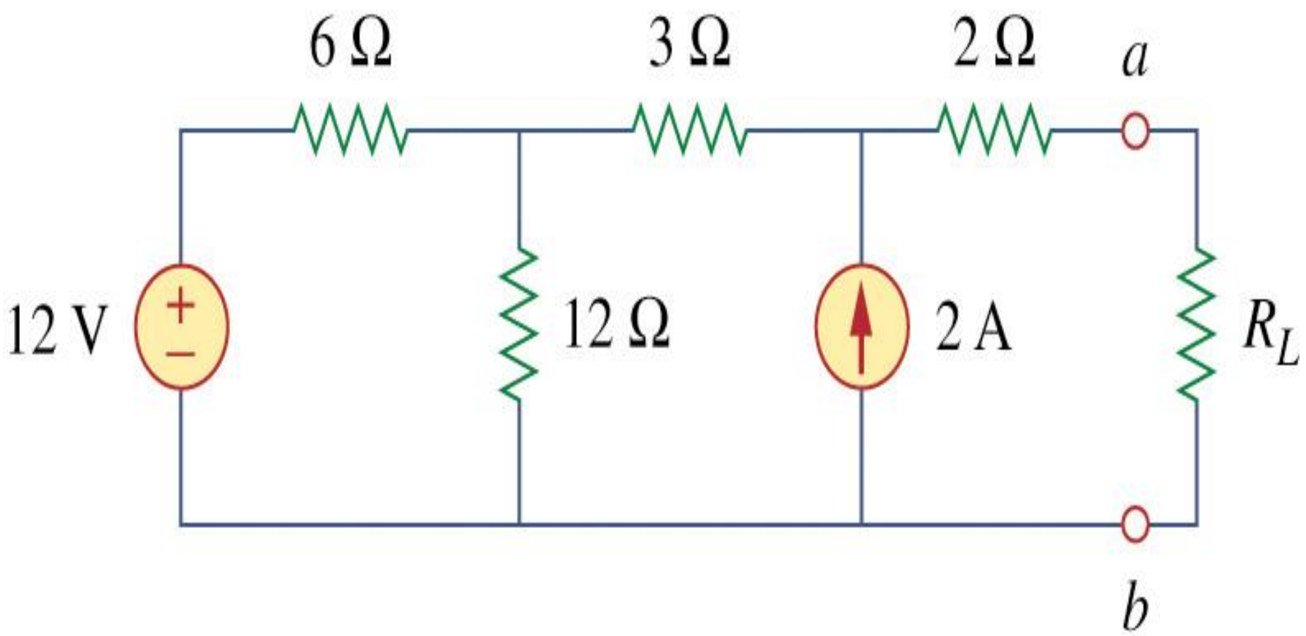
Thus, at best, one-half of the power is dissipated in the internal resistance and one-half in the load.

Maximum Power Transfer

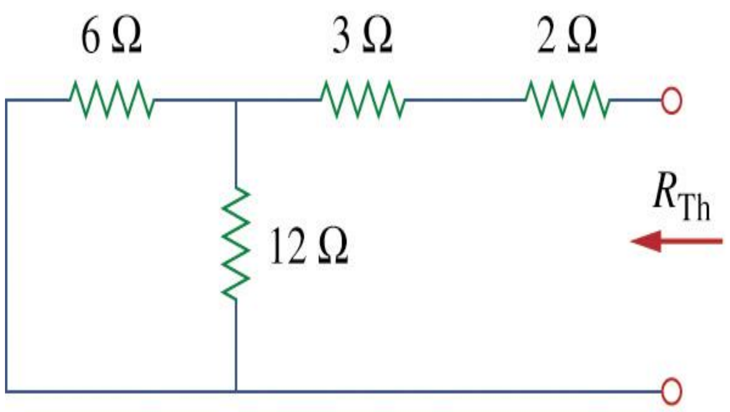
- *The total power delivered by a supply such as E_{Th} is absorbed by both the Thevenin's equivalent resistance and the load resistance.*
- *Any power delivered by the source that does not get to the load is lost to the Thevenin's resistance.*

Example

- Find the value of R_L for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.



$$R_{TH} = 2 + 3 + 6 \| 12 = 5 + \frac{6 \times 12}{18} = 9\Omega$$



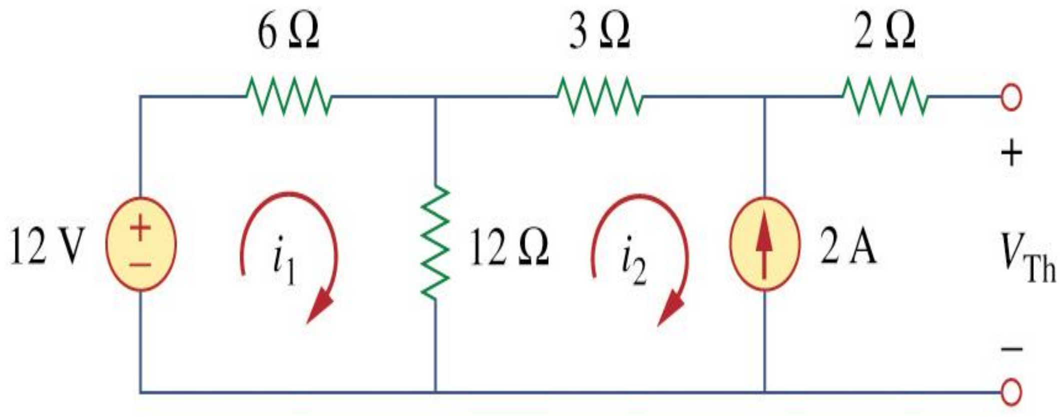
(a)

$$-12+18i_1-12i_2, \quad i_2=-2A$$

$$-12+6i_i1+3i_2+2(0)+V_{TH}=0 \Rightarrow V_{TH}=22V$$

$$R_L=R_{TH}=9\Omega$$

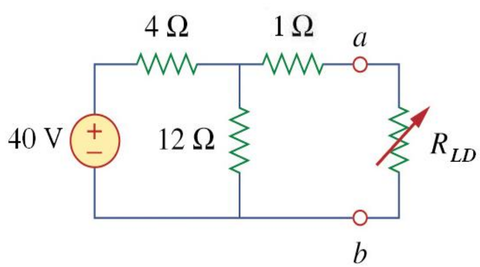
$$p_{\max}=\frac{V_{TH}^2}{4R_L}=\frac{22^2}{4\times 9}=13.44W$$

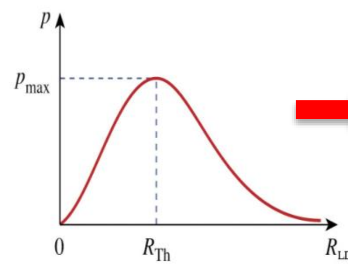


(b)

Example

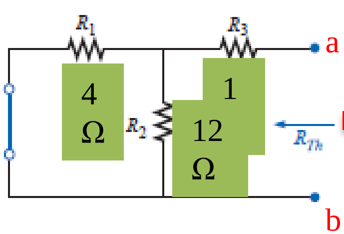
a) Find the Thevenin's equivalent circuit to the left of terminals *a*-*b*.
b) Calculate the maximum power transfer to the load if $\boldsymbol{R_{LD} = R_{TH}}$.
c) Determine the power dissipated by $\boldsymbol{R_{LD}}$ for load resistances of 2Ω and 6Ω.

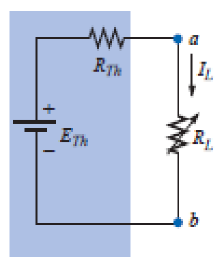




b) When $R_{LD} = R_{TH}$

$$\frac{V_{Th}^2}{4R_{Th}} = P_{MAX}$$





c) When $R_{LD} \neq R_{TH}$

$$P_L = I_L^2 * R_L \text{ OR } P_L = \left(\frac{V_{Th}}{R_{Th} + R_{LD}}\right)^2 R_{LD}$$

For the 2Ω load and because we already calculated V_{TH} and R_{TH} let's use:

$$P_L = \left(\frac{V_{Th}}{R_{Th} + R_{LD}}\right)^2 R_{LD} = \left(\frac{30V}{4\Omega + 2\Omega}\right)^2 * 2\Omega = 50W$$

Now, for the 6Ω load, and just to show it works let's use:

$$P_L = (I_L)^2 * R_{LD} = \left(\frac{30V}{4\Omega + 6\Omega}\right)^2 * 6\Omega = 54W$$

a) $R_{TH} = R_{ab} = \left(\frac{1}{4} + \frac{1}{12}\right)^{-1} + 1 = \boxed{4\Omega}$

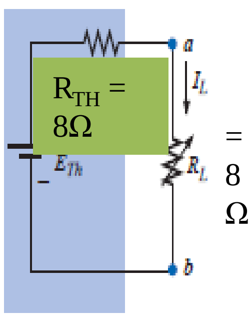
$$(VDR) \to E_{TH} = V_{ab} = V_{12\Omega} = E * \frac{R_{12\Omega}}{R_T} = 40V * \frac{12\Omega}{4\Omega + 12\Omega} = \boxed{30V}$$

Example

A stereo is rated for max output power of 150W per channel when $R_{LD} = 8\Omega$

a) Sketch the Thevenin's Equivalent circuit.

b) What would the output power be with two 8Ω speakers as the load and are connected in parallel to one of the channels?

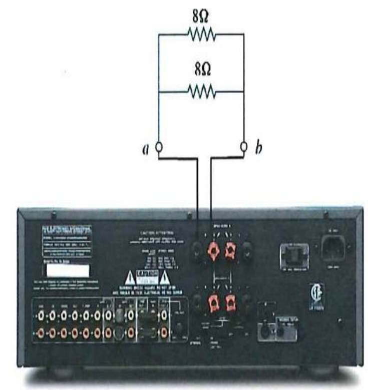


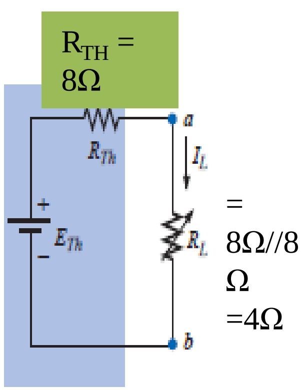
b) We know that $P_{MAX} = \dfrac{V_{Th}^2}{4R_{Th}}$

Rearrange the above and solve for V_{TH}:

$$V^2_{TH} = P_{MAX} * 4R_{Th}$$

$$V_{TH} = \sqrt{P_{MAX} * 4R_{Th}} = \sqrt{150W * 4 * 8\Omega} = \boxed{69.28V}$$





Now, to caculate output power for the two 8Ω resistors in parallel,

calculate R_L (8//8 = 4Ω),

and because we already know V_{TH} and R_{TH} let's use:

$$P_L = \left(\frac{V_{Th}}{R_{Th} + R_{LD}}\right)^2 R_{LD} = \left(\frac{69.28V}{8\Omega + 4\Omega}\right)^2 * 4\Omega = \boxed{133.3W}$$

You should note, P_L for this case is less than P_{MAX}.

Network Theorems

- Introduction
- Linear Circuits and Superposition
- Thevenin’s theorem
- Norton’s theorem
- Maximum power transfer theorem
- Millman's theorem
- Reciprocity theorem
- Substitution theorem
- Compensation theorem
- Tellegen’s theorem

Millman's theorem

- The application of **Millman's theorem,** any number of parallel voltage sources can be reduced to one.
- In Figure, the three voltage sources can be reduced to one. This permits finding the current through or voltage across *RL* without having to apply a method such as mesh analysis, nodal an

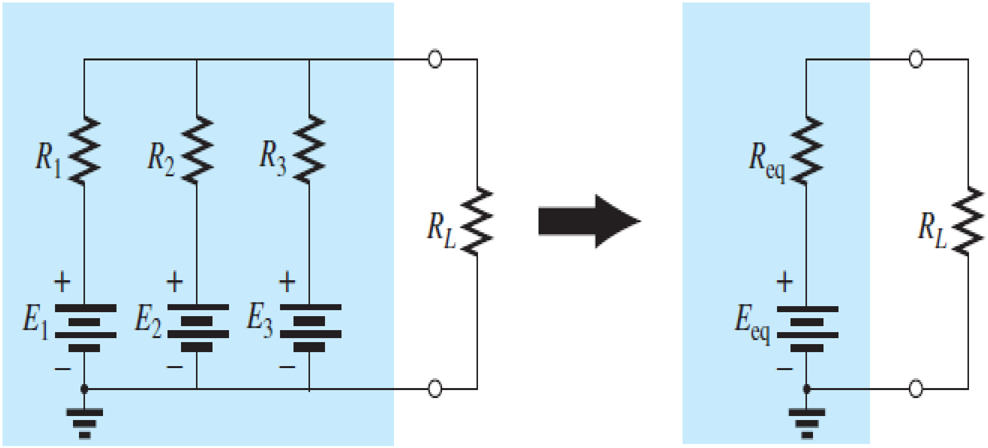


Figure: Demonstrating the effect of applying Millman's theorem.

Millman's theorem

Step 1: Convert all voltage sources to current sources.

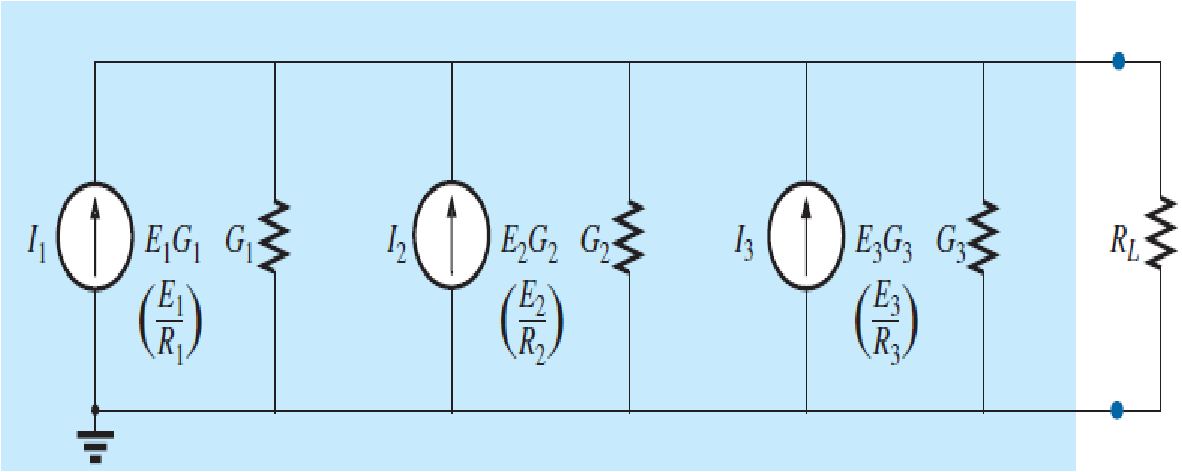


Figure: Converting all the voltage sources to current sources.

Millman's theorem

Step 2: Combine parallel current sources.

$$I_T = I_1 + I_2 + I_3 \text{ and } G_T = G_1 + G_2 + G_3$$

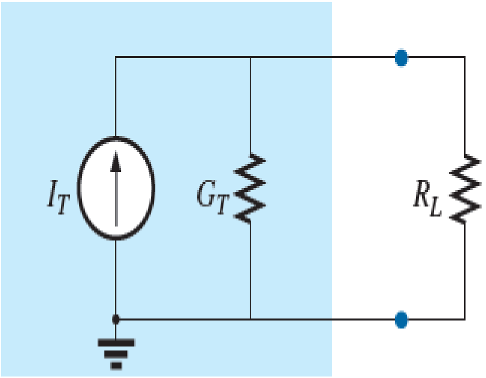


Figure: Reducing all the current sources into a single current source.

Millman's theorem

Step 3: Convert the resulting current source to a voltage source, and the desired single-source network is obtained.

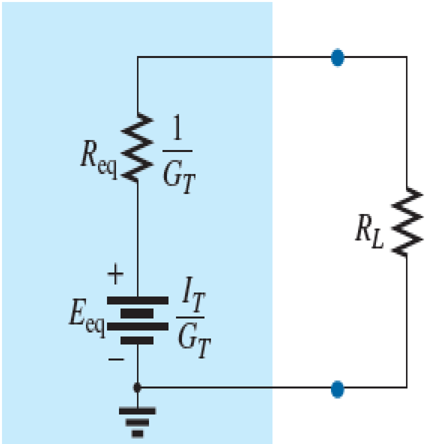


Figure: Converting the single current source into a single voltage source.

Millman's theorem

In general, Millman's theorem states that for any number of parallel voltage sources,

$$E_{eq} = \frac{I_T}{G_T} = \frac{\pm I_1 \pm I_2 \pm I_3 \pm \cdots \pm I_N}{G_1 + G_2 + G_3 + \cdots + G_N}$$

or

$$E_{eq} = \frac{\pm E_1 G_1 \pm E_2 G_2 \pm E_3 G_3 \pm \cdots \pm E_N G_N}{G_1 + G_2 + G_3 + \cdots + G_N}$$

The equivalent resistance is,

$$R_{eq} = \frac{1}{G_T} = \frac{1}{G_1 + G_2 + G_3 + \cdots + G_N}$$

Millman's theorem

In terms of the resistance values,

$$E_{eq} = \frac{\pm\frac{E_1}{R_1} \pm \frac{E_2}{R_2} \pm \frac{E_3}{R_3} \pm \cdots \pm \frac{E_N}{R_N}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}}$$

and

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}}$$

Millman's theorem

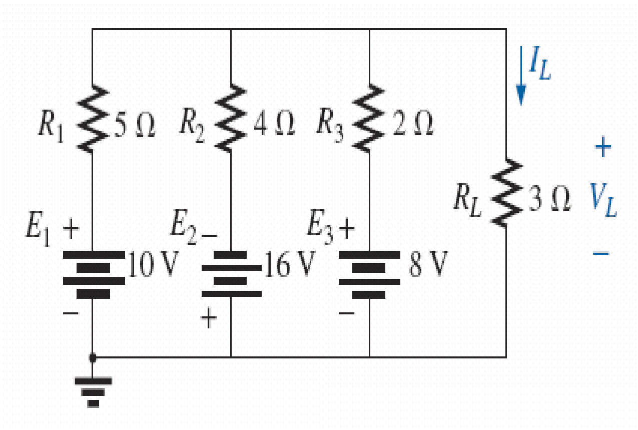


FIG. 1 *Example (9)*

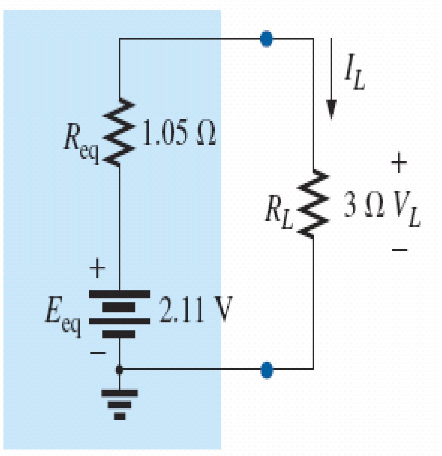


FIG. 2 *The result of applying Millman's theorem to the network in Fig. 1.*

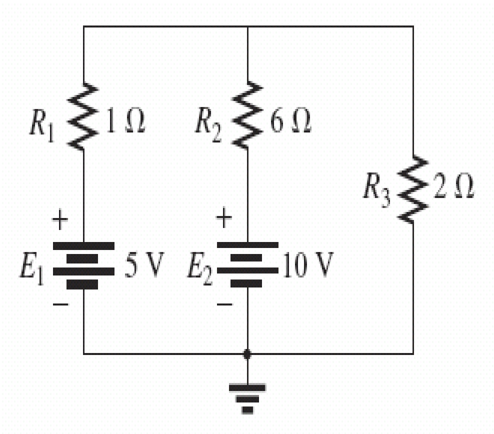


FIG. 3 ***Example (10).***

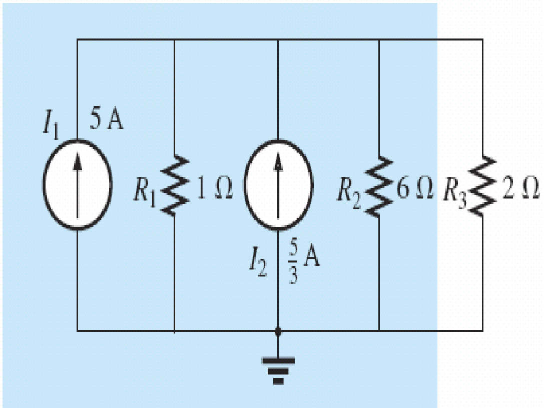


FIG. 4 ***Converting the sources in Fig. 3 to current sources.***

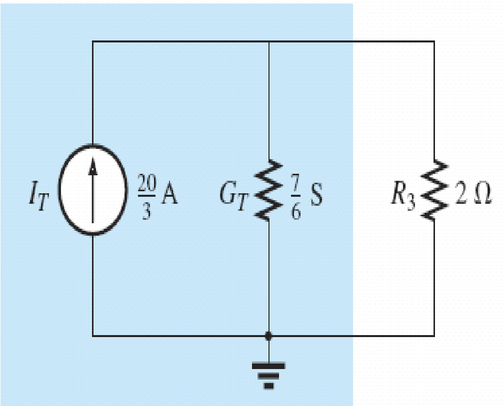


FIG. 5 ***Reducing the current sources in Fig. 4 to a single source.***

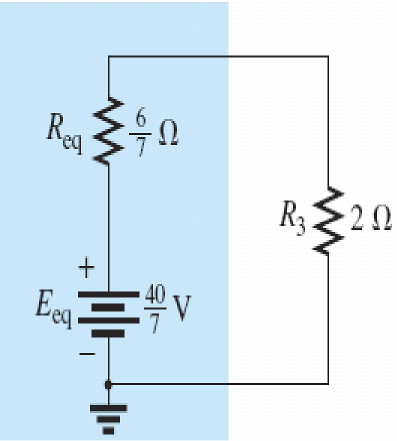


FIG. 6 ***Converting the current source in Fig. 5 to a voltage source.***

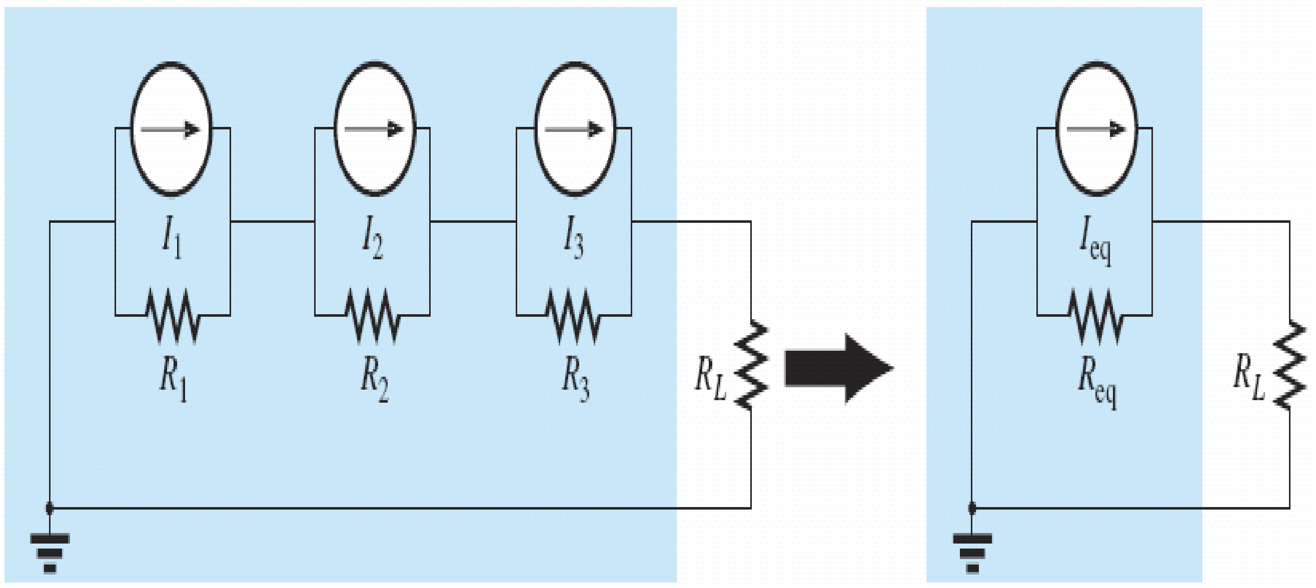


FIG. 7 ***The dual effect of Millman's theorem.***

Network Theorems

- Introduction
- Linear Circuits and Superposition
- Thevenin’s theorem
- Norton’s theorem
- Maximum power transfer theorem
- Millman's theorem
- Reciprocity theorem
- Substitution theorem
- Compensation theorem
- Tellegen’s theorem

Reciprocity theorem

- The **reciprocity theorem** is applicable only to single-source networks.
- The theorem states the following:

"The current I in any branch of a network due to a single voltage source E anywhere else in the network will equal the current through the branch in which the source was originally located if the source is placed in the branch in which th

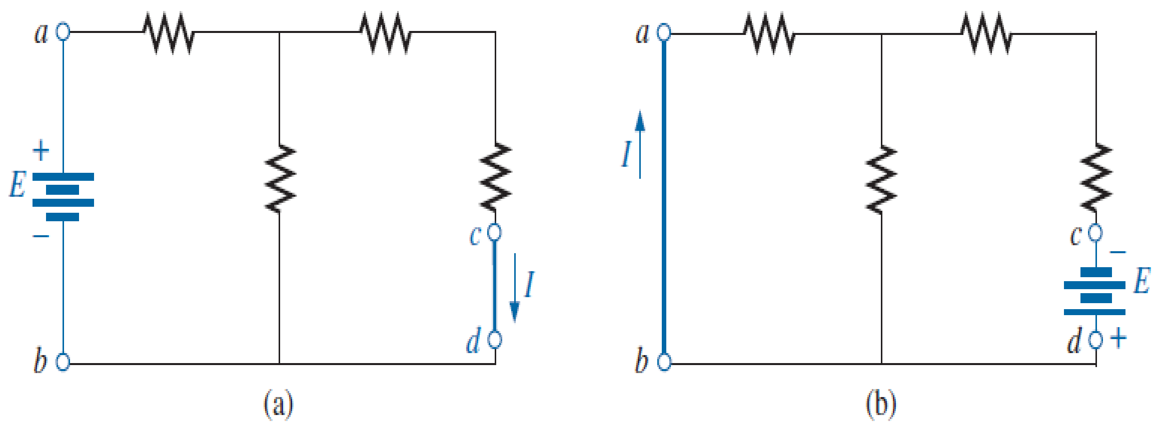
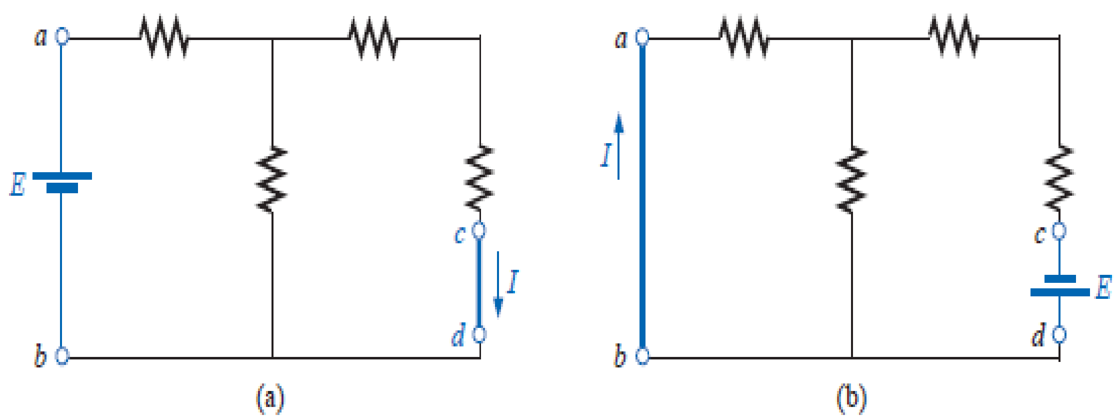


Figure 8: Demonstrating the impact of the reciprocity theorem.

Reciprocity theorem

- The location of the voltage source and the resulting current may be interchanged without a change in current.
- The theorem requires that the polarity of the voltage source have the same correspondence with the direction of the branch current in each position.
- In the representative network in Figure(a), the current *I* due to the voltage source *E* was determined. If the position of each i rrent *I* will b



- **To demonstrate the validity of this statement and the theorem, consider the network of Fig-9, in which values for the elements have been assigned.**

The total resistance is

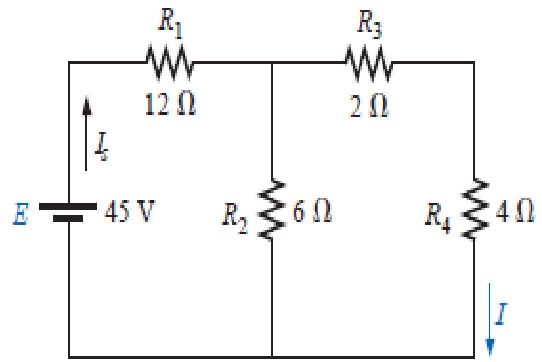


Fig-

$$R_T = R_1 + R_2 \parallel (R_3 + R_4) = 12\ \Omega + 6\ \Omega \parallel (2\ \Omega + 4\ \Omega)$$
$$= 12\ \Omega + 6\ \Omega \parallel 6\ \Omega = 12\ \Omega + 3\ \Omega = 15\ \Omega$$

And

$$I_s = \frac{E}{R_T} = \frac{45\ \text{V}}{15\ \Omega} = 3\ \text{A}$$

$$I = \frac{3\ \text{A}}{2} = \mathbf{1.5\ A}$$

For the network of Fig-10,

$$R_T = R_4 + R_3 + R_1 \parallel R_2$$

$$= 4\,\Omega + 2\,\Omega + 12\,\Omega \parallel 6\,\Omega = 10\,\Omega$$

$$I_s = \frac{E}{R_T} = \frac{45\text{ V}}{10\,\Omega} = 4.5\text{ A}$$

$$I = \frac{(6\,\Omega)(4.5\text{ A})}{12\,\Omega + 6\,\Omega} = \frac{4.5\text{ A}}{3} = \mathbf{1.5\text{ A}}$$

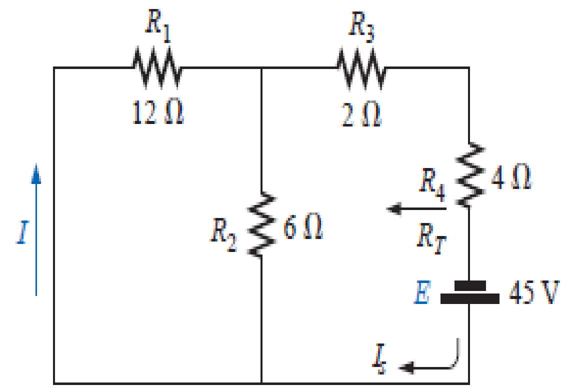


Fig-10

Network Theorems

- Introduction
- Linear Circuits and Superposition
- Thevenin’s theorem
- Norton’s theorem
- Maximum power transfer theorem
- Millman's theorem
- Reciprocity theorem
- Substitution theorem
- Compensation theorem
- Tellegens theorem

Substitution theorem

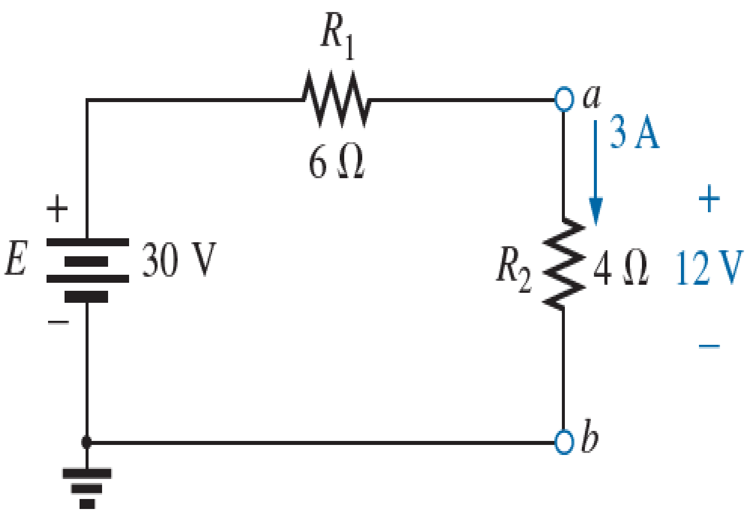
The **substitution theorem** states the following:

- If the voltage across and the current through any branch of a dc bilateral network are known, this branch can be replaced by any combination of elements that will maintain the same voltage across and current through the chosen branch.

- The theorem states that for branch equivalence, the terminal voltage and current must be the same.

Substitution theorem

- Consider the circuit in Figure11, in which the voltage across and current through the branch *a*-*b* are determined.



Substitution theorem

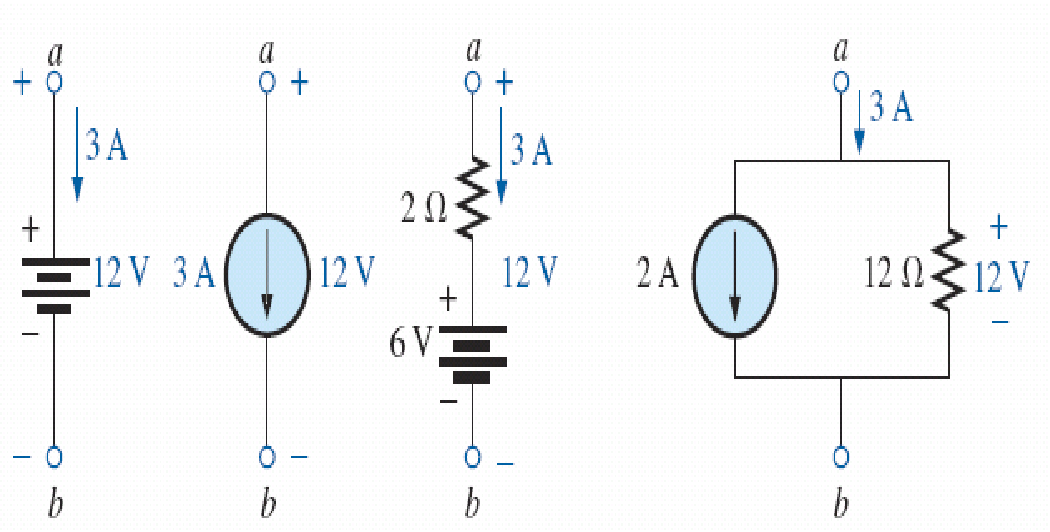


FIG. 12. ***Equivalent branches for the branch a-b in Fig.11 .***

Substitution theorem

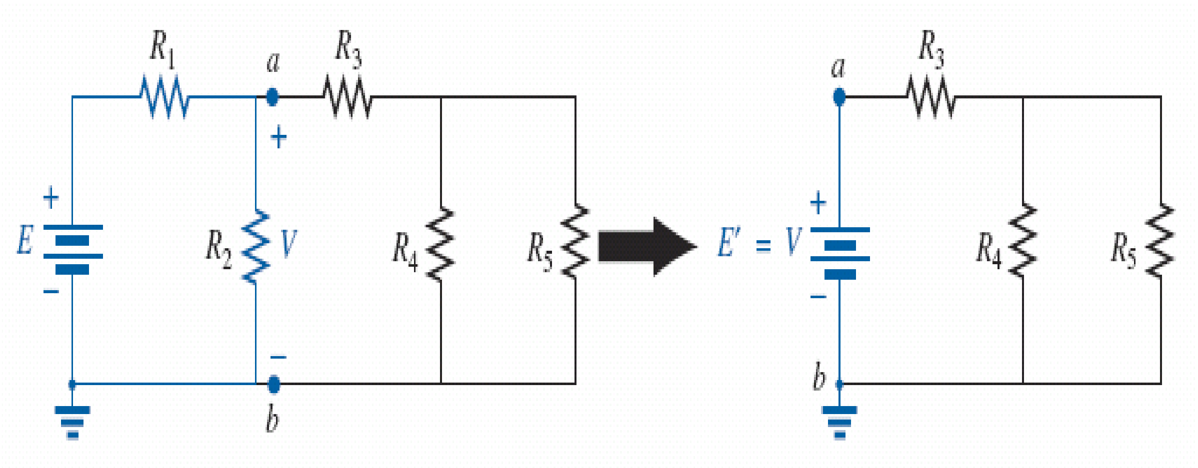


FIG. 13 ***Demonstrating the effect of knowing a voltage at some point in a complex network.***

Substitution theorem

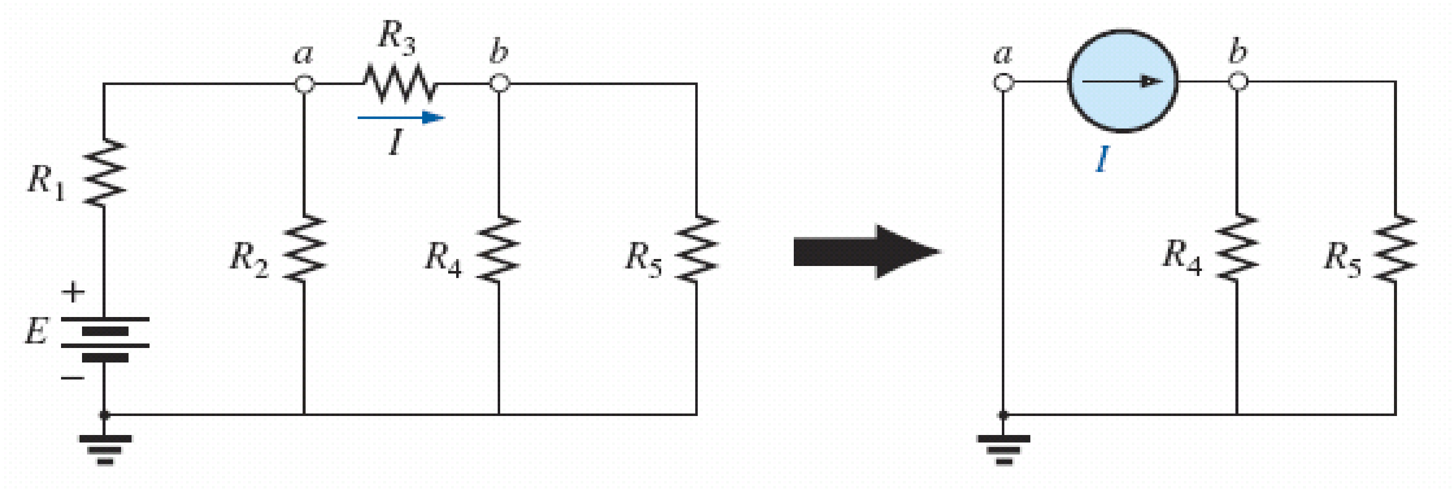


FIG. 14 ***Demonstrating the effect of knowing a current at some point in a complex network.***

Network Theorems

- Introduction
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- Reciprocity theorem
- Substitution theorem
- Compensation theorem
- Tellegen’s theorem

Compensation theorem

- It is one of the important theorems in Network Analysis , which finds it’s application mostly in calculating the sensitivity of electrical networks & bridges and solving electrical networks.
- The Compensation Theorem states that :-

“For the sake of branch responses calculations ; Any resistance in a branch of an linear bilateral electrical network can be replaced by a voltage source which provides the same voltage as the voltage dropped in the resistance.”

- In any linear bilateral Electrical Network, If in any Branch have it’s initial resistance (or impedance in case of AC) “R” conducting a current of “I” through it, and if the resistance of the branch is changed by a factor of R , with it’s final resistance R+ R , the final effect in various branches due to the change in the resistance of the branch can be calculated by injecting an extra voltage source along with the

- The above statement can be clarified with the following illustration.
- In Fig(a) , the current “I” flows through R3 when V1 acts upon it. In Fig (b) , the R3 is changed to R4 where R4=R3+dR , or R3 is increased by dR. This can also be thought of as an extra dR added in series with R3. Now , we don’t know how much current flows through the branch when R3 is increased by dR , so to calculate the current flowing through the branch due to the effect of dR , as per Compensation theorem in Fig (c). we add an extra V=-I.dR along with R4 and calculate the cu ... ne

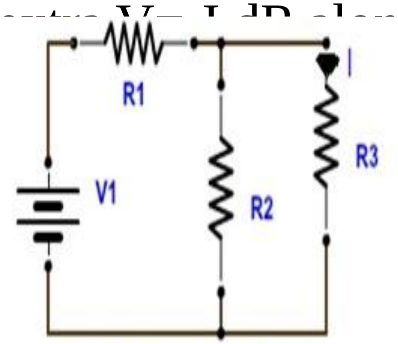


fig: .a Electrical Network

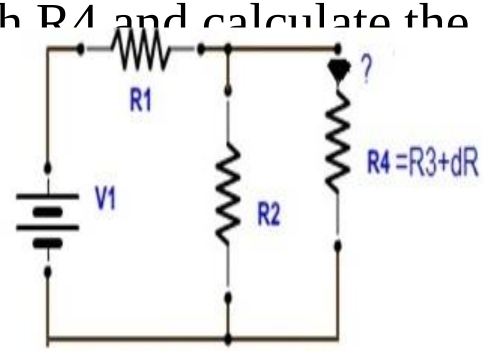


fig: b Addition of dR to R3

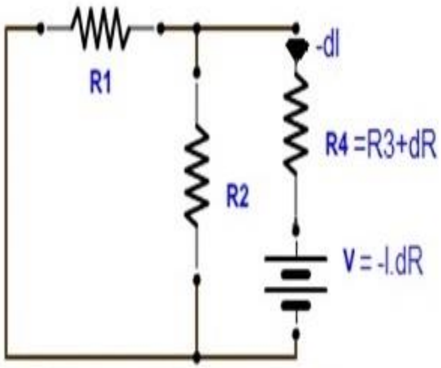


fig: c Calculation of branch response due to change in R3

- Now in Fig (d) we add the currents in Fig. (a) and (c) using superposition theorem to find the new current to be I-dI.

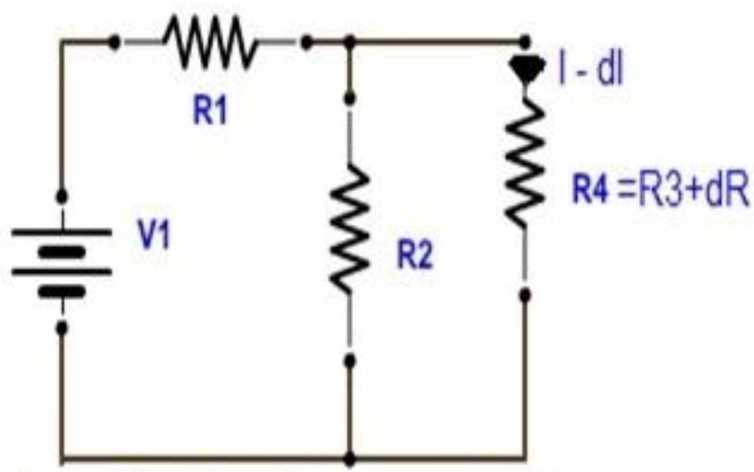


fig: d New branch response due to change in R3

Problem : Calculate the values of new currents in the network illustrated below wh
increased by 30 %.

Solution: In the given circuit, the values of various branch currents are

$$I_1 = 75/(5+10) = 5 \text{ A}$$

$$I_2 = I_3 = 2.5 \text{ A}$$

Now, value of

$$R_3 = 20 + (0.3 \times 20) = 26\ \Omega$$

$$\therefore \quad \Delta R = 6\ \Omega$$

$$V = -I_3\, \Delta R$$

$$= -2.5 \times 6 = -15 \text{ V}$$

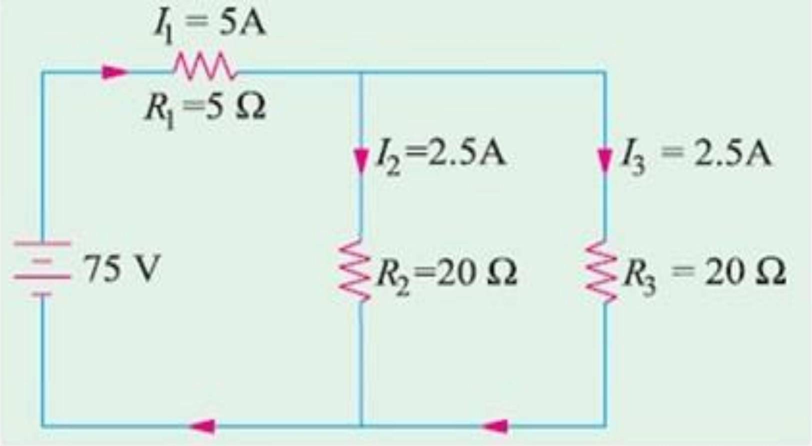
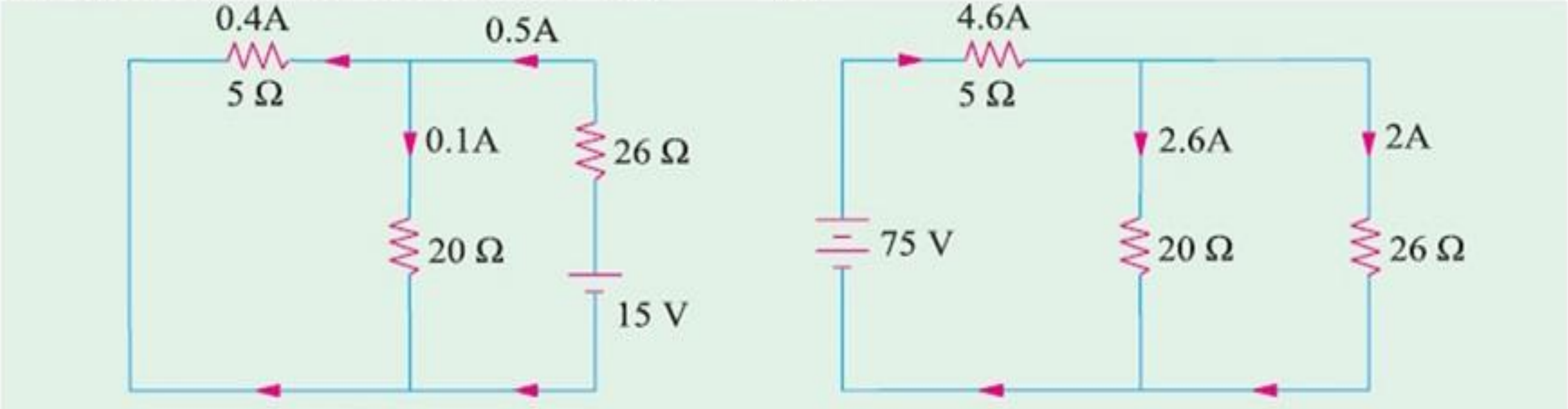


Fig.

The compensating currents produced by this voltage are as shown in Fig. (*a*).

When these currents are added to the original currents in their respective branches the new current distribution becomes as shown in Fig. (*b*)



(*a*) (*b*)

Network Theorems

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- Tellegen’s theorem

- **Tellegen's Theorem** states that the summation of power delivered is zero for each branch of any electrical network at any instant of time. It is mainly applicable for designing the filters in signal processings.
- It is also used in complex operation systems for regulating the stability. It is mostly used in the chemical and biological system and for finding the dynamic behaviour of the physical network.

Tellegen's Theorem

- If there are b branches in a lumped circuit, and the voltage u_k, current i_k of each branch apply passive sign convention, then we have

$$\sum_{k=1}^{b} u_k i_k = 0$$

Inference of Tellegen's Theorem

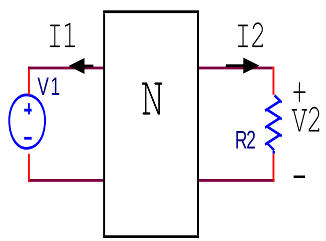
- If two lumped circuits N and $\hat{N}$ have the same topological graph with b branches, and the voltage, current of each branch apply passive sign convention, then we have not only

$$\sum_{k=1}^{b} u_k i_k = 0 \quad \sum_{k=1}^{b} \hat{u}_k \hat{i}_k = 0$$

$$\textit{but also} \quad \sum_{k=1}^{b} \hat{u}_k i_k = 0 \quad \sum_{k=1}^{b} u_k \hat{i}_k = 0$$

Example

N *is a network including resistors only. When* $R_2 = 2\Omega, V_1 = 6V$, *We can get* $I_1 = -2A$, $V_2 = 2V$; *When* $R_2' = 4\Omega, V_1' = 10V$, *We can get* $I_1' = -3A$, *find out* V_2' *then.*



According to the Tellegen Theorem

$$V_1 I_1' + V_2 I_2' + \sum_{k=3}^{b} V_k I_k' = 0\,;\; V_1' I_1 + V_2' I_2 + \sum_{k=3}^{b} V_k' I_k = 0$$

$$\text{and } V_k I_k' = R I_k I_k' = R I_k' I_k = V_k' I_k$$

$$\therefore \quad \sum_{k=3}^{b} V_k I_k' = \sum_{k=3}^{b} V_k' I_k$$

$$\therefore \quad V_1 I_1' + V_2 I_2' = V_1' I_1 + V_2' I_2$$

$$6 \times (-3) + 2 \times \frac{V_2'}{4} = 10 \times (-2) + V_2' \times \frac{2}{2}$$

$$\therefore \quad V_2' = 4V$$

PREVIOUS QUESTION PAPERS

I B. Tech II Semester Regular Examinations, December - 2020
ELECTRICAL CIRCUIT ANALYSIS-I
(Electrical and Electronics Engineering)

Time: 3 hours Max. Marks: 75

Answer any five Questions one Question from Each Unit
All Questions Carry Equal Marks

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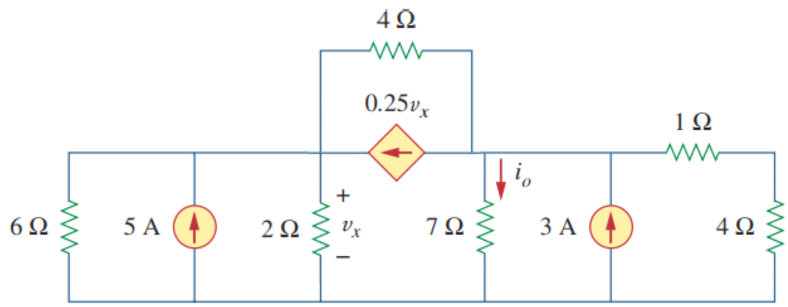
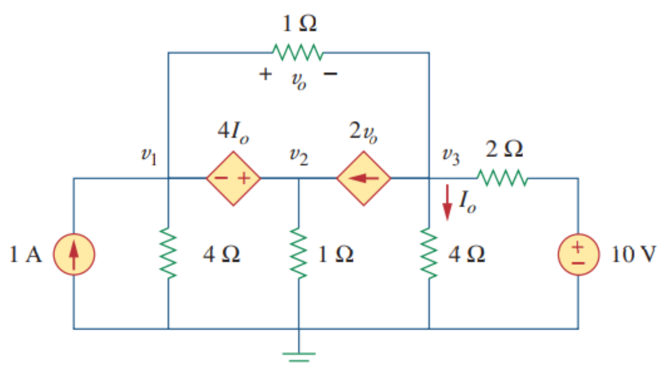
1. a) Find i_0 in the circuit of figure by using source transformation method. (8M)



Figure

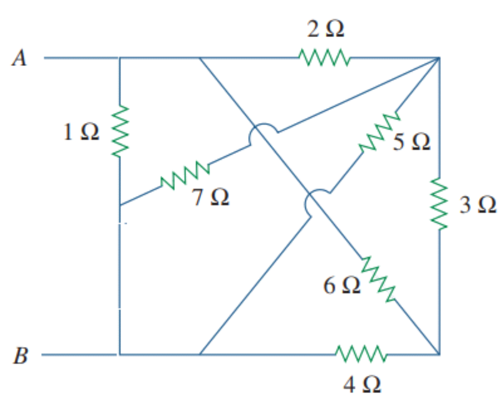
b) By using nodal analysis, find all the node voltages of the circuit of figure. (7M)



Figure

Or

2. a) What is the equivalent resistance between the terminals AB of the figure? (8M)



Figure
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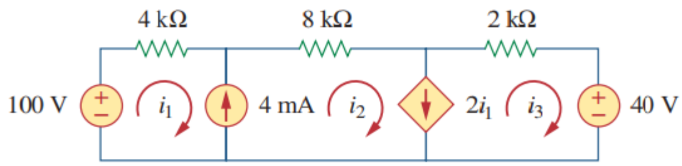
b) Find the mesh currents i_1, i_2, and i_3 in the network of figure. (7M)



Figure

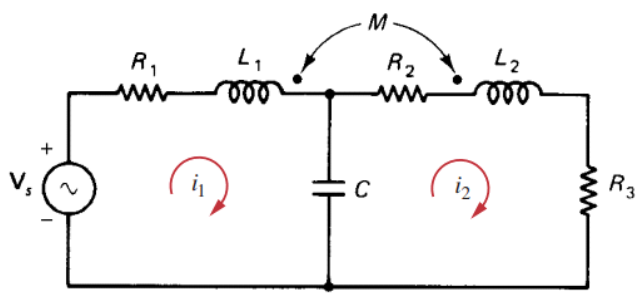
3. a) Two identical coupled coils have an equivalent inductance of 80 mH when connected series aiding and 35 mH in series opposing. Find L_1, L_2, M and K. (7M)

b) A teroidal core made of cast steel has a mean circumference of 1.2 m and cross sectional area of $4x10^{-4}$ m^2. A coil of 600 turns is wound on the core and the current in the coil is 6A. (i) determine the flux in the core (ii) calculate the width of an air gap to be cut radially in the core if it is desired to limit the flux in the core to 0.4 mWb. (8M)

Or

4. a) Explain self and mutual inductance in coupled magnetic circuits. (8M)

b) Calculate the phasor currents i_1 and i_2 in the circuit of figure, assume R_1=10 Ω, R_2=15 Ω, L_1 = 0.1 H, L_2=0.5 H, M=0.12, ω=100 rad/sec, Vs=120 V∠0^0 and $1/j\omega C = -j50$ Ω. (7M)



Figure

5. a) Three circuit elements are connected in series and the voltages across them are given byv_1 = 50 sin ωt; v_2=40 sin (ωt+60^0) and v_3=60 sin (ωt – 30^0). Determine the total voltage across the series combination and its phase angle with respect to v_1. (8M)

b) A capacitor of 80 μF capacitance takes a current of 1 A when supplied with 250 V (rms). Determine (i) the frequency of the supply, and (ii) the resistance which must be connected in series with this capacitor in order to reduce the current to 0.5 A at this frequency. (7M)

Or

6. a) A circuit consists of a 100 Ω resistance in parallel with a 25 μF capacitor connected to a 250V, 50Hz supply. Calculate (i) the current flowing in each branch, (ii) the total current drawn from the supply, (iii) the impedance of the circuit, and (iv) the phase angle of the circuit. (8M)

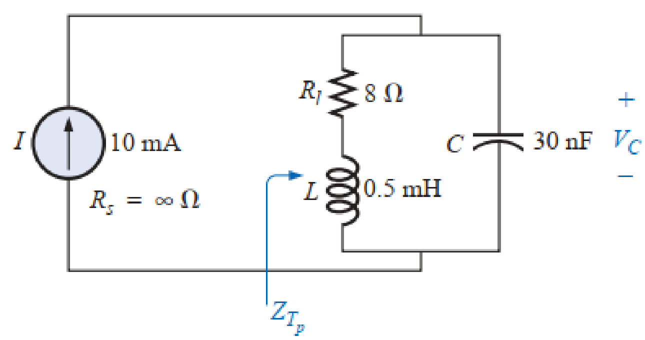
b) Three coils are connected in parallel across a 250V supply. Their impedances are (10+j30) Ω, (20+j0) Ω and (1–j20) Ω. Determine (i) the current drawn from the supply and (ii) the power factor of the circuit. (7M)

7. a) A coil of inductance L and resistance R in series with a capacitor is supplied at a constant voltage from a variable frequency source. If the frequency is ω_r, find in terms of L, R and ω_r the values of those frequencies at which the circuit current would be half as much as that at resonance and determine the bandwidth and selectivity of the circuit. (8M)

b) The impedance $Z_1 = (5 + j3)\ \Omega$ and $Z_2 = (10 - j30)\ \Omega$ are connected in parallel. This parallel branch is connected in series with impedance of $(R_3 - jX_3)\ \Omega$. Find the value of X_3 which will produce resonance. (7M)

Or

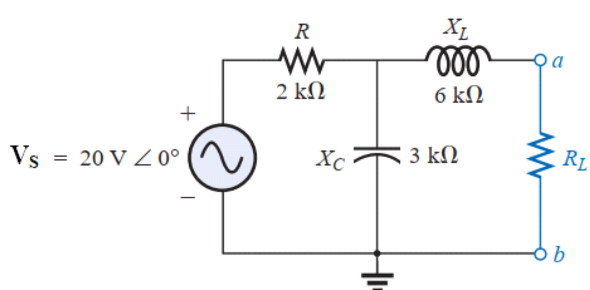
8. a) For the parallel resonant network of figure (i) Determine the resonant frequency f_p. (ii) Find the total impedance at resonance. (iii) Calculate the quality factor, bandwidth, and cutoff frequencies f_1 and f_2 of the system. (8M)



Figure

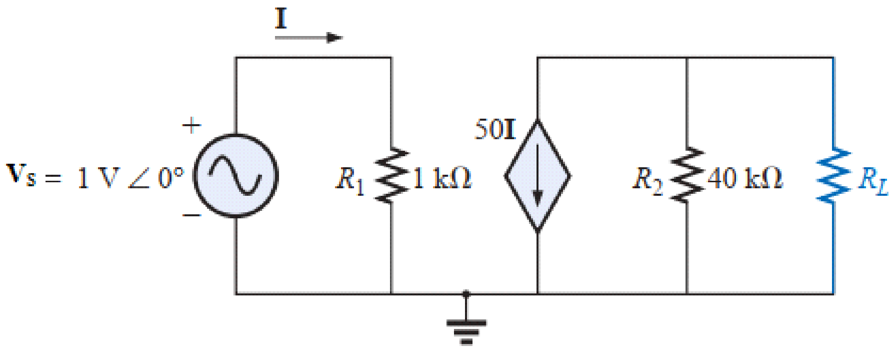
b) Given a series RLC circuit resonant at 250 kHz with $R = 10^4\ \Omega$ and L = 200 mH. (i) Calculate the value of C, (ii) band width (iii) half power frequencies (iv) the frequency at which the impedance has an angle of $+30^0$. (7M)

9. a) Find the Norton's equivalent circuit for the network external to the elements between *a* and *b* for the network of figure. (8M)



Figure

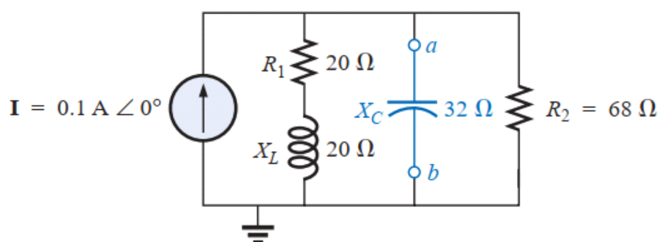
b) Find the load impedance R_L for the network of figure for maximum power to the load, and find the maximum power to the load. (7M)



Figure

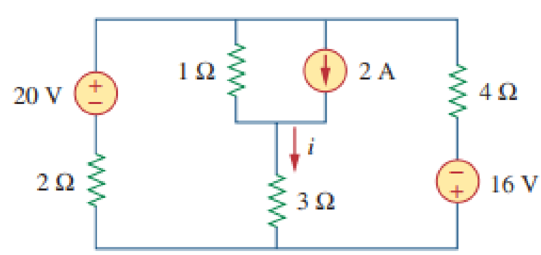
Or

10. a) Find the Thevenin's equivalent circuit for the portions of the networks of figure. external to the elements between points *a* and *b* (8M)



Figure

b) For the circuit in figure use super position theorem to find i, Calculate power delivered to the 3 Ω resistor. (7M)



Figure

R19

SET - 2

I B. Tech II Semester Regular Examinations, December - 2020
ELECTRICAL CIRCUIT ANALYSIS-I
(Electrical and Electronics Engineering)

Time: 3 hours Max. Marks: 75

Answer any five Questions one Question from Each Unit
All Questions Carry Equal Marks

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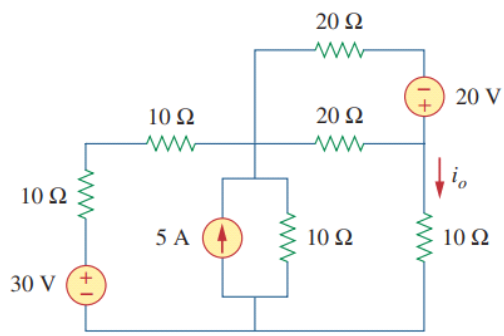
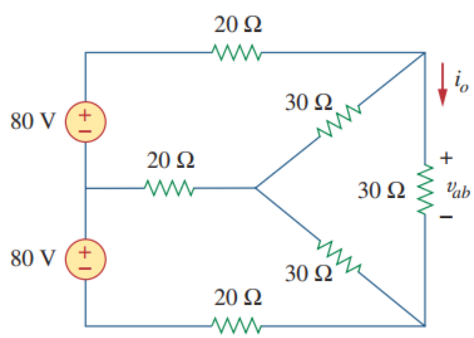
1. a) Find i_0 in the circuit of figure by using source transformation method. (8M)



Figure

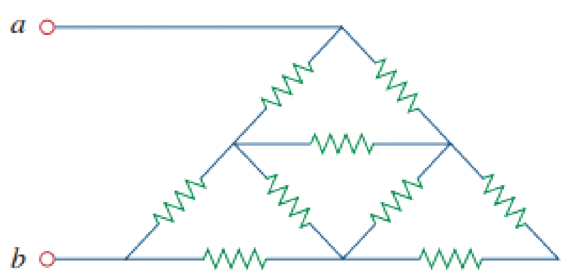
b) By using mesh analysis find i_0 and v_{ab} in the circuit of figure. (7M)



Figure

Or

2. a) Determine equivalent resistance R_{ab} in the circuit shown in figure all resistors have a value of 30 Ω. (8M)



Figure
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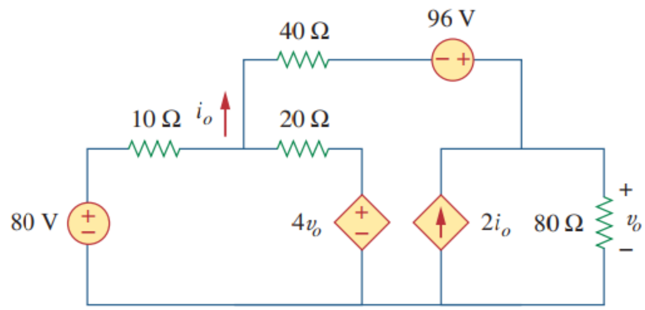
b) By using nodal analysis, find v_0 and i_0 in the circuit of figure. (7M)



Figure

3. a) Two coupled coils have these data: $L_1 = 0.1$ H, $L_2=0.4$ H, $M=0.15$ H (coupling additive) $\omega=100$ rad/sec, when $V_1=1000\ V\angle 0^0$, $I_1=50\ \angle 36.9^0$ A. Calculate V_2. (7M)

b) A magnetic circuit consists of an iron ring of mean circumference 80 cm with cross-sectional area of 12 cm^2 throughout. A current of 2A in the magnetizing coil of 200 turns produce a total flux of 1.2 mWb in the iron. Calculate: (i) the flux density in the iron (ii) the absolute and relative permeability of iron. (iii) the reluctance of the circuit. (8M)

Or

4. a) Derive the expression for equivalent inductance of a magnetically coupled two coils connected in series, when two coils are (i) aiding each other (ii) Opposing each other. (8M)

b) Two coils have a mutual inductance of 0.5 H. if the current in one coil is varied from 4A to 2A in 0.4 sec, calculate (i) The average e.m.f. induced in the second coil (ii) The rate of change of flux linked with the second coil assuming that it is wound with 400 turns. (7M)

5. a) A coil of inductance 15.9 mH and resistance 9 Ω is connected in parallel with a coil of inductance 38.2 mH and resistance 6 Ω across a 250 V, 50 Hz supply. Determine (i) the conductance, susceptance and admittance of the circuit, (ii) the current drawn from the supply and (iii) the total power consumed in kW. (8M)

b) Two coils A and B are connected in series across a 250 V, 50 Hz supply. The resistance of coil A is 10 Ω and inductance of coil B is 0.015 H. If input from the supply is 3 kW and 2 kVAR, find the resistance of coil B and inductance of coil A. Also calculate voltage across each coil. (7M)

Or

6. a) A coil of inductance 0.08 H takes a current of 5 A when connected in series with a 50 μF loss-free capacitor across a 250 V, 50 Hz supply. Calculate (i) resistance of the coil (ii) power factor of the coil (iii) the overall power factor. Sketch the phasor diagram. (8M)

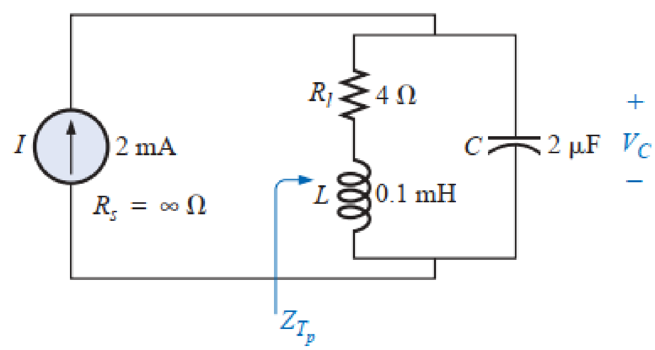
b) Three impedances (2+j4) Ω, (3–j5) Ω and (1–j3)Ω are connected in parallel. The combination is in series with a coil of resistance 3 Ω and inductance 0.02H to 250V, 50Hz supply. Find (i) The complex expression for the total impedance of the circuit. (ii) Current taken from the supply. (7M)

7. a) Show that resonant frequency ω_r of RLC series circuit is geometric mean of lower and upper half-frequencies ω_1 and ω_2. (8M)

b) A series circuit consists of a 40Ω resistor, a 0.5 H inductor and a variable capacitor connected across a 100 V, 50 Hz supply. Calculate (i) the value of the capacitance required to give resonance, (ii) the voltages across the resistor, the inductor and the capacitor at resonance, and (iii) the Q-factor of the circuit. (7M)

Or

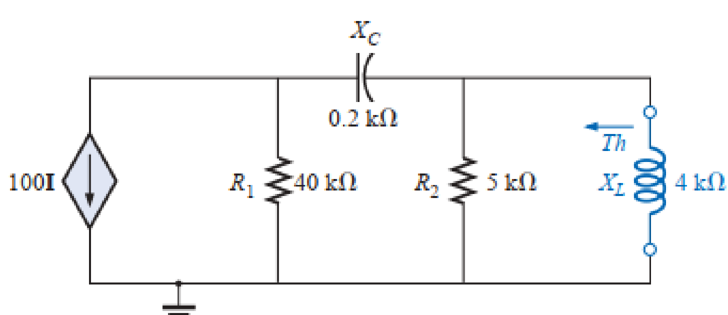
8. a) For the parallel resonant network of figure. (i) Determine the resonant frequency, f_p. (ii) Find the total impedance at resonance. (iii) Calculate the quality factor, bandwidth, and cutoff frequencies f_1 and f_2 of the system. (8M)



Figure

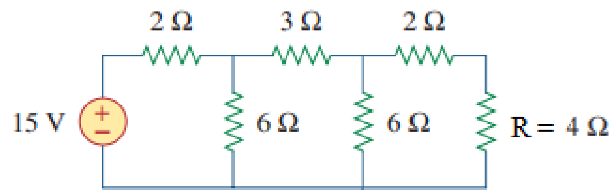
b) A series RC circuit having variable R and C= 15 μF is supplied from AC source having voltage V=250 V at ω=1000 rad/sec. Draw current locus for sample values of R=0,5, 15, 25, 35, 50Ω. (7M)

9. a) Determine the Thevenin's equivalent circuit for the network external to the 4-kΩ inductive reactance of figure (in terms of I). (8M)



Figure

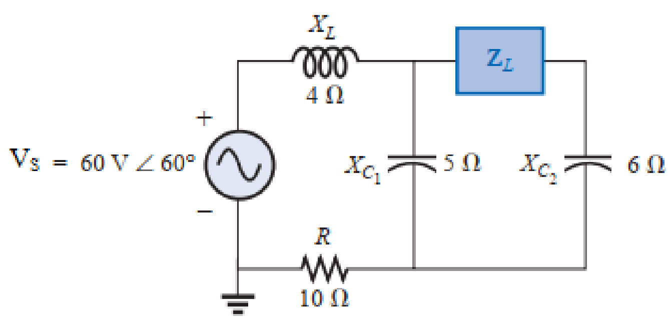
b) Verify the compensation theorem of the circuit in figure, when resistance, R is changed from 4 Ω to 2 Ω. (7M)



Figure

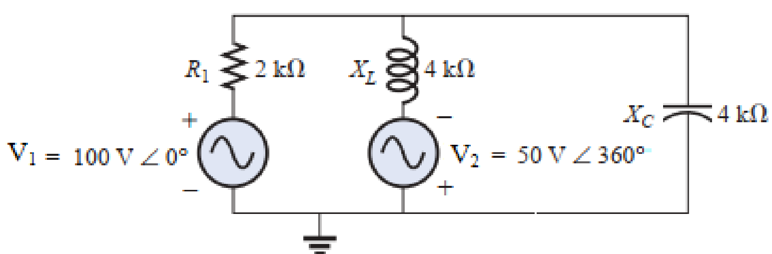
Or

10. a) Find the load impedance Z_L for the networks of figure for maximum power to the load, and find the maximum power to the load. (8M)



Figure

b) Using Millman's theorem, determine the current through the 4-kΩ capacitive reactance of figure. (7M)



Figure

I B. Tech II Semester Regular Examinations, December - 2020
ELECTRICAL CIRCUIT ANALYSIS-I
(Electrical and Electronics Engineering)

Time: 3 hours Max. Marks: 75

Answer any five Questions one Question from Each Unit
All Questions Carry Equal Marks

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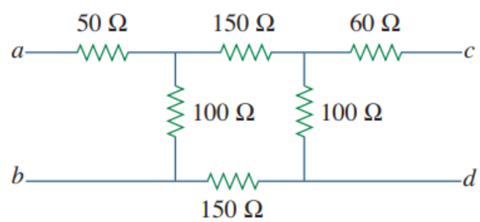
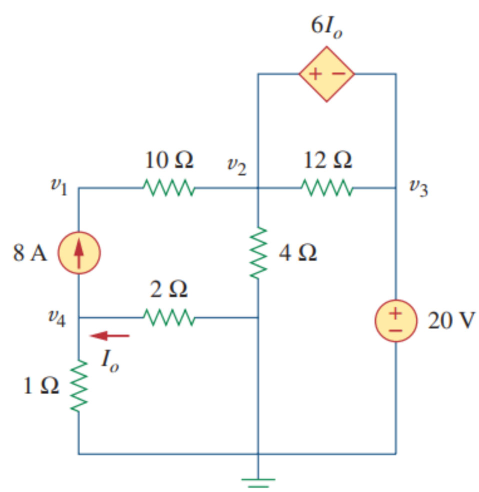
1. a) Consider the circuit in figure, find the equivalent resistance across the terminals: (i) a-b and (ii) c-d. (8M)



Figure

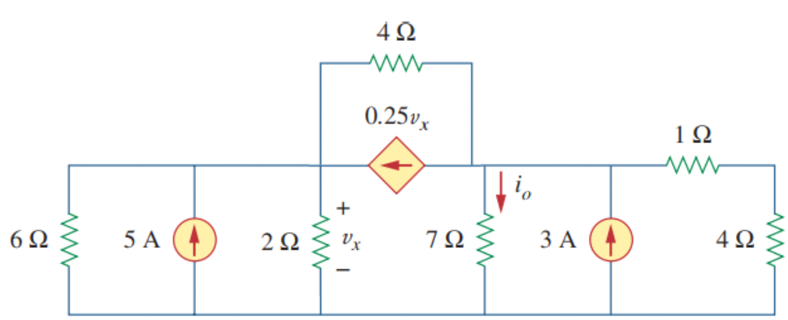
b) By using node analysis, find all the node voltages of the circuit of figure. (7M)



Figure

Or

2. a) Find i_0 in the circuit of figure by using source transformation method. (8M)



Figure
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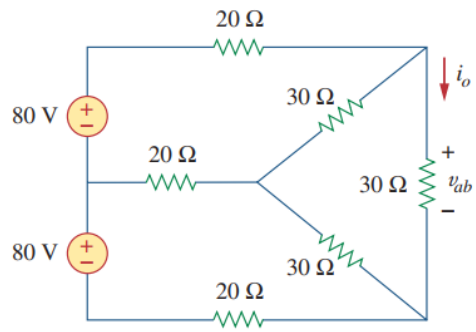
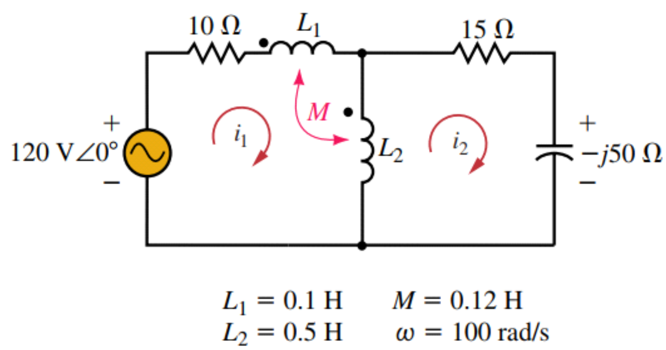
b) By using mesh analysis find i_0 and v_{ab} in the circuit of figure. (7M)



Figure

3\. a) The total inductance of two coils is measured to be 13 mH. If one of the coil is reversed total inductance is found to be 8 mH. If inductance of one coil is known to be 5 mH, calculate inductance of the other coil, the mutual inductance, and the coefficient of coupling between the two coils. (8M)

b) Calculate the phasor currents i_1 and i_2 in the circuit of figure. (7M)



Figure

Or

4\. a) Derive the expression for equivalent inductance of a magnetically coupled two coils connected in parallel, when the two coils are (i) Aiding each other (ii) Opposing each other. (8M)

b) Two coupled coils have self-inductances L_1=15mH and L_2=25mH.The coefficient of coupling (K) being 0.8 in the air, find voltage in the second coil and the flux of first coil provided the second coil has 500 turns and the circuit current is given by i_1= 10 sin 314t A. (7M)

5\. a) A resistance of 50 Ω is connected in series with a variable capacitor across a 200 V, 50 Hz supply. (8M)
(a) When the capacitance is set to 50 μF calculate (i) the current drawn from the supply, (ii) the voltage across the two elements and (iii) the power factor.
(b) Find the value of the capacitance when the current is 2 A.
(c) Determine the value of the capacitance required to give a power factor of 0.866 leading.

b) A resistance R, L=0.02H and capacitance C are connected in series. When a voltage V=200 cos(2000t–20^0) V is applied to the series combination, the current flowing is 5√2 cos(2000t – 65^0) A, find R & C. (7M)

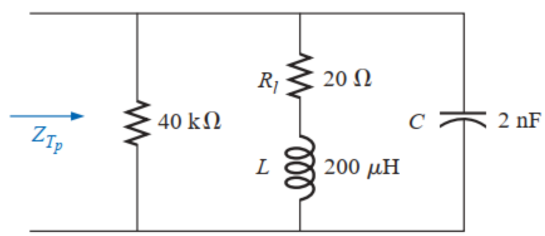
Or

6. a) A series circuit consisting of a 10 Ω resistor, 100mH inductance is driven by a 50 Hz. AC voltage source of maximum value 100V. Calculate the equivalent impedance, rms value of the voltage, form factor of the voltage, current in the circuit, the power factor and power dissipated in the circuit. (5M)

   b) A capacitor having a reactance of 5 Ω is connected in series with a resistor of 10 Ω. This circuit is then connected (a) in series and (b) in parallel with a coil of impedance (5+j7) Ω. Calculate for each case (i) the current drawn from the supply, (ii) the power supplied, and (iii) the power factor of the whole circuit. (10M)

7. a) A coil of inductance 80 mH and negligible resistance is connected in series with a capacitance of 0.25 µF and a resistor of resistance 12.5Ω across a 100V, variable frequency supply. Determine (a) the resonant frequency, and (b) the current at resonance. How many times greater than the supply voltage is the voltage across the reactances at resonance? (8M)

   b) For the parallel resonant circuit of figure, (i) Determine the resonant frequency, (ii)Find the total impedance at resonance, (iii) Find quality factor, and (iv) Calculate the band width. (7M)



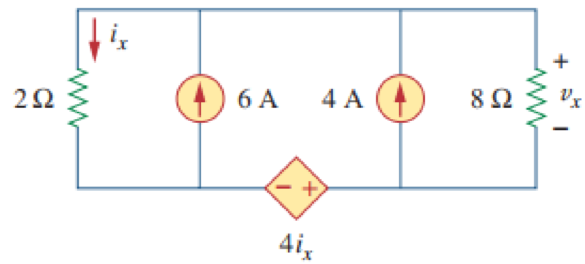
Figure

Or

8. a) A circuit consisting of a coil of inductance 250 mH, having a resistance of 20 Ω, in parallel with a variable capacitor C is connected to a 200 V, 50 Hz supply. Determine (i) the value of C required for the circuit to resonate, (ii) the power absorbed at resonance, and (iii) the ratio of the current through the capacitor to the supply current at resonance. (8M)

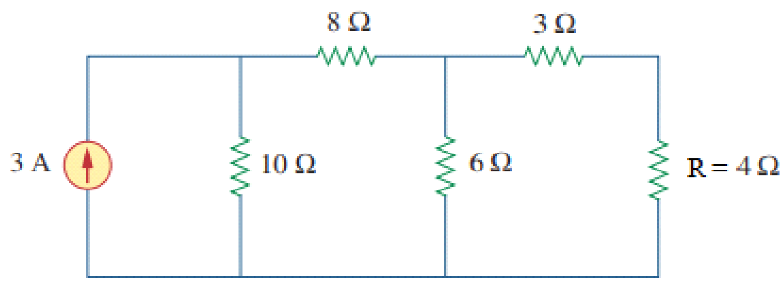
   b) A series circuit consisting of an inductance of 0.3 H, having a resistance of 10 Ω, and a variable capacitor C_1 is supplied from a 100 V, variable frequency source. (i) Determine the value of C_1 necessary for the circuit to operate resonantly at 50 Hz. (ii) A second variable capacitor, C_2, is now connected in parallel with the original circuit and the supply frequency is adjusted to 60 Hz. Determine the value of C_2 in order that the circuit still operates with minimum current. (7M)

9. a) Use superposition theorem to solve for v_x in the circuit of figure. (8M)



Figure

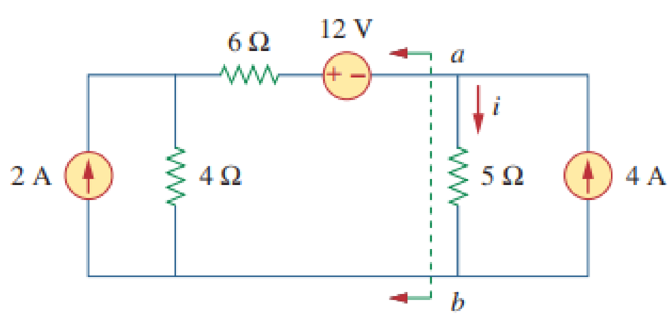
b) Verify the compensation theorem of the circuit in figure, when resistance, R is changed from 4 Ω to 2 Ω. (7M)



Figure

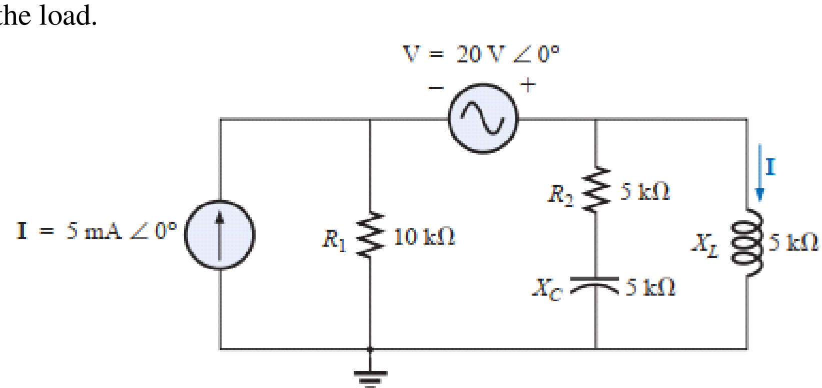
Or

10. a) Obtain Norton's equivalent of the circuit in figure to the left of terminals *a-b*. Use the result to find *i*. (8M)



Figure

b) Determine the load impedance to replace the inductor, X_L of figure to ensure maximum power to the load. Using the results, determine the maximum power to the load. (7M)



Figure

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I B. Tech II Semester Regular Examinations, December - 2020
ELECTRICAL CIRCUIT ANALYSIS-I
(Electrical and Electronics Engineering)

Time: 3 hours Max. Marks: 75

Answer any five Questions one Question from Each Unit
All Questions Carry Equal Marks

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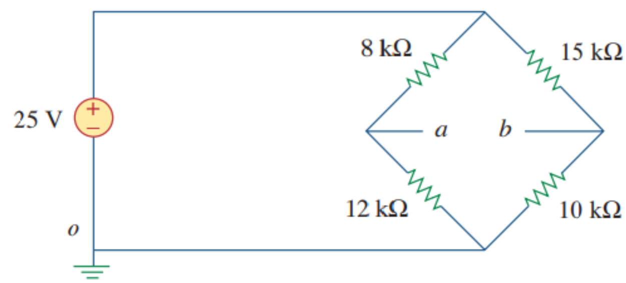
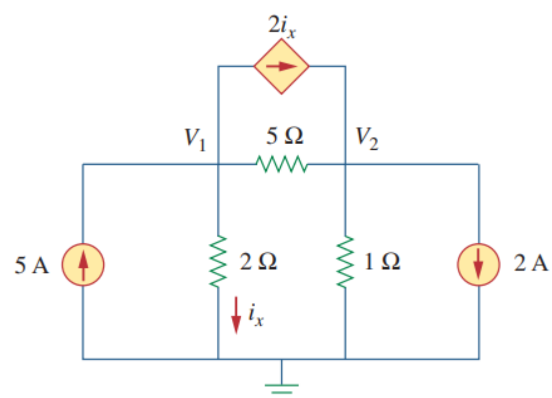
1. a) (i) Consider the circuit shown in figure, calculate v_a, v_b and v_{ab}. (10M)



Figure

(ii) If the ground is placed at ‘a’ instead of ‘o’ again calculate v_a, v_b and v_{ab}.

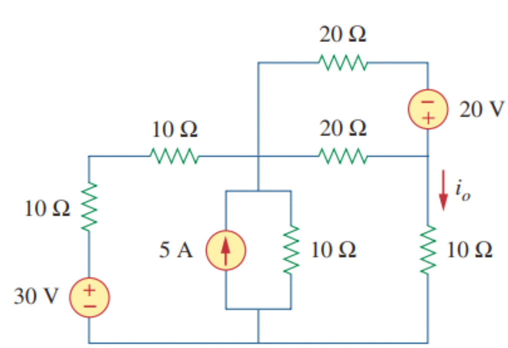
b) Solve for V_1 and V_2 in the circuit of figure. (5M)



Figure

Or

2. a) Find i_0 in the circuit of figure by using source transformation method. (8M)



Figure
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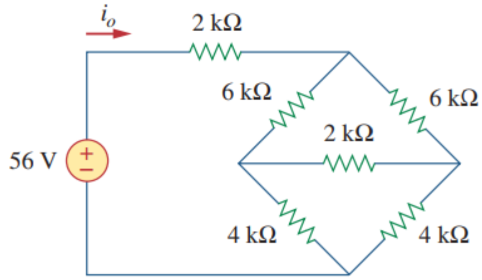
b) For the bridge network in figure, find i_0 using mesh analysis. (7M)



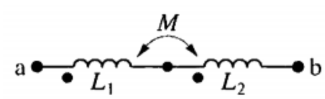
Figure

3. a) Define coefficient of coupling and derive its expression. (7M)

b) A ring has a mean diameter of 21 cm and cross sectional area of 10 cm^2. The ring is made up of semi-circular sections of cast iron and cast steel with each joint having reluctance equal to an air gap of 0.2 mm. Find the ampere turns required to produce a flux of 0.8 mWb. The relative permeability of cast steel and cast iron are 800 & 166 respectively. Neglect fringing and leakage effects. (8M)

Or

4. a) Show that the two coupled coils in figure can be replaced by a single coil having an inductance of $L_{ab} = L_1 + L_2 + 2M$. (8M)



Figure

b) Two magnetically coupled coils have self-inductances of 60 mH and 9.6 mH, respectively. The mutual inductance between the coils is 22.8 mH. (7M)

(i) What is the coefficient of coupling? (ii) For these two coils, what is the largest value that Mutual inductance can have?

5. a) A circuit is supplied from 50Hz mains whose voltage has a maximum value of 250V and takes a current whose maximum value is 5A. At a particular instant (t = 0) the voltage has a value of 200V and the current is then 2A. Obtain expressions for the instantaneous values of voltage and current as functions of time and determine their values at an instant t = 0.015 s. Determine also the phase difference between them. (7M)

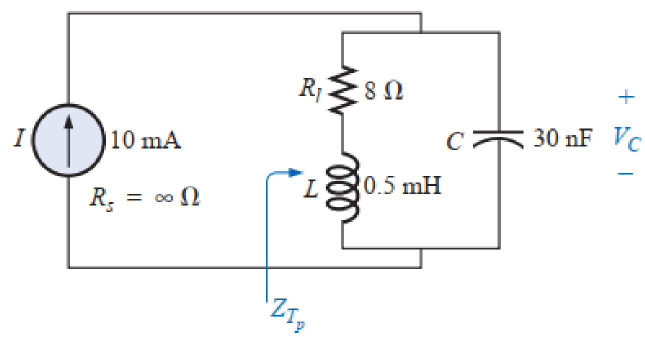
b) A coil of inductance 15.9 mH and resistance 9 Ω is connected in parallel with a coil of inductance 38.2 mH and resistance 6 Ω across a 250 V, 50 Hz supply. Determine (i) the conductance, susceptance and admittance of the circuit, (ii) the current drawn from the supply and (iii) the total power consumed in kW. (8M)

Or

6. a) A coil is connected in series with a 20 μF capacitor across 250 V, 50 Hz supply. The current drawn by the circuit is 8A and power consumed is 200W. Calculate inductance of the coil if power factor is (i) lagging and (ii) leading. (8M)

b) Two coils A and B are connected in series across a 250 V, 50 Hz supply. The resistance of coil A is 10 Ω and inductance of coil B is 0.015 H. If input from the supply is 3 kW and 2 kVAR, find the resistance of coil B and inductance of coil A. Also calculate voltage across each coil. (7M)

7. a) For the parallel resonant network of figure. (i) Determine the resonant frequency, f_p. (ii) Find the total impedance at resonance. (iii) Calculate the quality factor, bandwidth, and cutoff frequencies f_1 and f_2 of the system. (8M)



Figure

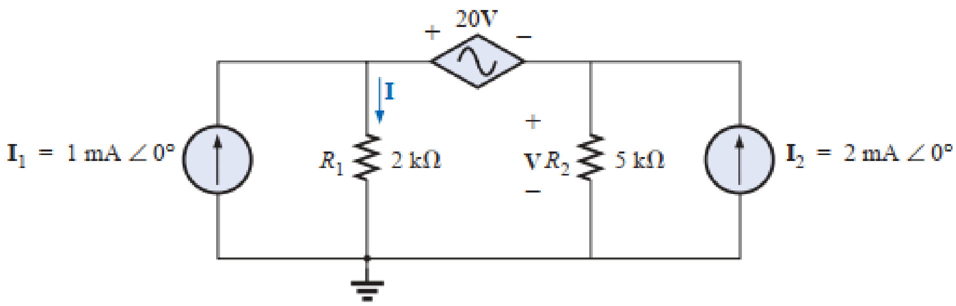
b) A series circuit consisting of an inductance of 0.3 H, having a resistance of 10 Ω, and a variable capacitor C_1 is supplied from a 100 V, variable frequency source. (i) Determine the value of C_1 necessary for the circuit to operate resonantly at 50 Hz. (ii) A second variable capacitor, C_2, is now connected in parallel with the original circuit and the supply frequency is adjusted to 60 Hz. Determine the value of C_2 in order that the circuit still operates with minimum current. (7M)

Or

8. a) A coil having a Q-factor of 100 is connected in parallel with a capacitor of 100 pF. The circuit resonates at a frequency of 5 MHz. Determine (i) the bandwidth of the circuit, (ii) the amount of resistance required to be placed in parallel with the capacitor in order to increase the bandwidth to 250 kHz, and (iii) the amount of resistance required to be placed in series with the inductor in order to produce the same bandwidth. (8M)

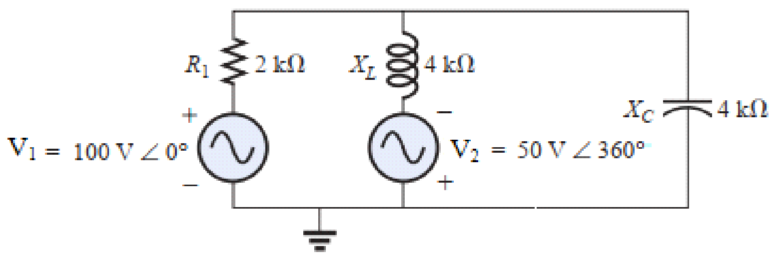
b) A resistor of 90 Ω resistance is connected in series with a coil of inductance 500mH, having a resistance of 10 Ω. This series circuit is connected in parallel with a 20 µF capacitance across a 250 V variable frequency supply. Determine (i) the resonant frequency of the circuit, (ii) the resonant frequency if the 90 Ω resistor is short circuited and (iii) the current drawn from the supply in each case. (7M)

9. a) Calculate the current, I for the network of figure by using superposition theorem. (8M)



Figure

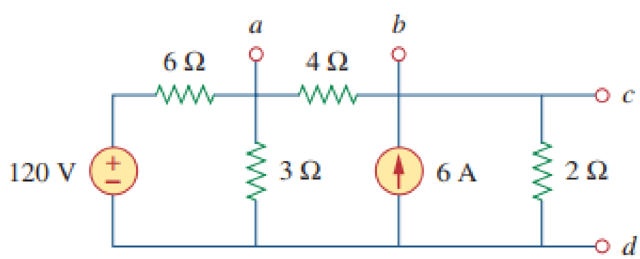
b) Using Millman's theorem, determine the current through the 4-kΩ capacitive reactance of figure. (7M)



Figure

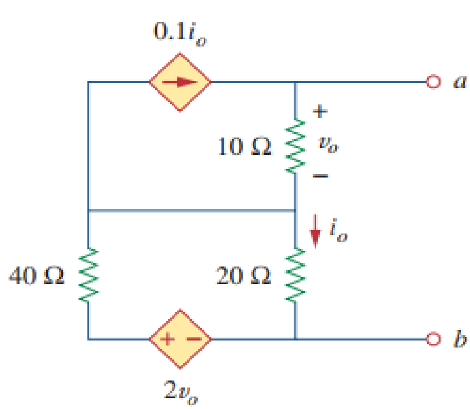
Or

10. a) Given the circuit in figure, Obtain Norton's equivalent circuit as viewed from (i) *a-b* (ii) *c-d*. (10M)



Figure

b) Find the Thevenin's equivalent circuit shown in figure. (5M)



Figure

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Code No: R201209 **R20** SET - 1

I B. Tech II Semester Regular Examinations, September- 2021

ELECTRICAL CIRCUIT ANALYSIS –I

(Only for EEE)

Time: 3 hours Max. Marks: 70

Answer any five Questions one Question from Each Unit
All Questions Carry Equal Marks

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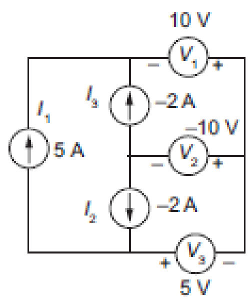
UNIT-I

1. a) Explain the following dependent sources: (7M)
   i) Voltage controlled voltage source ii) Voltage controlled current source
   iii) Current controlled current source iv) Current controlled voltage source

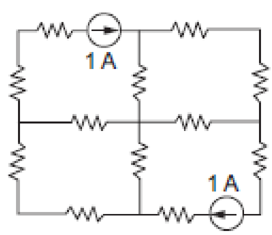
   b) Find the power delivered by all the sources in the following circuit: (7M)



Or

2. a) Three equal resistors are connected across a voltage source in series first and in parallel later. Find the ratio of power delivered by the source in the two cases. (7M)

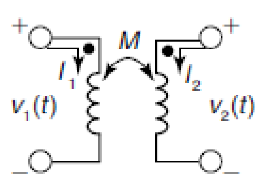
   b) All resistors in the circuit are of 4Ω. Find currents in all resistors and voltage across current sources by mesh analysis. (7M)



UNIT-II

3. a) Explain the following terms with respect to magnetic circuits: (7M)
   i) Self-inductance ii) Mutual inductance
   ii) Series and parallel magnetic circuits

   b) For the circuit shown below, if L_1 =0.4 H, L_2= 2.5 H, k =0.6, and $i_1 = 4i_2$= 20cos $(500t - 20^0)$ mA. (7M)



Evaluate the following quantities at t = 0:
(i) i_2,
(ii) V_1, and
(iii) the total energy stored in the system.
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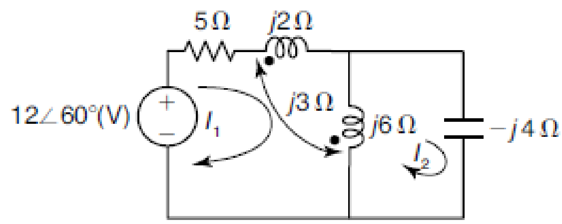
Or

4. a) Prove that when two coils of self-inductances L_1 and L_2 are connected in series aiding connection with a mutual inductance M then the total inductance is equal to $L_{eqv} = (L_1 + L_2 + 2M)$. (7M)

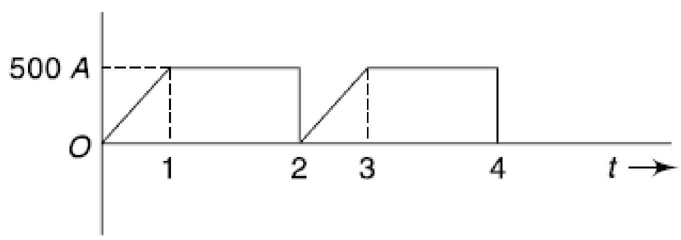
   b) For the circuit shown below, determine the phasor currents I_1 and I_2 (7M)



UNIT-III

5. a) A 200 V, 50 Hz. inductive circuit takes a current of 15 A, lagging the voltage by 45°. Calculate the resistance and inductance of the circuit. (7M)

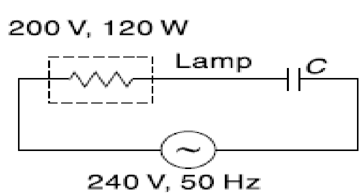
   b) Find the average and rms value for the following waveform: (7M)



Or

6. a) Prove that the active power over a complete cycle of current in a purely capacitive circuit is zero. (7M)

   b) A 200 V, 120 W lamp is to be operated on 240 V, 50 Hz. supply. Calculate the value of the capacitor that would be placed in series with the lamp in order that it may be used at its rated voltage. (7M)



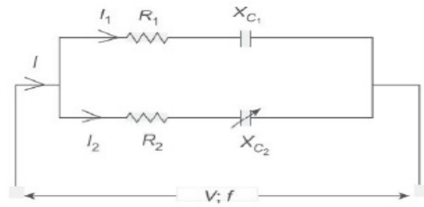
UNIT-IV

7. a) Explain the effect of band width and selectivity in series resonance circuit. (7M)

   b) A circuit consists of a coil of resistance 100 Ω and inductance 1 H in series with a capacitor of capacitance 1 µF. Calculate (i) the resonant frequency, (ii) current at resonant frequency and (iii) voltage across each element when the supply voltage is 50 V. (7M)

Or

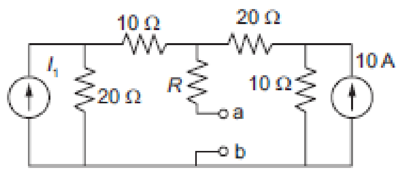
8. a) Draw the locus of I_2 and I for the parallel circuit shown below with neat step by step explanation: (7M)



b) A coil of resistance 5 Ω and inductance 0.1 H is connected in parallel with a circuit containing a coil of resistance 4 Ω and inductance 0.05 H in series with a capacitor C and a pure resistor R. Calculate the values of C and R so that currents in either branch are equal but differ in phase by 90°. (7M)

UNIT-V

9. A resistor of 20 Ω connected across a – b for the circuit shown below, draws maximum power from the circuit and the power drawn is 100W. i) Find the value of R and I_1. ii) With 20 Ω across a- b find the value of I_1 such that power transferred to it is 0 W. (14M)



Or

10. a) State and explain Thevenin's theorem. (7M)

b) Find the power dissipated in the resistor R_2 for the circuit shown below by applying superposition theorem (7M)